CHAPTER 13

MAGNETICALLY COUPLED CIRCUITS

People want success but keep running away from problems, and yet it is only in tackling problems that success is achieved.

— Josiah J. Bonire

Enhancing Your Career

Career in Electromagnetics Electromagnetics is the branch of electrical engineering (or physics) that deals with the analysis and application of electric and magnetic fields. In electromagnetics, electric circuit analysis is applied at low frequencies.

The principles of electromagnetics (EM) are applied in various allied disciplines, such as electric machines, electromechanical energy conversion, radar meteorology, remote sensing, satellite communications, bioelectromagnetics, electromagnetic interference and compatibility, plasmas, and fiber optics. EM devices include electric motors and generators, transformers, electromagnets, magnetic levitation, antennas, radars, microwave ovens, microwave dishes, superconductors, and electrocardiograms. The design of these devices requires a thorough knowledge of the laws and principles of EM.

EM is regarded as one of the more difficult disciplines in electrical engineering. One reason is that EM phenomena are rather abstract. But if one enjoys working with mathematics and can visualize the invisible, one should consider being a specialist in EM, since few electrical engineers specialize in this area. Electrical engineers who specialize in EM are needed in microwave industries, radio/TV broadcasting stations, electromagnetic research laboratories, and several communications industries.

Telemetry receiving station for space satellites. Source: T. J. Maloney, Modern Industrial Electronics, 3rd ed. Englewood Cliffs, NJ: Prentice Hall, 1996, p. 718.

13.1 INTRODUCTION

The circuits we have considered so far may be regarded as *conductively coupled*, because one loop affects the neighboring loop through current conduction. When two loops with or without contacts between them affect each other through the magnetic field generated by one of them, they are said to be *magnetically coupled*.

The transformer is an electrical device designed on the basis of the concept of magnetic coupling. It uses magnetically coupled coils to transfer energy from one circuit to another. Transformers are key circuit elements. They are used in power systems for stepping up or stepping down ac voltages or currents. They are used in electronic circuits such as radio and television receivers for such purposes as impedance matching, isolating one part of a circuit from another, and again for stepping up or down ac voltages and currents.

We will begin with the concept of mutual inductance and introduce the dot convention used for determining the voltage polarities of inductively coupled components. Based on the notion of mutual inductance, we then introduce the circuit element known as the *transformer*. We will consider the linear transformer, the ideal transformer, the ideal autotransformer, and the three-phase transformer. Finally, among their important applications, we look at transformers as isolating and matching devices and their use in power distribution.

13.2 MUTUAL INDUCTANCE

When two inductors (or coils) are in a close proximity to each other, the magnetic flux caused by current in one coil links with the other coil, thereby inducing voltage in the latter. This phenomenon is known as *mutual inductance*.

Let us first consider a single inductor, a coil with N turns. When current i flows through the coil, a magnetic flux ϕ is produced around it (Fig. 13.1). According to Faraday's law, the voltage v induced in the coil is proportional to the number of turns N and the time rate of change of the magnetic flux ϕ; that is,

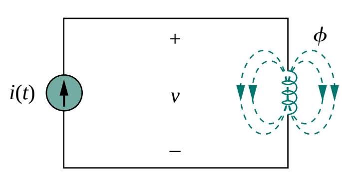


Figure 13.1 Magnetic flux produced by a single coil with N turns.

$$v = N\frac{d\phi}{dt} \tag{13.1}$$

But the flux ϕ is produced by current i so that any change in ϕ is caused by a change in the current. Hence, Eq. (13.1) can be written as

$$v = N\frac{d\phi}{di}\frac{di}{dt} \tag{13.2}$$

or

$$v = L\frac{di}{dt} \tag{13.3}$$

which is the voltage-current relationship for the inductor. From Eqs. (13.2) and (13.3), the inductance L of the inductor is thus given by

$$L = N\frac{d\phi}{di} \tag{13.4}$$

This inductance is commonly called *self-inductance*, because it relates the voltage induced in a coil by a time-varying current in the same coil.

Now consider two coils with self-inductances L_1 and L_2 that are in close proximity with each other (Fig. 13.2). Coil 1 has N_1 turns, while coil 2 has N_2 turns. For the sake of simplicity, assume that the second inductor carries no current. The magnetic flux ϕ_1 emanating from coil 1 has two components: one component ϕ_{11} links only coil 1, and another component ϕ_{12} links both coils. Hence,

$$\phi_1 = \phi_{11} + \phi_{12} \tag{13.5}$$

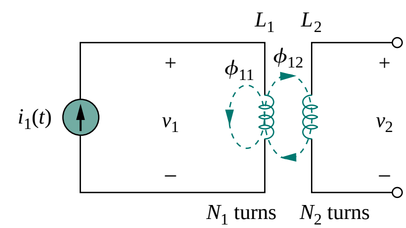


Figure 13.2 Mutual inductance M_{21} of coil 2 with respect to coil 1.

Although the two coils are physically separated, they are said to be *magnetically coupled*. Since the entire flux ϕ_1 links coil 1, the voltage induced in coil 1 is

$$v_1 = N_1 \frac{d\phi_1}{dt} \tag{13.6}$$

Only flux ϕ_{12} links coil 2, so the voltage induced in coil 2 is

$$v_2 = N_2 \frac{d\phi_{12}}{dt} \tag{13.7}$$

Again, as the fluxes are caused by the current i_1 flowing in coil 1, Eq. (13.6) can be written as

$$v_1 = N_1 \frac{d\phi_1}{di_1}\frac{di_1}{dt} = L_1 \frac{di_1}{dt} \tag{13.8}$$

where $L_1 = N_1\, d\phi_1/di_1$ is the self-inductance of coil 1. Similarly, Eq. (13.7) can be written as

$$v_2 = N_2 \frac{d\phi_{12}}{di_1}\frac{di_1}{dt} = M_{21} \frac{di_1}{dt} \tag{13.9}$$

where

$$M_{21} = N_2 \frac{d\phi_{12}}{di_1} \tag{13.10}$$

M_{21} is known as the *mutual inductance* of coil 2 with respect to coil 1. Subscript 21 indicates that the inductance M_{21} relates the voltage induced in coil 2 to the current in coil 1. Thus, the open-circuit *mutual voltage* (or induced voltage) across coil 2 is

$$\boxed{v_2 = M_{21} \frac{di_1}{dt}} \tag{13.11}$$

Suppose we now let current i_2 flow in coil 2, while coil 1 carries no current (Fig. 13.3). The magnetic flux ϕ_2 emanating from coil 2 comprises flux ϕ_{22} that links only coil 2 and flux ϕ_{21} that links both coils. Hence,

$$\phi_2 = \phi_{21} + \phi_{22} \tag{13.12}$$

The entire flux ϕ_2 links coil 2, so the voltage induced in coil 2 is

$$v_2 = N_2 \frac{d\phi_2}{dt} = N_2 \frac{d\phi_2}{di_2}\frac{di_2}{dt} = L_2 \frac{di_2}{dt} \tag{13.13}$$

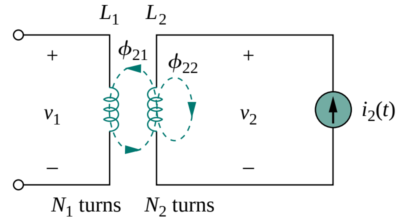


Figure 13.3 Mutual inductance M_{12} of coil 1 with respect to coil 2.

where $L_2 = N_2\, d\phi_2/di_2$ is the self-inductance of coil 2. Since only flux ϕ_{21} links coil 1, the voltage induced in coil 1 is

$$v_1 = N_1 \frac{d\phi_{21}}{dt} = N_1 \frac{d\phi_{21}}{di_2}\frac{di_2}{dt} = M_{12}\frac{di_2}{dt} \tag{13.14}$$

where

$$M_{12} = N_1 \frac{d\phi_{21}}{di_2} \tag{13.15}$$

which is the *mutual inductance* of coil 1 with respect to coil 2. Thus, the open-circuit *mutual voltage* across coil 1 is

$$\boxed{v_1 = M_{12}\frac{di_2}{dt}} \tag{13.16}$$

We will see in the next section that M_{12} and M_{21} are equal, that is,

$$M_{12} = M_{21} = M \tag{13.17}$$

and we refer to M as the mutual inductance between the two coils. Like self-inductance L, mutual inductance M is measured in henrys (H). Keep in mind that mutual coupling only exists when the inductors or coils are in close proximity, and the circuits are driven by time-varying sources. We recall that inductors act like short circuits to dc.

From the two cases in Figs. 13.2 and 13.3, we conclude that mutual inductance results if a voltage is induced by a time-varying current in another circuit. It is the property of an inductor to produce a voltage in reaction to a time-varying current in another inductor near it. Thus,

> **Mutual inductance** is the ability of one inductor to induce a voltage across a neighboring inductor, measured in henrys (H).

Although mutual inductance M is always a positive quantity, the mutual voltage $M\, di/dt$ may be negative or positive, just like the self-induced voltage $L\, di/dt$. However, unlike the self-induced $L\, di/dt$, whose polarity is determined by the reference direction of the current and the reference polarity of the voltage (according to the passive sign convention), the polarity of mutual voltage $M\, di/dt$ is not easy to determine, because four terminals are involved. The choice of the correct polarity for $M\, di/dt$ is made by examining the orientation or particular way in which both coils are physically wound and applying Lenz's law in conjunction with the right-hand rule. Since it is inconvenient to show the construction details of coils on a circuit schematic, we apply the *dot convention* in circuit analysis. By this convention, a dot is placed in the circuit at one end of each of the two magnetically coupled coils to indicate the direction of the magnetic flux if current enters that dotted terminal of the coil. This is illustrated in Fig. 13.4. Given a circuit, the dots are already placed beside the coils so that we need not bother about how to place them. The dots are used along with the dot convention to determine the polarity of the mutual voltage. The dot convention is stated as follows:

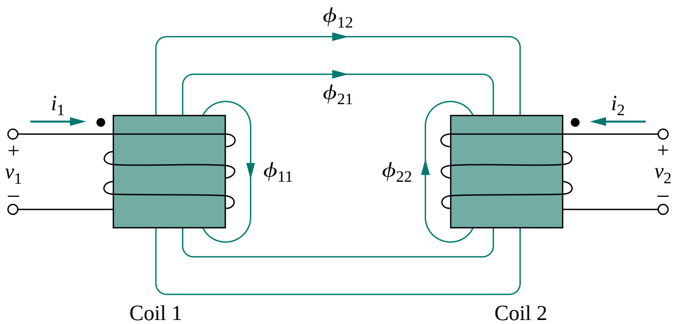


Figure 13.4 Illustration of the dot convention.

> If a current **enters** the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is **positive** at the dotted terminal of the second coil.

Alternatively,

> If a current **leaves** the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is **negative** at the dotted terminal of the second coil.

Thus, the reference polarity of the mutual voltage depends on the reference direction of the inducing current and the dots on the coupled coils. Application of the dot convention is illustrated in the four pairs of mutually coupled coils in Fig. 13.5. For the coupled coils in Fig. 13.5(a), the sign of the mutual voltage v_2 is determined by the reference polarity for v_2 and the direction of i_1. Since i_1 enters the dotted terminal of coil 1 and v_2 is positive at the dotted terminal of coil 2, the mutual voltage is $+M\, di_1/dt$. For the coils in Fig. 13.5(b), the current i_1 enters the dotted terminal of coil 1 and v_2 is negative at the dotted terminal of coil 2. Hence, the mutual voltage is $-M\, di_1/dt$. The same reasoning applies to the coils in Fig. 13.5(c) and 13.5(d). Figure 13.6 shows the dot convention for coupled coils in series. For the coils in Fig. 13.6(a), the total inductance is

$$\boxed{L = L_1 + L_2 + 2M \qquad \text{(Series-aiding connection)}} \tag{13.18}$$

For the coil in Fig. 13.6(b),

$$\boxed{L = L_1 + L_2 - 2M \qquad \text{(Series-opposing connection)}} \tag{13.19}$$

Now that we know how to determine the polarity of the mutual voltage, we are prepared to analyze circuits involving mutual inductance.

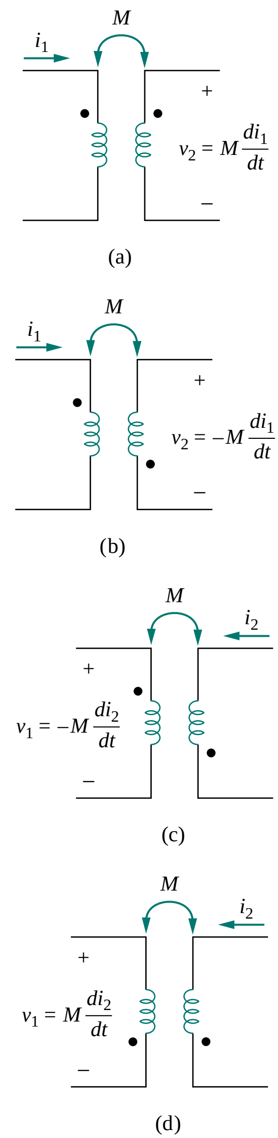


Figure 13.5 Examples illustrating how to apply the dot convention.

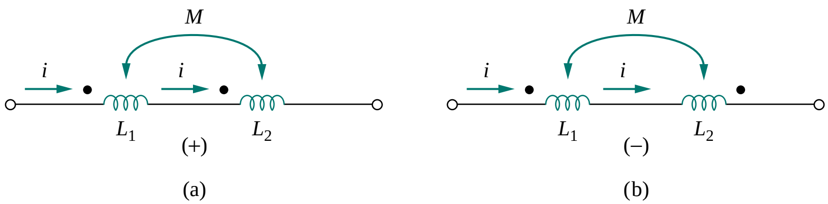


Figure 13.6 Dot convention for coils in series; the sign indicates the polarity of the mutual voltage: (a) series-aiding connection, (b) series-opposing connection.

As the first example, consider the circuit in Fig. 13.7. Applying KVL to coil 1 gives

$$v_1 = i_1 R_1 + L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \tag{13.20a}$$

For coil 2, KVL gives

$$v_2 = i_2 R_2 + L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} \tag{13.20b}$$

We can write Eq. (13.20) in the frequency domain as

$$\mathbf{V}_1 = (R_1 + j\omega L_1)\mathbf{I}_1 + j\omega M \mathbf{I}_2 \tag{13.21a}$$

$$\mathbf{V}_2 = j\omega M \mathbf{I}_1 + (R_2 + j\omega L_2)\mathbf{I}_2 \tag{13.21b}$$

As a second example, consider the circuit in Fig. 13.8. We analyze this in the frequency domain. Applying KVL to coil 1, we get

$$\mathbf{V} = (\mathbf{Z}_1 + j\omega L_1)\mathbf{I}_1 - j\omega M \mathbf{I}_2 \tag{13.22a}$$

For coil 2, KVL yields

$$0 = -j\omega M \mathbf{I}_1 + (\mathbf{Z}_L + j\omega L_2)\mathbf{I}_2 \tag{13.22b}$$

Equations (13.21) and (13.22) are solved in the usual manner to determine the currents.

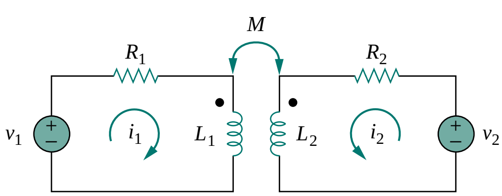


Figure 13.7 Time-domain analysis of a circuit containing coupled coils.

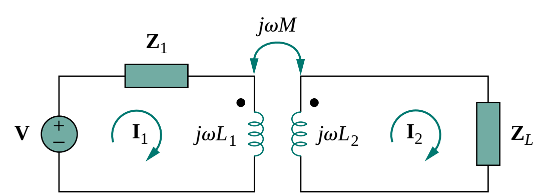


Figure 13.8 Frequency-domain analysis of a circuit containing coupled coils.

At this introductory level we are not concerned with the determination of the mutual inductances of the coils and their dot placements. Like R, L, and C, calculation of M would involve applying the theory of electromagnetics to the actual physical properties of the coils. In this text, we assume that the mutual inductance and the dots placement are the "givens" of the circuit problem, like the circuit components R, L, and C.

EXAMPLE 13.1

Calculate the phasor currents $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit of Fig. 13.9.

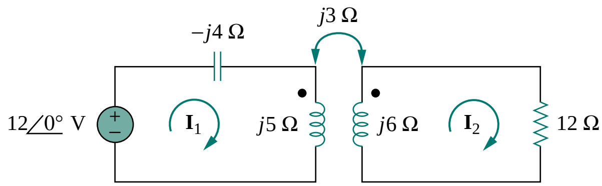


Figure 13.9 For Example 13.1.

Solution:

For coil 1, KVL gives

$$-12 + (-j4 + j5)\mathbf{I}_1 - j3\mathbf{I}_2 = 0$$

or

$$j\mathbf{I}_1 - j3\mathbf{I}_2 = 12 \tag{13.1.1}$$

For coil 2, KVL gives

$$-j3\mathbf{I}_1 + (12 + j6)\mathbf{I}_2 = 0$$

or

$$\mathbf{I}_1 = \frac{(12 + j6)\mathbf{I}_2}{j3} = (2 - j4)\mathbf{I}_2 \tag{13.1.2}$$

Substituting this in Eq. (13.1.1), we get

$$(j2 + 4 - j3)\mathbf{I}_2 = (4 - j)\mathbf{I}_2 = 12$$

or

$$\mathbf{I}_2 = \frac{12}{4 - j} = 2.91\underline{/14.04^\circ}\ \text{A} \tag{13.1.3}$$

From Eqs. (13.1.2) and (13.1.3),

$$\mathbf{I}_1 = (2 - j4)\mathbf{I}_2 = (4.472\underline{/-63.43^\circ})(2.91\underline{/14.04^\circ})$$
$$= 13.01\underline{/-49.39^\circ}\ \text{A}$$

PRACTICE PROBLEM 13.1

Determine the voltage $\mathbf{V}_o$ in the circuit of Fig. 13.10.

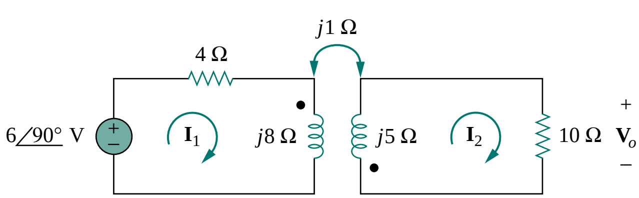


Figure 13.10 For Practice Prob. 13.1.

Answer: $0.6\underline{/-90^\circ}$ V.

EXAMPLE 13.2

Calculate the mesh currents in the circuit of Fig. 13.11.

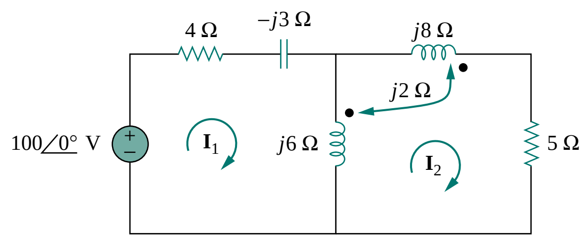


Figure 13.11 For Example 13.2.

Solution:

The key to analyzing a magnetically coupled circuit is knowing the polarity of the mutual voltage. We need to apply the dot rule. In Fig. 13.11, suppose coil 1 is the one whose reactance is 6 Ω, and coil 2 is the one whose reactance is 8 Ω. To figure out the polarity of the mutual voltage in coil 1 due to current $\mathbf{I}_2$, we observe that $\mathbf{I}_2$ leaves the dotted terminal of coil 2. Since we are applying KVL in the clockwise direction, it implies that the mutual voltage is negative, that is, $-j2\mathbf{I}_2$.

Alternatively, it might be best to figure out the mutual voltage by redrawing the relevant portion of the circuit, as shown in Fig. 13.12(a), where it becomes clear that the mutual voltage is $\mathbf{V}_1 = -2j\mathbf{I}_2$.

Thus, for mesh 1 in Fig. 13.11, KVL gives

$$-100 + \mathbf{I}_1(4 - j3 + j6) - j6\mathbf{I}_2 - j2\mathbf{I}_2 = 0$$

or

$$100 = (4 + j3)\mathbf{I}_1 - j8\mathbf{I}_2 \tag{13.2.1}$$

Similarly, to figure out the mutual voltage in coil 2 due to current $\mathbf{I}_1$, consider the relevant portion of the circuit, as shown in Fig. 13.12(b). Applying the dot convention gives the mutual voltage as $\mathbf{V}_2 = -2j\mathbf{I}_1$. Also, current $\mathbf{I}_2$ sees the two coupled coils in series in Fig. 13.11; since it leaves the dotted terminals in both coils, Eq. (13.18) applies. Therefore, for mesh 2, KVL gives

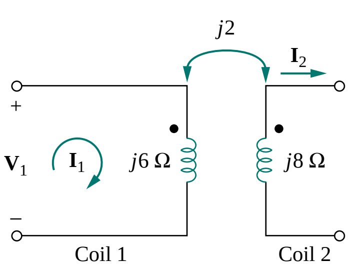
$$0 = -2j\mathbf{I}_1 - j6\mathbf{I}_1 + (j6 + j8 + j2 \times 2 + 5)\mathbf{I}_2$$

or

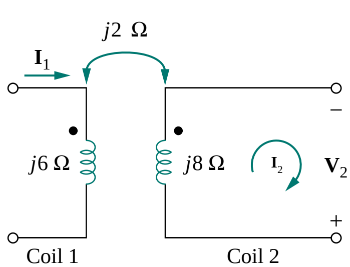
$$0 = -j8\mathbf{I}_1 + (5 + j18)\mathbf{I}_2 \tag{13.2.2}$$

Putting Eqs. (13.2.1) and (13.2.2) in matrix form, we get

$$\begin{bmatrix} 100 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 + j3 & -j8 \\ -j8 & 5 + j18 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$



(a) $\mathbf{V}_1 = -2j\mathbf{I}_2$



(b) $\mathbf{V}_2 = -2j\mathbf{I}_1$

Figure 13.12 For Example 13.2; redrawing the relevant portion of the circuit in Fig. 13.11 to find mutual voltages by the dot convention.

The determinants are

$$\Delta = \begin{vmatrix} 4+j3 & -j8 \\ -j8 & 5+j18 \end{vmatrix} = 30 + j87$$

$$\Delta_1 = \begin{vmatrix} 100 & -j8 \\ 0 & 5+j18 \end{vmatrix} = 100(5 + j18)$$

$$\Delta_2 = \begin{vmatrix} 4+j3 & 100 \\ -j8 & 0 \end{vmatrix} = j800$$

Thus, we obtain the mesh currents as

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \frac{100(5+j18)}{30+j87} = \frac{1868.2\underline{/74.5^\circ}}{92.03\underline{/71^\circ}} = 20.3\underline{/3.5^\circ}\text{ A}$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{j800}{30+j87} = \frac{800\underline{/90^\circ}}{92.03\underline{/71^\circ}} = 8.693\underline{/19^\circ}\text{ A}$$

PRACTICE PROBLEM 13.2

Determine the phasor currents $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit of Fig. 13.13.

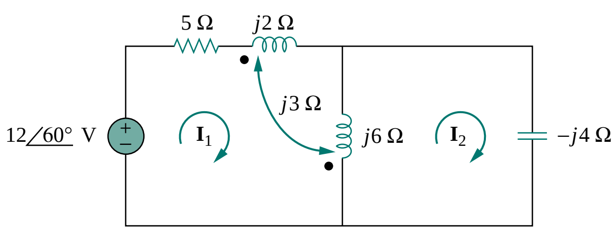


Figure 13.13 For Practice Prob. 13.2.

Answer: $2.15\underline{/86.56^\circ}$, $3.23\underline{/86.56^\circ}$ A.

13.3 ENERGY IN A COUPLED CIRCUIT

In Chapter 6, we saw that the energy stored in an inductor is given by

$$w = \frac{1}{2}Li^2 \tag{13.23}$$

We now want to determine the energy stored in magnetically coupled coils.

Consider the circuit in Fig. 13.14. We assume that currents i_1 and i_2 are zero initially, so that the energy stored in the coils is zero. If we let i_1 increase from zero to I_1 while maintaining $i_2 = 0$, the power in coil 1 is

$$p_1(t) = v_1 i_1 = i_1 L_1 \frac{di_1}{dt} \tag{13.24}$$

and the energy stored in the circuit is

$$w_1 = \int p_1\,dt = L_1 \int_0^{I_1} i_1\,di_1 = \frac{1}{2}L_1 I_1^2 \tag{13.25}$$

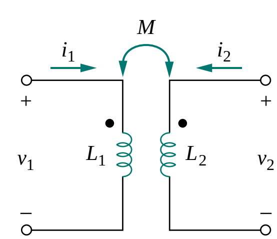


Figure 13.14 The circuit for deriving energy stored in a coupled circuit.

If we now maintain $i_1 = I_1$ and increase i_2 from zero to I_2, the mutual voltage induced in coil 1 is $M_{12}\,di_2/dt$, while the mutual voltage induced in coil 2 is zero, since i_1 does not change. The power in the coils is now

$$p_2(t) = i_1 M_{12}\frac{di_2}{dt} + i_2 v_2 = I_1 M_{12}\frac{di_2}{dt} + i_2 L_2 \frac{di_2}{dt} \tag{13.26}$$

and the energy stored in the circuit is

$$\begin{aligned} w_2 &= \int p_2\,dt = M_{12} I_1 \int_0^{I_2} di_2 + L_2 \int_0^{I_2} i_2\,di_2 \\ &= M_{12} I_1 I_2 + \frac{1}{2} L_2 I_2^2 \end{aligned} \tag{13.27}$$

The total energy stored in the coils when both i_1 and i_2 have reached constant values is

$$w = w_1 + w_2 = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + M_{12} I_1 I_2 \tag{13.28}$$

If we reverse the order by which the currents reach their final values, that is, if we first increase i_2 from zero to I_2 and later increase i_1 from zero to I_1, the total energy stored in the coils is

$$w = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + M_{21} I_1 I_2 \tag{13.29}$$

Since the total energy stored should be the same regardless of how we reach the final conditions, comparing Eqs. (13.28) and (13.29) leads us to conclude that

$$M_{12} = M_{21} = M \tag{13.30a}$$

and

$$w = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + M I_1 I_2 \tag{13.30b}$$

This equation was derived based on the assumption that the coil currents both entered the dotted terminals. If one current enters one dotted terminal while the other current leaves the other dotted terminal, the mutual voltage is negative, so that the mutual energy $M I_1 I_2$ is also negative. In that case,

$$w = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 - M I_1 I_2 \tag{13.31}$$

Also, since I_1 and I_2 are arbitrary values, they may be replaced by i_1 and i_2, which gives the instantaneous energy stored in the circuit the general expression

$$\boxed{w = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 \pm M i_1 i_2} \tag{13.32}$$

The positive sign is selected for the mutual term if both currents enter or leave the dotted terminals of the coils; the negative sign is selected otherwise.

We will now establish an upper limit for the mutual inductance M. The energy stored in the circuit cannot be negative because the circuit is

passive. This means that the quantity $1/2L_1i_1^2 + 1/2L_2i_2^2 - Mi_1i_2$ must be greater than or equal to zero,

$$\frac{1}{2}L_1i_1^2 + \frac{1}{2}L_2i_2^2 - Mi_1i_2 \ge 0 \tag{13.33}$$

To complete the square, we both add and subtract the term $i_1i_2\sqrt{L_1L_2}$ on the right-hand side of Eq. (13.33) and obtain

$$\frac{1}{2}(i_1\sqrt{L_1} - i_2\sqrt{L_2})^2 + i_1i_2(\sqrt{L_1L_2} - M) \ge 0 \tag{13.34}$$

The squared term is never negative; at its least it is zero. Therefore, the second term on the right-hand side of Eq. (13.34) must be greater than zero; that is,

$$\sqrt{L_1L_2} - M \ge 0$$

or

$$M \le \sqrt{L_1L_2} \tag{13.35}$$

Thus, the mutual inductance cannot be greater than the geometric mean of the self-inductances of the coils. The extent to which the mutual inductance M approaches the upper limit is specified by the *coefficient of coupling* k, given by

$$k = \frac{M}{\sqrt{L_1L_2}} \tag{13.36}$$

or

$$\boxed{M = k\sqrt{L_1L_2}} \tag{13.37}$$

where $0 \le k \le 1$ or equivalently $0 \le M \le \sqrt{L_1L_2}$. The coupling coefficient is the fraction of the total flux emanating from one coil that links the other coil. For example, in Fig. 13.2,

$$k = \frac{\phi_{12}}{\phi_1} = \frac{\phi_{12}}{\phi_{11} + \phi_{12}} \tag{13.38}$$

and in Fig. 13.3,

$$k = \frac{\phi_{21}}{\phi_2} = \frac{\phi_{21}}{\phi_{21} + \phi_{22}} \tag{13.39}$$

If the entire flux produced by one coil links another coil, then $k = 1$ and we have 100 percent coupling, or the coils are said to be *perfectly coupled*. Thus,

> The coupling coefficient k is a measure of the magnetic coupling between two coils; $0 \le k \le 1$.

For $k < 0.5$, coils are said to be *loosely coupled*; and for $k > 0.5$, they are said to be *tightly coupled*.

We expect k to depend on the closeness of the two coils, their core, their orientation, and their windings. Figure 13.15 shows loosely coupled

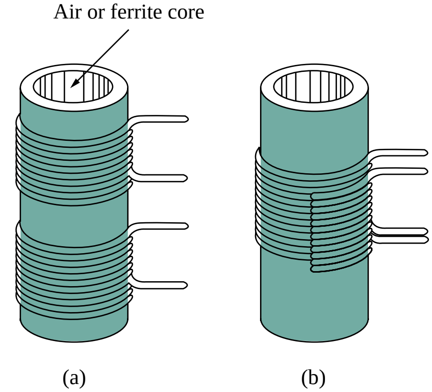


Figure 13.15 Windings: (a) loosely coupled, (b) tightly coupled; cutaway view demonstrates both windings.

windings and tightly coupled windings. The air-core transformers used in radio frequency circuits are loosely coupled, whereas iron-core transformers used in power systems are tightly coupled. The linear transformers discussed in Section 3.4 are mostly air-core; the ideal transformers discussed in Sections 13.5 and 13.6 are principally iron-core.

EXAMPLE 13.3

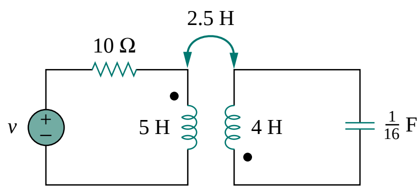


Figure 13.16 For Example 13.3.

Consider the circuit in Fig. 13.16. Determine the coupling coefficient. Calculate the energy stored in the coupled inductors at time $t = 1$ s if $v = 60\cos(4t + 30°)$ V.

Solution:

The coupling coefficient is

$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{2.5}{\sqrt{20}} = 0.56$$

indicating that the inductors are tightly coupled. To find the energy stored, we need to obtain the frequency-domain equivalent of the circuit.

$$\begin{aligned}
60\cos(4t + 30°) \quad &\Longrightarrow \quad 60\underline{/30°}, \quad \omega = 4 \text{ rad/s} \\
5 \text{ H} \quad &\Longrightarrow \quad j\omega L_1 = j20\ \Omega \\
2.5 \text{ H} \quad &\Longrightarrow \quad j\omega M = j10\ \Omega \\
4 \text{ H} \quad &\Longrightarrow \quad j\omega L_2 = j16\ \Omega \\
\frac{1}{16} \text{ F} \quad &\Longrightarrow \quad \frac{1}{j\omega C} = -j4\ \Omega
\end{aligned}$$

The frequency-domain equivalent is shown in Fig. 13.17. We now apply mesh analysis. For mesh 1,

$$(10 + j20)\mathbf{I}_1 + j10\mathbf{I}_2 = 60\underline{/30°} \tag{13.3.1}$$

For mesh 2,

$$j10\mathbf{I}_1 + (j16 - j4)\mathbf{I}_2 = 0$$

or

$$\mathbf{I}_1 = -1.2\mathbf{I}_2 \tag{13.3.2}$$

Substituting this into Eq. (13.3.1) yields

$$\mathbf{I}_2(-12 - j14) = 60\underline{/30°} \quad \Longrightarrow \quad \mathbf{I}_2 = 3.254\underline{/-160.6°} \text{ A}$$

and

$$\mathbf{I}_1 = -1.2\mathbf{I}_2 = 3.905\underline{/-19.4°} \text{ A}$$

In the time-domain,

$$i_1 = 3.905\cos(4t - 19.4°), \qquad i_2 = 3.254\cos(4t - 199.4°)$$

At time $t = 1$ s, $4t = 4$ rad $= 229.2°$, and

$$i_1 = 3.905\cos(229.2° - 19.4°) = -3.389 \text{ A}$$
$$i_2 = 3.254\cos(229.2° + 160.6°) = 2.824 \text{ A}$$

The total energy stored in the coupled inductors is

$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + M i_1 i_2$$
$$= \frac{1}{2}(5)(-3.389)^2 + \frac{1}{2}(4)(2.824)^2 + 2.5(-3.389)(2.824) = 20.73 \text{ J}$$

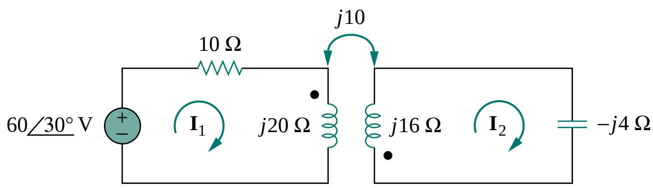


Figure 13.17 Frequency-domain equivalent of the circuit in Fig. 13.16.

PRACTICE PROBLEM 13.3

For the circuit in Fig. 13.18, determine the coupling coefficient and the energy stored in the coupled inductors at $t = 1.5$ s.

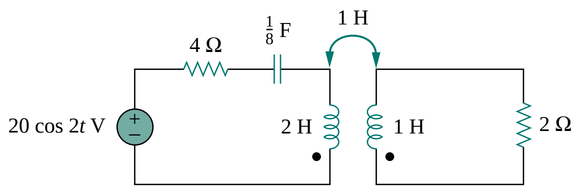


Figure 13.18 For Practice Prob. 13.3.

Answer: 0.7071, 9.85 J.

13.4 LINEAR TRANSFORMERS

Here we introduce the transformer as a new circuit element. A transformer is a magnetic device that takes advantage of the phenomenon of mutual inductance.

> A transformer is generally a four-terminal device comprising two (or more) magnetically coupled coils.

As shown in Fig. 13.19, the coil that is directly connected to the voltage source is called the *primary winding*. The coil connected to the load is called the *secondary winding*. The resistances R_1 and R_2 are included to account for the losses (power dissipation) in the coils. The transformer is said to be *linear* if the coils are wound on a magnetically linear

A linear transformer may also be regarded as one whose flux is proportional to the currents in its windings.

material—a material for which the magnetic permeability is constant. Such materials include air, plastic, Bakelite, and wood. In fact, most materials are magnetically linear. Linear transformers are sometimes called *air-core transformers*, although not all of them are necessarily air-core. They are used in radio and TV sets. Figure 13.20 portrays different types of transformers.

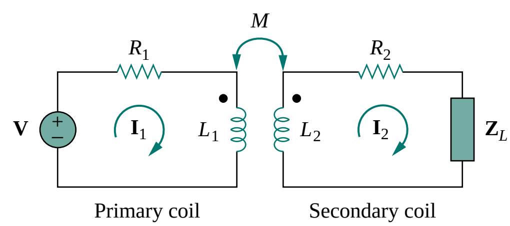
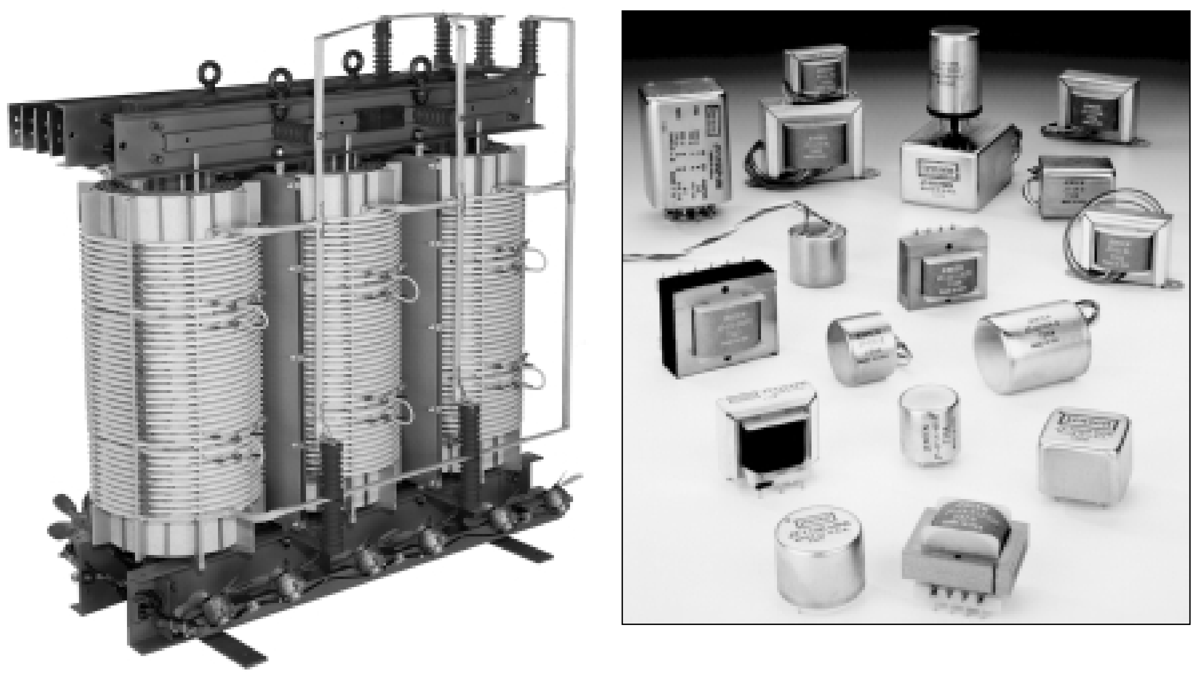


Figure 13.19 A linear transformer.



(a) (b)

Figure 13.20 Different types of transformers: (a) copper wound dry power transformer, (b) audio transformers. *(Courtesy of: (a) Electric Service Co., (b) Jensen Transformers.)*

We would like to obtain the input impedance $\mathbf{Z}_{\text{in}}$ as seen from the source, because $\mathbf{Z}_{\text{in}}$ governs the behavior of the primary circuit. Applying KVL to the two meshes in Fig. 13.19 gives

$$\mathbf{V} = (R_1 + j\omega L_1)\mathbf{I}_1 - j\omega M\mathbf{I}_2 \tag{13.40a}$$

$$0 = -j\omega M\mathbf{I}_1 + (R_2 + j\omega L_2 + \mathbf{Z}_L)\mathbf{I}_2 \tag{13.40b}$$

In Eq. (13.40b), we express $\mathbf{I}_2$ in terms of $\mathbf{I}_1$ and substitute it into Eq. (13.40a). We get the input impedance as

$$\mathbf{Z}_{\text{in}} = \frac{\mathbf{V}}{\mathbf{I}_1} = R_1 + j\omega L_1 + \frac{\omega^2 M^2}{R_2 + j\omega L_2 + \mathbf{Z}_L} \tag{13.41}$$

Notice that the input impedance comprises two terms. The first term, $(R_1 + j\omega L_1)$, is the primary impedance. The second term is due to the coupling between the primary and secondary windings. It is as though this impedance is reflected to the primary. Thus, it is known as the *reflected impedance* $\mathbf{Z}_R$, and

Some authors call this the *coupled impedance*.

$$\boxed{\mathbf{Z}_R = \frac{\omega^2 M^2}{R_2 + j\omega L_2 + \mathbf{Z}_L}} \tag{13.42}$$

It should be noted that the result in Eq. (13.41) or (13.42) is not affected by the location of the dots on the transformer, because the same result is produced when M is replaced by $-M$.

The little bit of experience gained in Sections 13.2 and 13.3 in analyzing magnetically coupled circuits is enough to convince anyone that analyzing these circuits is not as easy as circuits in previous chapters. For this reason, it is sometimes convenient to replace a magnetically coupled circuit by an equivalent circuit with no magnetic coupling. We want to replace the linear transformer in Fig. 13.19 by an equivalent T or Π circuit, a circuit that would have no mutual inductance. Ignore the resistances of the coils and assume that the coils have a common ground as shown in Fig. 13.21. The assumption of a common ground for the two coils is a major restriction of the equivalent circuits. A common ground is imposed on the linear transformer in Fig. 13.21 in view of the necessity of having a common ground in the equivalent T or Π circuit; see Figs. 13.22 and 13.23.

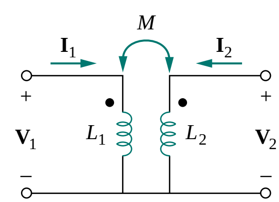


Figure 13.21 Determining the equivalent circuit of a linear transformer.

The voltage-current relationships for the primary and secondary coils give the matrix equation

$$\begin{bmatrix}\mathbf{V}_1\\ \mathbf{V}_2\end{bmatrix} = \begin{bmatrix} j\omega L_1 & j\omega M\\ j\omega M & j\omega L_2\end{bmatrix}\begin{bmatrix}\mathbf{I}_1\\ \mathbf{I}_2\end{bmatrix} \tag{13.43}$$

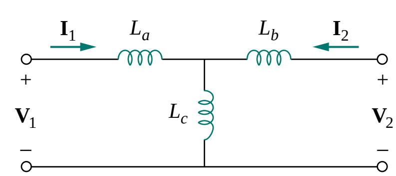


Figure 13.22 An equivalent T circuit.

By matrix inversion, this can be written as

$$\begin{bmatrix}\mathbf{I}_1\\ \mathbf{I}_2\end{bmatrix} = \begin{bmatrix} \dfrac{L_2}{j\omega(L_1L_2 - M^2)} & \dfrac{-M}{j\omega(L_1L_2 - M^2)}\\ \dfrac{-M}{j\omega(L_1L_2 - M^2)} & \dfrac{L_1}{j\omega(L_1L_2 - M^2)}\end{bmatrix}\begin{bmatrix}\mathbf{V}_1\\ \mathbf{V}_2\end{bmatrix} \tag{13.44}$$

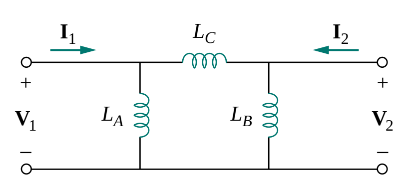


Figure 13.23 An equivalent Π circuit.

Our goal is to match Eqs. (13.43) and (13.44) with the corresponding equations for the T and Π networks.

For the T (or Y) network of Fig. 13.22, mesh analysis provides the terminal equations as

$$\begin{bmatrix}\mathbf{V}_1\\ \mathbf{V}_2\end{bmatrix} = \begin{bmatrix} j\omega(L_a + L_c) & j\omega L_c\\ j\omega L_c & j\omega(L_b + L_c)\end{bmatrix}\begin{bmatrix}\mathbf{I}_1\\ \mathbf{I}_2\end{bmatrix} \tag{13.45}$$

If the circuits in Figs. 13.21 and 13.22 are equivalents, Eqs. (13.43) and (13.45) must be identical. Equating terms in the impedance matrices of

Eqs. (13.43) and (13.45) leads to

$$\boxed{L_a = L_1 - M, \qquad L_b = L_2 - M, \qquad L_c = M} \tag{13.46}$$

For the Π (or Δ) network in Fig. 13.23, nodal analysis gives the terminal equations as

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \dfrac{1}{j\omega L_A} + \dfrac{1}{j\omega L_C} & -\dfrac{1}{j\omega L_C} \\ -\dfrac{1}{j\omega L_C} & \dfrac{1}{j\omega L_B} + \dfrac{1}{j\omega L_C} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} \tag{13.47}$$

Equating terms in admittance matrices of Eqs. (13.44) and (13.47), we obtain

$$\boxed{\begin{gathered} L_A = \frac{L_1L_2 - M^2}{L_2 - M}, \qquad L_B = \frac{L_1L_2 - M^2}{L_1 - M} \\ L_C = \frac{L_1L_2 - M^2}{M} \end{gathered}} \tag{13.48}$$

Note that in Figs. 13.23 and 13.24, the inductors are not magnetically coupled. Also note that changing the locations of the dots in Fig. 13.21 can cause M to become $-M$. As Example 13.6 illustrates, a negative value of M is physically unrealizable but the equivalent model is still mathematically valid.

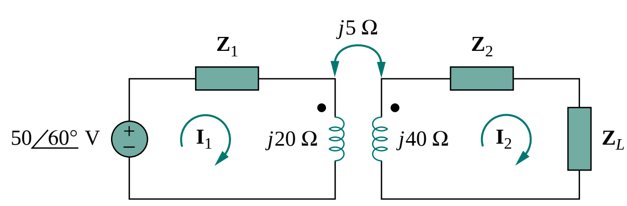


Figure 13.24 For Example 13.4.

EXAMPLE 13.4

In the circuit of Fig. 13.24, calculate the input impedance and current $\mathbf{I}_1$. Take $\mathbf{Z}_1 = 60 - j100\ \Omega$, $\mathbf{Z}_2 = 30 + j40\ \Omega$, and $\mathbf{Z}_L = 80 + j60\ \Omega$.

Solution:

From Eq. (13.41),

$$\begin{aligned} \mathbf{Z}_{\text{in}} &= \mathbf{Z}_1 + j20 + \frac{(5)^2}{j40 + \mathbf{Z}_2 + \mathbf{Z}_L} \\ &= 60 - j100 + j20 + \frac{25}{110 + j140} \\ &= 60 - j80 + 0.14\angle -51.84° \\ &= 60.09 - j80.11 = 100.14\angle -53.1°\ \Omega \end{aligned}$$

Thus,

$$\mathbf{I}_1 = \frac{\mathbf{V}}{\mathbf{Z}_{\text{in}}} = \frac{50\underline{/60^\circ}}{100.14\underline{/-53.1^\circ}} = 0.5\underline{/113.1^\circ} \text{ A}$$

PRACTICE PROBLEM 13.4

Find the input impedance of the circuit of Fig. 13.25 and the current from the voltage source.

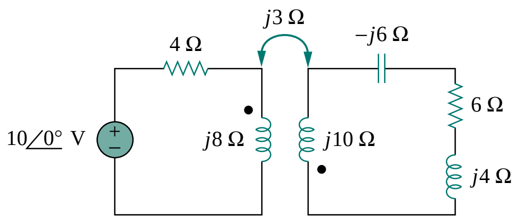


Figure 13.25 For Practice Prob. 13.4.

Answer: $8.58\underline{/58.05^\circ}$ Ω, $1.165\underline{/-58.05^\circ}$ A.

EXAMPLE 13.5

Determine the T-equivalent circuit of the linear transformer in Fig. 13.26(a).

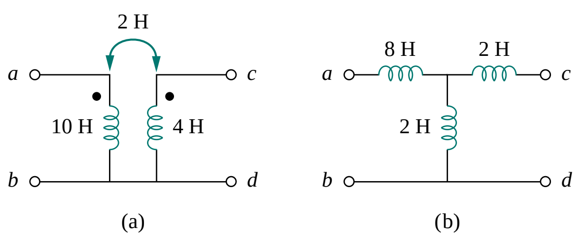


Figure 13.26 For Example 13.5: (a) a linear transformer, (b) its T-equivalent circuit.

Solution:

Given that $L_1 = 10$, $L_2 = 4$, and $M = 2$, the T equivalent network has the following parameters:

$$L_a = L_1 - M = 10 - 2 = 8 \text{ H}$$
$$L_b = L_2 - M = 4 - 2 = 2 \text{ H}, \qquad L_c = M = 2 \text{ H}$$

The T-equivalent circuit is shown in Fig. 13.26(b). We have assumed that reference directions for currents and voltage polarities in the primary and secondary windings conform to those in Fig. 13.21. Otherwise, we may need to replace M with $-M$, as Example 13.6 illustrates.

PRACTICE PROBLEM 13.5

For the linear transformer in Fig. 13.26 (a), find the Π equivalent network.

Answer: $L_A = 18$ H, $L_B = 4.5$ H, $L_C = 18$ H.

EXAMPLE 13.6

Solve for $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{V}_o$ in Fig. 13.27 (the same circuit as for Practice Prob. 13.1) using the T-equivalent circuit for the linear transformer.

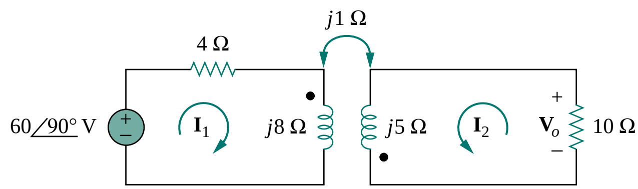


Figure 13.27 For Example 13.6.

Solution:

Notice that the circuit in Fig. 13.27 is the same as that in Fig. 13.10 except that the reference direction for current $\mathbf{I}_2$ has been reversed, just to make the reference directions for the currents for the magnetically coupled coils conform with those in Fig. 13.21.

We need to replace the magnetically coupled coils with the T-equivalent circuit. The relevant portion of the circuit in Fig. 13.27 is shown in Fig. 13.28(a). Comparing Fig. 13.28(a) with Fig. 13.21 shows that there are two differences. First, due to the current reference directions and voltage polarities, we need to replace M by $-M$ to make Fig. 13.28(a) conform with Fig. 13.21. Second, the circuit in Fig. 13.21 is in the time-domain, whereas the circuit in Fig. 13.28(a) is in the frequency-domain. The difference is the factor $j\omega$; that is, L in Fig. 13.21 has been replaced with $j\omega L$ and M with $j\omega M$. Since ω is not specified, we can assume $\omega = 1$ or any other value; it really does not matter. With these two differences in mind,

$$L_a = L_1 - (-M) = 8 + 1 = 9 \text{ H}$$

$$L_b = L_2 - (-M) = 5 + 1 = 6 \text{ H}, \qquad L_c = -M = -1 \text{ H}$$

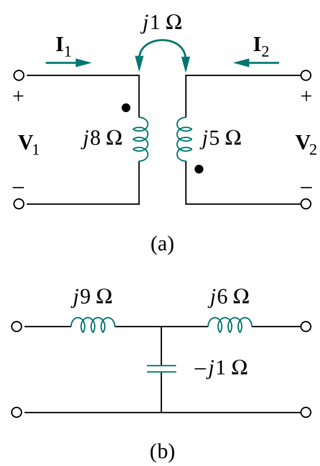


Figure 13.28 For Example 13.6: (a) circuit for coupled coils of Fig. 13.27, (b) T-equivalent circuit.

Thus, the T-equivalent circuit for the coupled coils is as shown in Fig. 13.28(b).

Inserting the T-equivalent circuit in Fig. 13.28(b) to replace the two coils in Fig. 13.27 gives the equivalent circuit in Fig. 13.29, which can be solved using nodal or mesh analysis. Applying mesh analysis, we obtain

$$j6 = \mathbf{I}_1(4 + j9 - j1) + \mathbf{I}_2(-j1) \tag{13.6.1}$$

and

$$0 = \mathbf{I}_1(-j1) + \mathbf{I}_2(10 + j6 - j1) \tag{13.6.2}$$

From Eq. (13.6.2),

$$\mathbf{I}_1 = \frac{(10 + j5)}{j}\mathbf{I}_2 = (5 - j10)\mathbf{I}_2 \tag{13.6.3}$$

Substituting Eq. (13.6.3) into Eq. (13.6.1) gives

$$j6 = (4 + j8)(5 - j10)\mathbf{I}_2 - j\mathbf{I}_2 = (100 - j)\mathbf{I}_2 \simeq 100\mathbf{I}_2$$

Since 100 is very large compared to 1, the imaginary part of $(100 - j)$ can be ignored so that $100 - j \simeq 100$. Hence,

$$\mathbf{I}_2 = \frac{j6}{100} = j0.06 = 0.06\underline{/90^\circ}\text{ A}$$

From Eq. (13.6.3),

$$\mathbf{I}_1 = (5 - j10)j0.06 = 0.6 + j0.3\text{ A}$$

and

$$\mathbf{V}_o = -10\mathbf{I}_2 = -j0.6 = 0.6\underline{/-90^\circ}\text{ V}$$

This agrees with the answer to Practice Prob. 13.1. Of course, the direction of $\mathbf{I}_2$ in Fig. 13.10 is opposite to that in Fig. 13.27. This will not affect $\mathbf{V}_o$, but the value of $\mathbf{I}_2$ in this example is the negative of that of $\mathbf{I}_2$ in Practice Prob. 13.1. The advantage of using the T-equivalent model for the magnetically coupled coils is that in Fig. 13.29 we do not need to bother with the dot on the coupled coils.

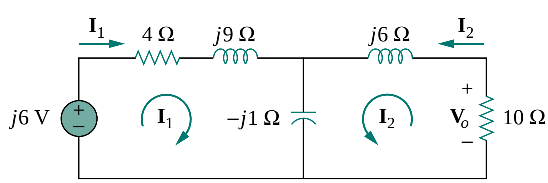


Figure 13.29 For Example 13.6.

PRACTICE PROBLEM 13.6

Solve the problem in Example 13.1 (see Fig. 13.9) using the T-equivalent model for the magnetically coupled coils.

Answer: $13\underline{/-49.4^\circ}$ A, $2.91\underline{/14.04^\circ}$ A.

13.5 IDEAL TRANSFORMERS

An ideal transformer is one with perfect coupling ($k = 1$). It consists of two (or more) coils with a large number of turns wound on a common core of high permeability. Because of this high permeability of the core, the flux links all the turns of both coils, thereby resulting in a perfect coupling.

To see how an ideal transformer is the limiting case of two coupled inductors where the inductances approach infinity and the coupling is perfect, let us reexamine the circuit in Fig. 13.14. In the frequency domain,

$$\mathbf{V}_1 = j\omega L_1\mathbf{I}_1 + j\omega M\mathbf{I}_2 \tag{13.49a}$$

$$\mathbf{V}_2 = j\omega M\mathbf{I}_1 + j\omega L_2\mathbf{I}_2 \tag{13.49b}$$

From Eq. (13.49a), $\mathbf{I}_1 = (\mathbf{V}_1 - j\omega M\mathbf{I}_2)/j\omega L_1$. Substituting this in Eq. (13.49b) gives

$$\mathbf{V}_2 = j\omega L_2\mathbf{I}_2 + \frac{M\mathbf{V}_1}{L_1} - \frac{j\omega M^2\mathbf{I}_2}{L_1}$$

But $M = \sqrt{L_1L_2}$ for perfect coupling ($k = 1$). Hence,

$$\mathbf{V}_2 = j\omega L_2\mathbf{I}_2 + \frac{\sqrt{L_1L_2}\mathbf{V}_1}{L_1} - \frac{j\omega L_1L_2\mathbf{I}_2}{L_1} = \sqrt{\frac{L_2}{L_1}}\mathbf{V}_1 = n\mathbf{V}_1$$

where $n = \sqrt{L_2/L_1}$ and is called the *turns ratio*. As $L_1, L_2, M \rightarrow \infty$ such that n remains the same, the coupled coils become an ideal transformer. A transformer is said to be ideal if it has the following properties:

1. Coils have very large reactances ($L_1, L_2, M \rightarrow \infty$).
2. Coupling coefficient is equal to unity ($k = 1$).
3. Primary and secondary coils are lossless ($R_1 = 0 = R_2$).

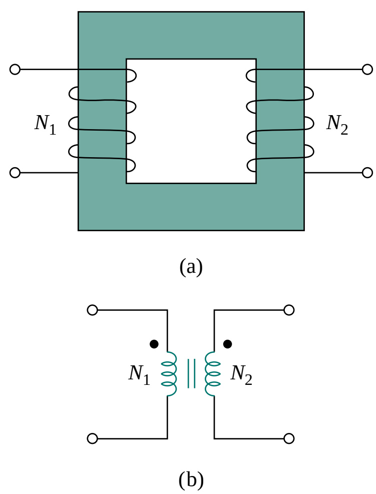


Figure 13.30 (a) Ideal transformer, (b) circuit symbol for ideal transformers.

> An ideal transformer is a unity-coupled, lossless transformer in which the primary and secondary coils have infinite self-inductances.

Iron-core transformers are close approximations to ideal transformers. These are used in power systems and electronics.

Figure 13.30(a) shows a typical ideal transformer; the circuit symbol is in Fig. 13.30(b). The vertical lines between the coils indicate an iron core as distinct from the air core used in linear transformers. The primary winding has N_1 turns; the secondary winding has N_2 turns.

When a sinusoidal voltage is applied to the primary winding as shown in Fig. 13.31, the same magnetic flux ϕ goes through both windings. According to Faraday's law, the voltage across the primary winding is

$$v_1 = N_1\frac{d\phi}{dt} \tag{13.50a}$$

while that across the secondary winding is

$$v_2 = N_2\frac{d\phi}{dt} \tag{13.50b}$$

Dividing Eq. (13.50b) by Eq. (13.50a), we get

$$\frac{v_2}{v_1} = \frac{N_2}{N_1} = n \tag{13.51}$$

where n is, again, the *turns ratio* or *transformation ratio*. We can use the phasor voltages $\mathbf{V}_1$ and $\mathbf{V}_2$ rather than the instantaneous values v_1 and v_2. Thus, Eq. (13.51) may be written as

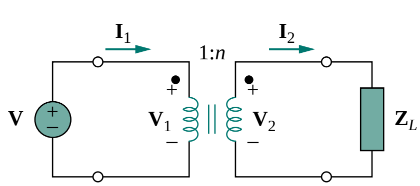


Figure 13.31 Relating primary and secondary quantities in an ideal transformer.

$$\boxed{\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{N_2}{N_1} = n} \tag{13.52}$$

For the reason of power conservation, the energy supplied to the primary must equal the energy absorbed by the secondary, since there are no losses in an ideal transformer. This implies that

$$v_1 i_1 = v_2 i_2 \tag{13.53}$$

In phasor form, Eq. (13.53) in conjunction with Eq. (13.52) becomes

$$\frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{\mathbf{V}_2}{\mathbf{V}_1} = n \tag{13.54}$$

showing that the primary and secondary currents are related to the turns ratio in the inverse manner as the voltages. Thus,

$$\boxed{\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{N_1}{N_2} = \frac{1}{n}} \tag{13.55}$$

When $n = 1$, we generally call the transformer an *isolation transformer*. The reason will become obvious in Section 13.9.1. If $n > 1$, we have a *step-up transformer*, as the voltage is increased from primary to secondary ($\mathbf{V}_2 > \mathbf{V}_1$). On the other hand, if $n < 1$, the transformer is a *step-down transformer*, since the voltage is decreased from primary to secondary ($\mathbf{V}_2 < \mathbf{V}_1$).

A **step-down transformer** is one whose secondary voltage is less than its primary voltage.

A **step-up transformer** is one whose secondary voltage is greater than its primary voltage.

The ratings of transformers are usually specified as V_1/V_2. A transformer with rating 2400/120 V should have 2400 V on the primary and 120 in the secondary (i.e., a step-down transformer). Keep in mind that the voltage ratings are in rms.

Power companies often generate at some convenient voltage and use a step-up transformer to increase the voltage so that the power can be transmitted at very high voltage and low current over transmission lines, resulting in significant cost savings. Near residential consumer premises, step-down transformers are used to bring the voltage down to 120 V. Section 13.9.3 will elaborate on this.

It is important that we know how to get the proper polarity of the voltages and the direction of the currents for the transformer in Fig. 13.31. If the polarity of $\mathbf{V}_1$ or $\mathbf{V}_2$ or the direction of $\mathbf{I}_1$ or $\mathbf{I}_2$ is changed, n in Eqs. (13.51) to (13.55) may need to be replaced by $-n$. The two simple rules to follow are:

1. If $\mathbf{V}_1$ and $\mathbf{V}_2$ are *both* positive or both negative at the dotted terminals, use $+n$ in Eq. (13.52). Otherwise, use $-n$.
2. If $\mathbf{I}_1$ and $\mathbf{I}_2$ *both* enter into or both leave the dotted terminals, use $-n$ in Eq. (13.55). Otherwise, use $+n$.

The rules are demonstrated with the four circuits in Fig. 13.32.

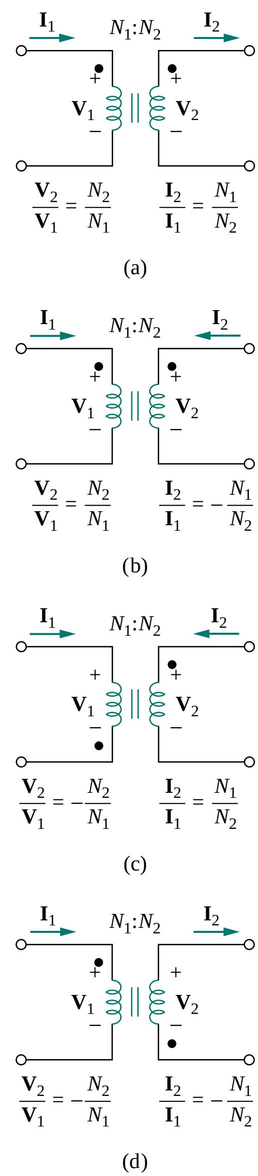


Figure 13.32 Typical circuits illustrating proper voltage polarities and current directions in an ideal transformer.

Using Eqs. (13.52) and (13.55), we can always express $\mathbf{V}_1$ in terms of $\mathbf{V}_2$ and $\mathbf{I}_1$ in terms of $\mathbf{I}_2$, or vice versa:

$$\mathbf{V}_1 = \frac{\mathbf{V}_2}{n} \qquad \text{or} \qquad \mathbf{V}_2 = n\mathbf{V}_1 \tag{13.56}$$

$$\mathbf{I}_1 = n\mathbf{I}_2 \qquad \text{or} \qquad \mathbf{I}_2 = \frac{\mathbf{I}_1}{n} \tag{13.57}$$

The complex power in the primary winding is

$$\boxed{\mathbf{S}_1 = \mathbf{V}_1\mathbf{I}_1^* = \frac{\mathbf{V}_2}{n}(n\mathbf{I}_2)^* = \mathbf{V}_2\mathbf{I}_2^* = \mathbf{S}_2} \tag{13.58}$$

showing that the complex power supplied to the primary is delivered to the secondary without loss. The transformer absorbs no power. Of course, we should expect this, since the ideal transformer is lossless. The input impedance as seen by the source in Fig. 13.31 is found from Eqs. (13.56) and (13.57) as

$$\mathbf{Z}_{\text{in}} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{1}{n^2}\frac{\mathbf{V}_2}{\mathbf{I}_2} \tag{13.59}$$

It is evident from Fig. 13.31 that $\mathbf{V}_2/\mathbf{I}_2 = \mathbf{Z}_L$, so that

$$\boxed{\mathbf{Z}_{\text{in}} = \frac{\mathbf{Z}_L}{n^2}} \tag{13.60}$$

Notice that an ideal transformer reflects an impedance as the square of the turns ratio.

The input impedance is also called the *reflected impedance*, since it appears as if the load impedance is reflected to the primary side. This ability of the transformer to transform a given impedance into another impedance provides us a means of *impedance matching* to ensure maximum power transfer. The idea of impedance matching is very useful in practice and will be discussed more in Section 13.9.2.

In analyzing a circuit containing an ideal transformer, it is common practice to eliminate the transformer by reflecting impedances and sources from one side of the transformer to the other. In the circuit of Fig. 13.33, suppose we want to reflect the secondary side of the circuit to the primary side. We find the Thevenin equivalent of the circuit to the right of the terminals a-b. We obtain $\mathbf{V}_{\text{Th}}$ as the open-circuit voltage at terminals a-b, as shown in Fig. 13.34(a). Since terminals a-b are open, $\mathbf{I}_1 = 0 = \mathbf{I}_2$ so that $\mathbf{V}_2 = \mathbf{V}_{s2}$. Hence, from Eq. (13.56),

$$\mathbf{V}_{\text{Th}} = \mathbf{V}_1 = \frac{\mathbf{V}_2}{n} = \frac{\mathbf{V}_{s2}}{n} \tag{13.61}$$

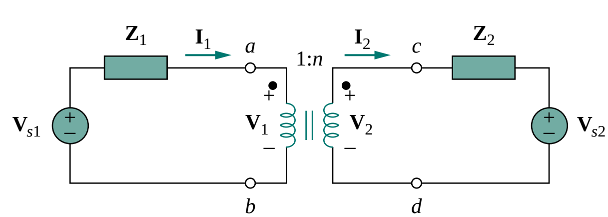


Figure 13.33 Ideal transformer circuit whose equivalent circuits are to be found.

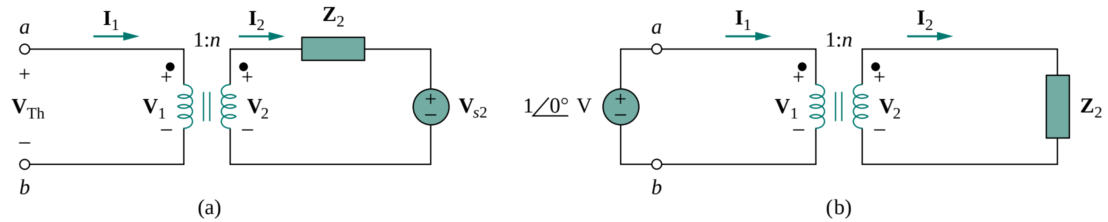


Figure 13.34 (a) Obtaining $\mathbf{V}_{\mathrm{Th}}$ for the circuit in Fig. 13.33, (b) obtaining $\mathbf{Z}_{\mathrm{Th}}$ for the circuit in Fig. 13.33.

To get $\mathbf{Z}_{\mathrm{Th}}$, we remove the voltage source in the secondary winding and insert a unit source at terminals a-b, as in Fig. 13.34(b). From Eqs. (13.56) and (13.57), $\mathbf{I}_1 = n\mathbf{I}_2$ and $\mathbf{V}_1 = \mathbf{V}_2/n$, so that

$$\mathbf{Z}_{\mathrm{Th}} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{\mathbf{V}_2/n}{n\mathbf{I}_2} = \frac{\mathbf{Z}_2}{n^2}, \qquad \mathbf{V}_2 = \mathbf{Z}_2\mathbf{I}_2 \tag{13.62}$$

which is what we should have expected from Eq. (13.60). Once we have $\mathbf{V}_{\mathrm{Th}}$ and $\mathbf{Z}_{\mathrm{Th}}$, we add the Thevenin equivalent to the part of the circuit in Fig. 13.33 to the left of terminals a-b. Figure 13.35 shows the result.

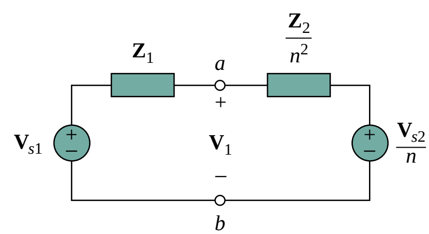


Figure 13.35 Equivalent circuit for Fig. 13.33 obtained by reflecting the secondary circuit to the primary side.

The general rule for eliminating the transformer and reflecting the secondary circuit to the primary side is: divide the secondary impedance by n^2, divide the secondary voltage by n, and multiply the secondary current by n.

We can also reflect the primary side of the circuit in Fig. 13.33 to the secondary side. Figure 13.36 shows the equivalent circuit.

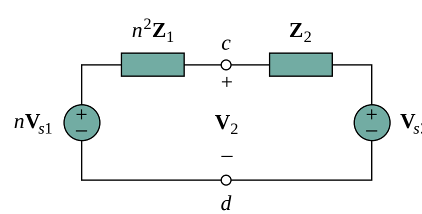


Figure 13.36 Equivalent circuit for Fig. 13.33 obtained by reflecting the primary circuit to the secondary side.

The rule for eliminating the transformer and reflecting the primary circuit to the secondary side is: multiply the primary impedance by n^2, multiply the primary voltage by n, and divide the primary current by n.

According to Eq. (13.58), the power remains the same, whether calculated on the primary or the secondary side. But realize that this reflection approach only applies if there are no external connections between the primary and secondary windings. When we have external connections between the primary and secondary windings, we simply use regular mesh and nodal analysis. Examples of circuits where there are external connections between the primary and secondary windings are in Figs. 13.39 and 13.40. Also note that if the locations of the dots in Fig. 13.33 are changed, we might have to replace n by $-n$ in order to obey the dot rule, illustrated in Fig. 13.32.

EXAMPLE 13.7

An ideal transformer is rated at 2400/120 V, 9.6 kVA, and has 50 turns on the secondary side. Calculate: (a) the turns ratio, (b) the number of

turns on the primary side, and (c) the current ratings for the primary and secondary windings.

Solution:

(a) This is a step-down transformer, since $V_1 = 2400$ V $> V_2 = 120$ V.

$$n = \frac{V_2}{V_1} = \frac{120}{2400} = 0.05$$

(b)

$$n = \frac{N_2}{N_1} \qquad \Longrightarrow \qquad 0.05 = \frac{50}{N_1}$$

or

$$N_1 = \frac{50}{0.05} = 1000 \text{ turns}$$

(c) $S = V_1 I_1 = V_2 I_2 = 9.6$ kVA. Hence,

$$I_1 = \frac{9600}{V_1} = \frac{9600}{2400} = 4 \text{ A}$$

$$I_2 = \frac{9600}{V_2} = \frac{9600}{120} = 80 \text{ A} \qquad \text{or} \qquad I_2 = \frac{I_1}{n} = \frac{4}{0.05} = 80 \text{ A}$$

PRACTICE PROBLEM 13.7

The primary current to an ideal transformer rated at 3300/110 V is 3 A. Calculate: (a) the turns ratio, (b) the kVA rating, (c) the secondary current.

Answer: (a) 1/30, (b) 9.9 kVA, (c) 90 A.

EXAMPLE 13.8

For the ideal transformer circuit of Fig. 13.37, find: (a) the source current $\mathbf{I}_1$, (b) the output voltage $\mathbf{V}_o$, and (c) the complex power supplied by the source.

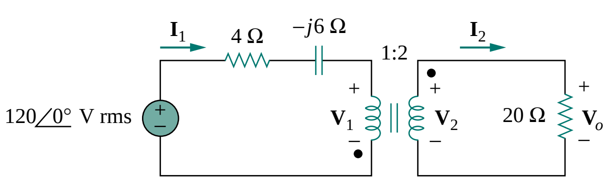


Figure 13.37 For Example 13.8.

Solution:

(a) The 20-Ω impedance can be reflected to the primary side and we get

$$\mathbf{Z}_R = \frac{20}{n^2} = \frac{20}{4} = 5 \ \Omega$$

Thus,

$$\mathbf{Z}_{\text{in}} = 4 - j6 + \mathbf{Z}_R = 9 - j6 = 10.82\underline{/-33.69^\circ} \ \Omega$$

$$\mathbf{I}_1 = \frac{120\underline{/0^\circ}}{\mathbf{Z}_{\text{in}}} = \frac{120\underline{/0^\circ}}{10.82\underline{/-33.69^\circ}} = 11.09\underline{/33.69^\circ} \text{ A}$$

(b) Since both $\mathbf{I}_1$ and $\mathbf{I}_2$ leave the dotted terminals,

$$\mathbf{I}_2 = -\frac{1}{n}\mathbf{I}_1 = -5.545\underline{/33.69^\circ} \text{ A}$$

$$\mathbf{V}_o = 20\mathbf{I}_2 = 110.9\underline{/213.69^\circ} \text{ V}$$

(c) The complex power supplied is

$$\mathbf{S} = \mathbf{V}_s\mathbf{I}_1^* = (120\underline{/0^\circ})(11.09\underline{/-33.69^\circ}) = 1330.8\underline{/-33.69^\circ} \text{ VA}$$

PRACTICE PROBLEM 13.8

In the ideal transformer circuit of Fig. 13.38, find $\mathbf{V}_o$ and the complex power supplied by the source.

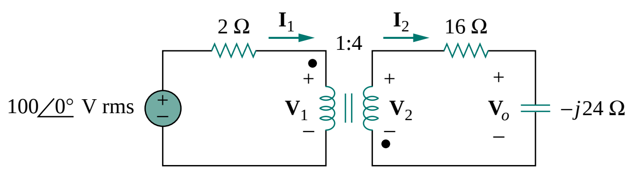


Figure 13.38 For Practice Prob. 13.8.

Answer: $178.9\underline{/116.56^\circ}$ V, $2981.5\underline{/-26.56^\circ}$ VA.

EXAMPLE 13.9

Calculate the power supplied to the 10-Ω resistor in the ideal transformer circuit of Fig. 13.39.

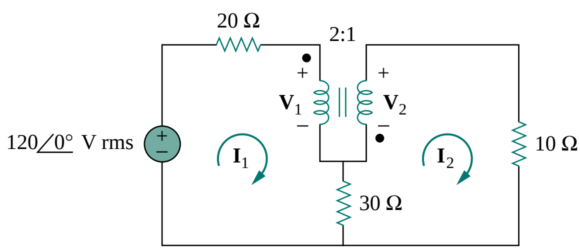


Figure 13.39 For Example 13.9.

Solution:

Reflection to the secondary or primary side cannot be done with this circuit: there is direct connection between the primary and secondary sides due to the 30-Ω resistor. We apply mesh analysis. For mesh 1,

$$-120 + (20 + 30)\mathbf{I}_1 - 30\mathbf{I}_2 + \mathbf{V}_1 = 0$$

or

$$50\mathbf{I}_1 - 30\mathbf{I}_2 + \mathbf{V}_1 = 120 \qquad (13.9.1)$$

For mesh 2,

$$-\mathbf{V}_2 + (10+30)\mathbf{I}_2 - 30\mathbf{I}_1 = 0$$

or

$$-30\mathbf{I}_1 + 40\mathbf{I}_2 - \mathbf{V}_2 = 0 \qquad (13.9.2)$$

At the transformer terminals,

$$\mathbf{V}_2 = -\frac{1}{2}\mathbf{V}_1 \qquad (13.9.3)$$

$$\mathbf{I}_2 = -2\mathbf{I}_1 \qquad (13.9.4)$$

(Note that $n = 1/2$.) We now have four equations and four unknowns, but our goal is to get $\mathbf{I}_2$. So we substitute for $\mathbf{V}_1$ and $\mathbf{I}_1$ in terms of $\mathbf{V}_2$ and $\mathbf{I}_2$ in Eqs. (13.9.1) and (13.9.2). Equation (13.9.1) becomes

$$-55\mathbf{I}_2 - 2\mathbf{V}_2 = 120 \qquad (13.9.5)$$

and Eq. (13.9.2) becomes

$$15\mathbf{I}_2 + 40\mathbf{I}_2 - \mathbf{V}_2 = 0 \quad \Longrightarrow \quad \mathbf{V}_2 = 55\mathbf{I}_2 \qquad (13.9.6)$$

Substituting Eq. (13.9.6) in Eq. (13.9.5),

$$-165\mathbf{I}_2 = 120 \quad \Longrightarrow \quad \mathbf{I}_2 = -\frac{120}{165} = -0.7272 \text{ A}$$

The power absorbed by the 10-Ω resistor is

$$P = (-0.7272)^2(10) = 5.3 \text{ W}$$

PRACTICE PROBLEM 13.9

Find $\mathbf{V}_o$ in the circuit in Fig. 13.40.

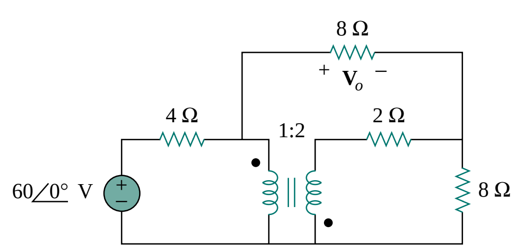


Figure 13.40 For Practice Prob. 13.9.

Answer: 24 V.

13.6 IDEAL AUTOTRANSFORMERS

Unlike the conventional two-winding transformer we have considered so far, an *autotransformer* has a single continuous winding with a connection point called a *tap* between the primary and secondary sides. The tap is

often adjustable so as to provide the desired turns ratio for stepping up or stepping down the voltage. This way, a variable voltage is provided to the load connected to the autotransformer.

An autotransformer is a transformer in which both the primary and the secondary are in a single winding.

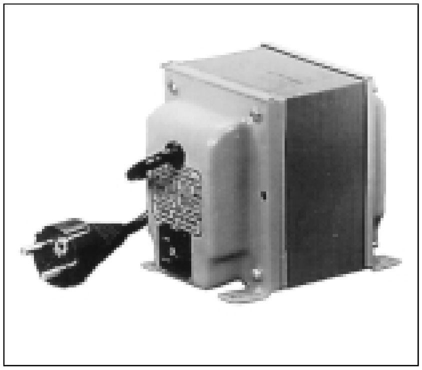


Figure 13.41 A typical autotransformer. *(Courtesy of Todd Systems, Inc.)*

Figure 13.41 shows a typical autotransformer. As shown in Fig. 13.42, the autotransformer can operate in the step-down or step-up mode. The autotransformer is a type of power transformer. Its major advantage over the two-winding transformer is its ability to transfer larger apparent power. Example 13.10 will demonstrate this. Another advantage is that an autotransformer is smaller and lighter than an equivalent two-winding transformer. However, since both the primary and secondary windings are one winding, *electrical isolation* (no direct electrical connection) is lost. (We will see how the property of electrical isolation in the conventional transformer is practically employed in Section 13.9.1.) The lack of electrical isolation between the primary and secondary windings is a major disadvantage of the autotransformer.

Some of the formulas we derived for ideal transformers apply to ideal autotransformers as well. For the step-down autotransformer circuit of Fig. 13.42(a), Eq. (13.52) gives

$$\boxed{\frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{N_1 + N_2}{N_2} = 1 + \frac{N_1}{N_2}} \tag{13.63}$$

As an ideal autotransformer, there are no losses, so the complex power remains the same in the primary and secondary windings:

$$\mathbf{S}_1 = \mathbf{V}_1\mathbf{I}_1^* = \mathbf{S}_2 = \mathbf{V}_2\mathbf{I}_2^* \tag{13.64}$$

Equation (13.64) can also be expressed with rms values as

$$V_1 I_1 = V_2 I_2$$

or

$$\frac{V_2}{V_1} = \frac{I_1}{I_2} \tag{13.65}$$

Thus, the current relationship is

$$\frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{N_2}{N_1 + N_2} \tag{13.66}$$

For the step-up autotransformer circuit of Fig. 13.42(b),

$$\frac{\mathbf{V}_1}{N_1} = \frac{\mathbf{V}_2}{N_1 + N_2}$$

or

$$\boxed{\frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{N_1}{N_1 + N_2}} \tag{13.67}$$

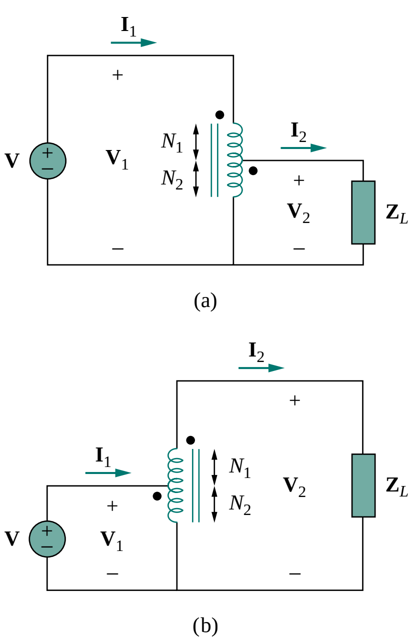


Figure 13.42 (a) Step-down autotransformer, (b) step-up autotransformer.

The complex power given by Eq. (13.64) also applies to the step-up autotransformer so that Eq. (13.65) again applies. Hence, the current relationship is

$$\frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{N_1 + N_2}{N_1} = 1 + \frac{N_2}{N_1} \tag{13.68}$$

A major difference between conventional transformers and autotransformers is that the primary and secondary sides of the autotransformer are not only coupled magnetically but also coupled conductively. The autotransformer can be used in place of a conventional transformer when electrical isolation is not required.

EXAMPLE 13.10

Compare the power ratings of the two-winding transformer in Fig. 13.43(a) and the autotransformer in Fig. 13.43(b).

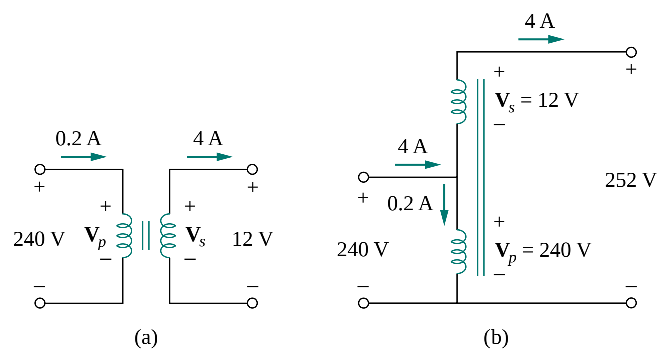


Figure 13.43 For Example 13.10.

Solution:

Although the primary and secondary windings of the autotransformer are together as a continuous winding, they are separated in Fig. 13.43(b) for clarity. We note that the current and voltage of each winding of the autotransformer in Fig. 13.43(b) are the same as those for the two-winding transformer in Fig. 13.43(a). This is the basis of comparing their power ratings.

For the two-winding transformer, the power rating is

$$S_1 = 0.2(240) = 48 \text{ VA} \qquad \text{or} \qquad S_2 = 4(12) = 48 \text{ VA}$$

For the autotransformer, the power rating is

$$S_1 = 4.2(240) = 1008 \text{ VA} \qquad \text{or} \qquad S_2 = 4(252) = 1008 \text{ VA}$$

which is 21 times the power rating of the two-winding transformer.

PRACTICE PROBLEM 13.10

Refer to Fig. 13.43. If the two-winding transformer is a 60-VA, 120 V/10 V transformer, what is the power rating of the autotransformer?

Answer: 780 VA.

EXAMPLE 13.11

Refer to the autotransformer circuit in Fig. 13.44. Calculate: (a) $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_o$ if $\mathbf{Z}_L = 8 + j6\ \Omega$, and (b) the complex power supplied to the load.

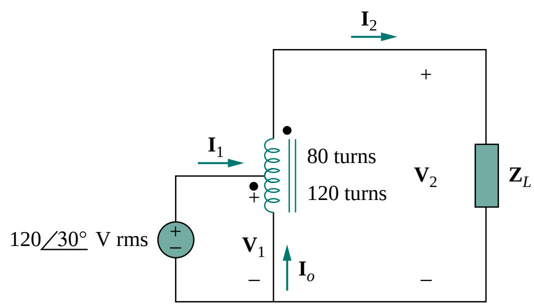


Figure 13.44 For Example 13.11.

Solution:

(a) This is a step-up autotransformer with $N_1 = 80$, $N_2 = 120$, $\mathbf{V}_1 = 120\underline{/30^\circ}$, so Eq. (13.67) can be used to find $\mathbf{V}_2$ by

$$\frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{N_1}{N_1 + N_2} = \frac{80}{200}$$

or

$$\mathbf{V}_2 = \frac{200}{80}\mathbf{V}_1 = \frac{200}{80}(120\underline{/30^\circ}) = 300\underline{/30^\circ}\ \text{V}$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_2}{\mathbf{Z}_L} = \frac{300\underline{/30^\circ}}{8 + j6} = \frac{300\underline{/30^\circ}}{10\underline{/36.87^\circ}} = 30\underline{/-6.87^\circ}\ \text{A}$$

But

$$\frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{N_1 + N_2}{N_1} = \frac{200}{80}$$

or

$$\mathbf{I}_1 = \frac{200}{80}\mathbf{I}_2 = \frac{200}{80}(30\underline{/-6.87^\circ}) = 75\underline{/-6.87^\circ}\ \text{A}$$

At the tap, KCL gives

$$\mathbf{I}_1 + \mathbf{I}_o = \mathbf{I}_2$$

or

$$\mathbf{I}_o = \mathbf{I}_2 - \mathbf{I}_1 = 30\underline{/-6.87^\circ} - 75\underline{/-6.87^\circ} = 45\underline{/173.13^\circ}\ \text{A}$$

(b) The complex power supplied to the load is

$$\mathbf{S}_2 = \mathbf{V}_2\mathbf{I}_2^* = |\mathbf{I}_2|^2\mathbf{Z}_L = (30)^2(10\underline{/36.87^\circ}) = 9\underline{/36.87^\circ}\ \text{kVA}$$

PRACTICE PROBLEM 13.11

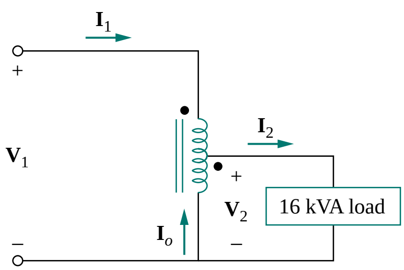


Figure 13.45 For Practice Prob. 13.11.

In the autotransformer circuit in Fig. 13.45, find currents I_1, I_2, and I_o. Take $V_1 = 1250$ V, $V_2 = 800$ V.

Answer: 12.8 A, 20 A, 7.2 A.

†13.7 THREE-PHASE TRANSFORMERS

To meet the demand for three-phase power transmission, transformer connections compatible with three-phase operations are needed. We can achieve the transformer connections in two ways: by connecting three single-phase transformers, thereby forming a so-called *transformer bank*, or by using a special three-phase transformer. For the same kVA rating, a three-phase transformer is always smaller and cheaper than three single-phase transformers. When single-phase transformers are used, one must ensure that they have the same turns ratio n to achieve a balanced three-phase system. There are four standard ways of connecting three single-phase transformers or a three-phase transformer for three-phase operations: Y-Y, Δ-Δ, Y-Δ, and Δ-Y.

For any of the four connections, the total apparent power S_T, real power P_T, and reactive power Q_T are obtained as

$$S_T = \sqrt{3} V_L I_L \tag{13.69a}$$

$$P_T = S_T \cos\theta = \sqrt{3} V_L I_L \cos\theta \tag{13.69b}$$

$$Q_T = S_T \sin\theta = \sqrt{3} V_L I_L \sin\theta \tag{13.69c}$$

where V_L and I_L are, respectively, equal to the line voltage V_{Lp} and the line current I_{Lp} for the primary side, or the line voltage V_{Ls} and the line current I_{Ls} for the secondary side. Notice from Eq. (13.69) that for each of the four connections, $V_{Ls} I_{Ls} = V_{Lp} I_{Lp}$, since power must be conserved in an ideal transformer.

For the Y-Y connection (Fig. 13.46), the line voltage V_{Lp} at the primary side, the line voltage V_{Ls} on the secondary side, the line current I_{Lp} on the primary side, and the line current I_{Ls} on the secondary side are related to the transformer per phase turns ratio n according to Eqs. (13.52) and (13.55) as

$$V_{Ls} = n V_{Lp} \tag{13.70a}$$

$$I_{Ls} = \frac{I_{Lp}}{n} \tag{13.70b}$$

For the Δ-Δ connection (Fig. 13.47), Eq. (13.70) also applies for the line voltages and line currents. This connection is unique in the sense that if one of the transformers is removed for repair or maintenance, the other two form an *open delta*, which can provide three-phase voltages at a reduced level of the original three-phase transformer.

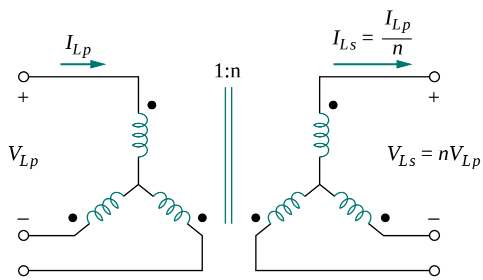


Figure 13.46 Y-Y three-phase transformer connection.

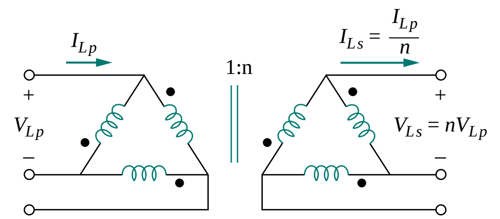


Figure 13.47 Δ-Δ three-phase transformer connection.

For the Y-Δ connection (Fig. 13.48), there is a factor of $\sqrt{3}$ arising from the line-phase values in addition to the transformer per phase turns ratio n. Thus,

$$V_{Ls} = \frac{nV_{Lp}}{\sqrt{3}} \tag{13.71a}$$

$$I_{Ls} = \frac{\sqrt{3}I_{Lp}}{n} \tag{13.71b}$$

Similarly, for the Δ-Y connection (Fig. 13.49),

$$V_{Ls} = n\sqrt{3}V_{Lp} \tag{13.72a}$$

$$I_{Ls} = \frac{I_{Lp}}{n\sqrt{3}} \tag{13.72b}$$

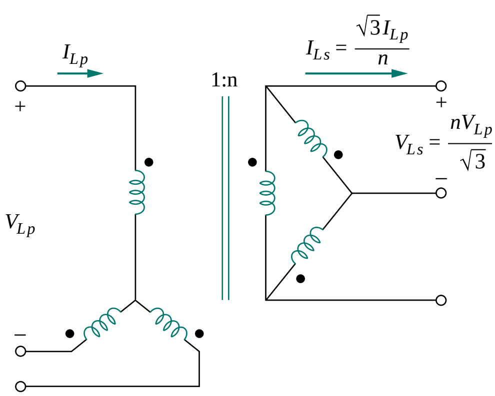


Figure 13.48 Y-Δ three-phase transformer connection.

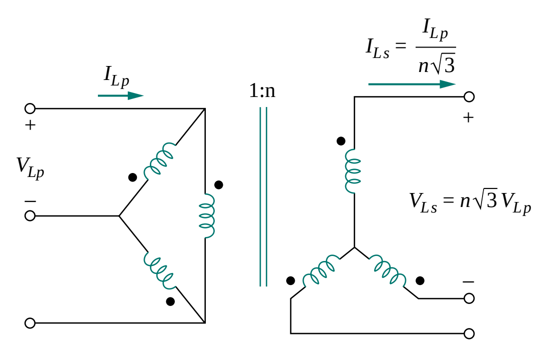


Figure 13.49 Δ-Y three-phase transformer connection.

EXAMPLE 13.12

The 42-kVA balanced load depicted in Fig. 13.50 is supplied by a three-phase transformer. (a) Determine the type of transformer connections. (b) Find the line voltage and current on the primary side. (c) Determine the kVA rating of each transformer used in the transformer bank. Assume that the transformers are ideal.

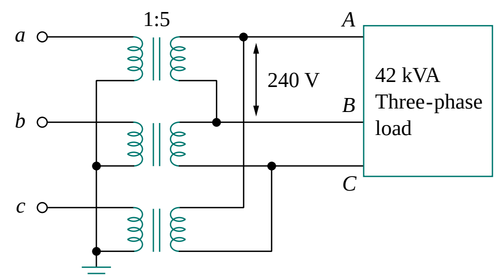


Figure 13.50 For Example 13.12.

Solution:

(a) A careful observation of Fig. 13.50 shows that the primary side is Y-connected, while the secondary side is Δ-connected. Thus, the three-phase transformer is Y-Δ, similar to the one shown in Fig. 13.48.

(b) Given a load with total apparent power $S_T = 42$ kVA, the turns ratio $n = 5$, and the secondary line voltage $V_{Ls} = 240$ V, we can find the secondary line current using Eq. (13.69a), by

$$I_{Ls} = \frac{S_T}{\sqrt{3}V_{Ls}} = \frac{42{,}000}{\sqrt{3}(240)} = 101 \text{ A}$$

From Eq. (13.71),

$$I_{Lp} = \frac{n}{\sqrt{3}} I_{Ls} = \frac{5 \times 101}{\sqrt{3}} = 292 \text{ A}$$

$$V_{Lp} = \frac{\sqrt{3}}{n} V_{Ls} = \frac{\sqrt{3} \times 240}{5} = 83.14 \text{ V}$$

(c) Because the load is balanced, each transformer equally shares the total load and since there are no losses (assuming ideal transformers), the kVA rating of each transformer is $S = S_T/3 = 14$ kVA. Alternatively, the transformer rating can be determined by the product of the phase current and phase voltage of the primary or secondary side. For the primary side, for example, we have a delta connection, so that the phase voltage is the same as the line voltage of 240 V, while the phase current is $I_{Lp}/\sqrt{3} = 58.34$ A. Hence, $S = 240 \times 58.34 = 14$ kVA.

PRACTICE PROBLEM 13.12

A three-phase Δ-Δ transformer is used to step down a line voltage of 625 kV, to supply a plant operating at a line voltage of 12.5 kV. The plant

draws 40 MW with a lagging power factor of 85 percent. Find: (a) the current drawn by the plant, (b) the turns ratio, (c) the current on the primary side of the transformer, and (d) the load carried by each transformer.

Answer: (a) 2.1736 kA, (b) 0.02, (c) 43.47 A, (d) 15.69 MVA.

13.8 *PSPICE* ANALYSIS OF MAGNETICALLY COUPLED CIRCUITS

PSpice analyzes magnetically coupled circuits just like inductor circuits except that the dot convention must be followed. In *PSpice* Schematic, the dot (not shown) is always next to pin 1, which is the left-hand terminal of the inductor when the inductor with part name L is placed (horizontally) without rotation on a schematic. Thus, the dot or pin 1 will be at the bottom after one 90° counterclockwise rotation, since rotation is always about pin 1. Once the magnetically coupled inductors are arranged with the dot convention in mind and their value attributes are set in henries, we use the coupling symbol K_LINEAR to define the coupling. For each pair of coupled inductors, take the following steps:

1. Select **Draw/Get New Part** and type K_LINEAR.
2. Hit ⟨enter⟩ or click **OK** and place the K_LINEAR symbol on the schematic, as shown in Fig. 13.51. (Notice that K_LINEAR is not a component and therefore has no pins.)
3. **DCLICKL** on COUPLING and set the value of the coupling coefficient k.
4. **DCLICKL** on the boxed K (the coupling symbol) and enter the reference designator names for the coupled inductors as values of Li, i = 1, 2, . . . , 6. For example, if inductors L20 and L23 are coupled, we set L1 = L20 and L2 = L23. L1 and at least one other Li must be assigned values; other Li's may be left blank.

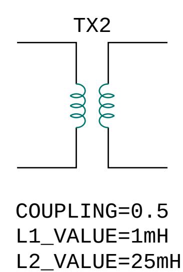


Figure 13.51 Linear transformer XFRM_LINEAR.

In step 4, up to six coupled inductors with equal coupling can be specified.

For the air-core transformer, the partname is XFRM_LINEAR. It can be inserted in a circuit by selecting **Draw/Get Part Name** and then typing in the part name or by selecting the part name from the analog.slb library. As shown typically in Fig. 13.51, the main attributes of the linear transformer are the coupling coefficient k and the inductance values L1 and L2 in henries. If the mutual inductance M is specified, its value must be used along with L1 and L2 to calculate k. Keep in mind that the value of k should lie between 0 and 1.

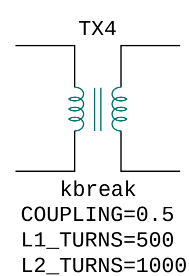


Figure 13.52 Ideal transformer XFRM_NONLINEAR.

For the ideal transformer, the part name is XFRM_NONLINEAR and is located in the breakout.slb library. Select it by clicking **Draw/Get Part Name** and then typing in the part name. Its attributes are the coupling coefficient and the numbers of turns associated with L1 and L2, as illustrated typically in Fig. 13.52. The value of the coefficient of mutual coupling must lie between 0 and 1.

PSpice has some additional transformer configurations that we will not discuss here.

EXAMPLE 13.13

Use *PSpice* to find i_1, i_2, and i_3 in the circuit displayed in Fig. 13.53.

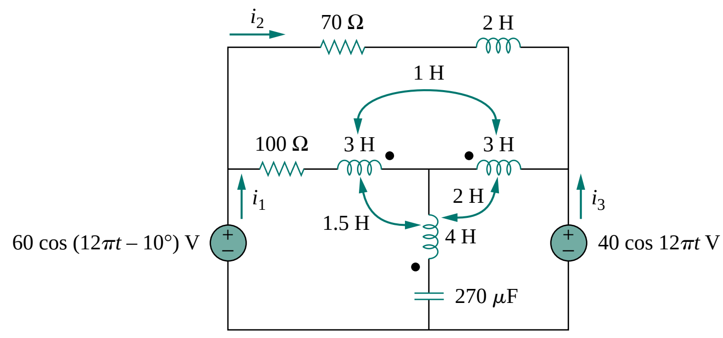


Figure 13.53 For Example 13.13.

Solution:

The coupling coefficients of the three coupled inductors are determined as follows.

$$k_{12} = \frac{M_{12}}{\sqrt{L_1 L_2}} = \frac{1}{\sqrt{3 \times 3}} = 0.3333$$

$$k_{13} = \frac{M_{13}}{\sqrt{L_1 L_3}} = \frac{1.5}{\sqrt{3 \times 4}} = 0.433$$

$$k_{23} = \frac{M_{23}}{\sqrt{L_2 L_3}} = \frac{2}{\sqrt{3 \times 4}} = 0.5774$$

The operating frequency f is obtained from Fig. 13.53 as $\omega = 12\pi = 2\pi f \rightarrow f = 6$ Hz.

The schematic of the circuit is portrayed in Fig. 13.54. Notice how the dot convention is adhered to. For L2, the dot (not shown) is on pin 1 (the left-hand terminal) and is therefore placed without rotation. For L1, in order for the dot to be on the right-hand side of the inductor, the inductor must be rotated through 180°. For L3, the inductor must be rotated through 90° so that the dot will be at the bottom. Note that the 2-H inductor (L_4) is not coupled. To handle the three coupled inductors, we use three K_LINEAR parts provided in the analog library and set the following attributes (by double-clicking on the symbol K in the box):

The right-hand values are the reference designators of the inductors on the schematic.

```
K1 - K_LINEAR
L1 = L1
L2 = L2
COUPLING = 0.3333

K2 - K_LINEAR
L1 = L2
L2 = L3
COUPLING = 0.433
```

```
K3 - K_LINEAR
L1 = L1
L2 = L3
COUPLING = 0.5774
```

Three IPRINT pseudocomponents are inserted in the appropriate branches to obtain the required currents i_1, i_2, and i_3. As an AC single-frequency analysis, we select **Analysis/Setup/AC Sweep** and enter *Total Pts* = 1, *Start Freq* = 6, and *Final Freq* = 6. After saving the schematic, we select **Analysis/Simulate** to simulate it. The output file includes:

```
FREQ         IM(V_PRINT2)  IP(V_PRINT2)
6.000E+00    2.114E-01     -7.575E+01
FREQ         IM(V_PRINT1)  IP(V_PRINT1)
6.000E+00    4.654E-01     -7.025E+01
FREQ         IM(V_PRINT3)  IP(V_PRINT3)
6.000E+00    1.095E-01      1.715E+01
```

From this we obtain

$$\mathbf{I}_1 = 0.4654\underline{/-70.25^\circ}$$

$$\mathbf{I}_2 = 0.2114\underline{/-75.75^\circ}, \qquad \mathbf{I}_3 = 0.1095\underline{/17.15^\circ}$$

Thus,

$$i_1 = 0.4654\cos(12\pi t - 70.25^\circ)\text{ A}$$
$$i_2 = 0.2114\cos(12\pi t - 75.75^\circ)\text{ A}$$
$$i_3 = 0.1095\cos(12\pi t + 17.15^\circ)\text{ A}$$

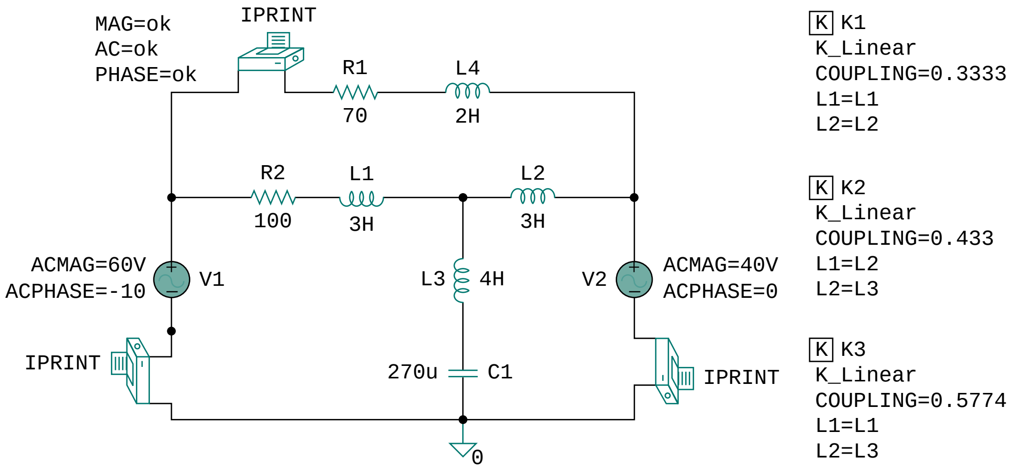


Figure 13.54 Schematic of the circuit of Fig. 13.53.

PRACTICE PROBLEM 13.13

Find i_o in the circuit of Fig. 13.55.

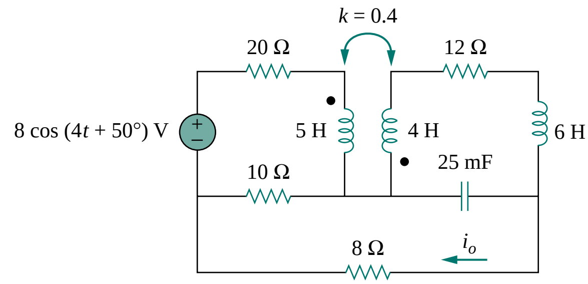


Figure 13.55 For Practice Prob. 13.13.

Answer: $0.1006\cos(4t + 68.52^\circ)$ A.

EXAMPLE 13.14

Find $\mathbf{V}_1$ and $\mathbf{V}_2$ in the ideal transformer circuit of Fig. 13.56 using *PSpice*.

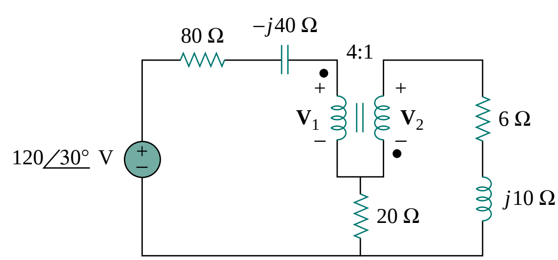


Figure 13.56 For Example 13.14.

Solution:

As usual, we assume $\omega = 1$ and find the corresponding values of capacitance and inductance of the elements:

$$j10 = j\omega L \qquad \Longrightarrow \qquad L = 10 \text{ H}$$

$$-j40 = \frac{1}{j\omega C} \qquad \Longrightarrow \qquad C = 25 \text{ mF}$$

Reminder: For an ideal transformer, the inductances of both the primary and secondary windings are infinitely large.

Figure 13.57 shows the schematic. For the ideal transformer, we set the coupling factor to 0.999 and the numbers of turns to 400,000 and 100,000. The two VPRINT2 pseudocomponents are connected across the transformer terminals to obtain $\mathbf{V}_1$ and $\mathbf{V}_2$. As a single-frequency analysis, we select **Analysis/Setup/AC Sweep** and enter *Total Pts* = 1, *Start Freq* = 0.1592, and *Final Freq* = 0.1592. After saving the schematic, we select **Analysis/Simulate** to simulate it. The output file includes:

```
FREQ        VM(C,A)     VP(C,A)
1.592E-01   1.212E+02   -1.435E+02
```

```
FREQ        VM(B,C)     VP(B,C)
1.592E-01   2.775E+02   2.789E+01
```

From this we obtain

$$\mathbf{V}_1 = -\mathtt{V(C,A)} = 121.1\underline{/36.5^\circ} \text{ V}$$

$$\mathbf{V}_2 = \mathtt{V(B,C)} = 27.75\underline{/27.89^\circ} \text{ V}$$

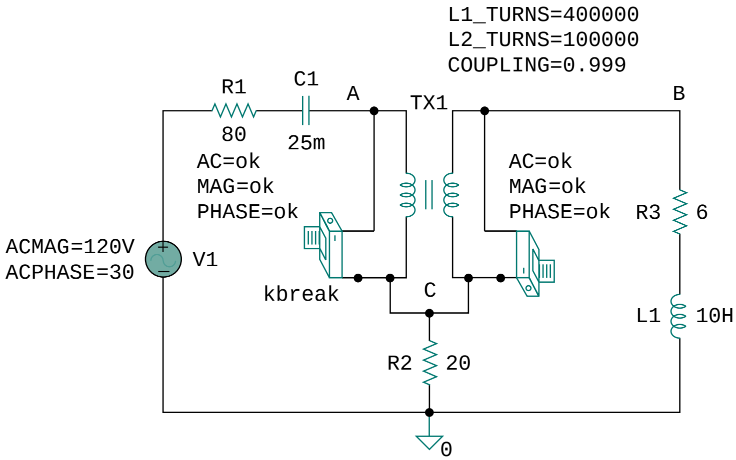


Figure 13.57 The schematic for the circuit in Fig. 13.56.

PRACTICE PROBLEM 13.14

Obtain $\mathbf{V}_1$ and $\mathbf{V}_2$ in the circuit of Fig. 13.58 using *PSpice*.

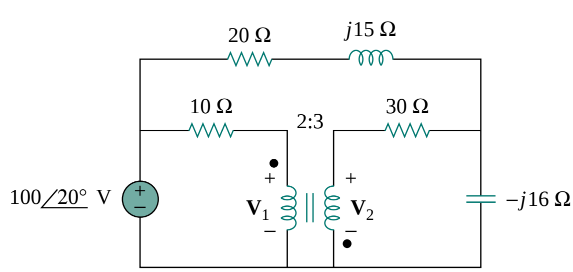


Figure 13.58 For Practice Prob. 13.14.

Answer: $63.1\underline{/28.65^\circ}$ V, $94.64\underline{/-151.4^\circ}$ V.

†13.9 APPLICATIONS

Transformers are the largest, the heaviest, and often the costliest of circuit components. Nevertheless, they are indispensable passive devices in

electric circuits. They are among the most efficient machines, 95 percent efficiency being common and 99 percent being achievable. They have numerous applications. For example, transformers are used:

- To step up or step down voltage and current, making them useful for power transmission and distribution.
- To isolate one portion of a circuit from another (i.e., to transfer power without any electrical connection).
- As an impedance-matching device for maximum power transfer.
- In frequency-selective circuits whose operation depends on the response of inductances.

Because of these diverse uses, there are many special designs for transformers (only some of which are discussed in this chapter): voltage transformers, current transformers, power transformers, distribution transformers, impedance-matching transformers, audio transformers, single-phase transformers, three-phase transformers, rectifier transformers, inverter transformers, and more. In this section, we consider three important applications: transformer as an isolation device, transformer as a matching device, and power distribution system.

For more information on the many kinds of transformers, a good text is W. M. Flanagan, *Handbook of Transformer Design and Applications*, 2nd ed. (New York: McGraw-Hill, 1993).

13.9.1 Transformer as an Isolation Device

Electrical isolation is said to exist between two devices when there is no physical connection between them. In a transformer, energy is transferred by magnetic coupling, without electrical connection between the primary circuit and secondary circuit. We now consider three simple practical examples of how we take advantage of this property.

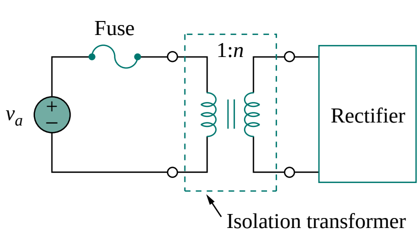


Figure 13.59 A transformer used to isolate an ac supply from a rectifier.

First, consider the circuit in Fig. 13.59. A rectifier is an electronic circuit that converts an ac supply to a dc supply. A transformer is often used to couple the ac supply to the rectifier. The transformer serves two purposes. First, it steps up or steps down the voltage. Second, it provides electrical isolation between the ac power supply and the rectifier, thereby reducing the risk of shock hazard in handling the electronic device.

As a second example, a transformer is often used to couple two stages of an amplifier, to prevent any dc voltage in one stage from affecting the dc bias of the next stage. Biasing is the application of a dc voltage to a transistor amplifier or any other electronic device in order to produce a desired mode of operation. Each amplifier stage is biased separately to operate in a particular mode; the desired mode of operation will be compromised without a transformer providing dc isolation. As shown in Fig. 13.60, only the ac signal is coupled through the transformer from one stage to the next. We recall that magnetic coupling does not exist with a dc voltage source. Transformers are used in radio and TV receivers to couple stages of high-frequency amplifiers. When the sole purpose of a transformer is to provide isolation, its turns ratio n is made unity. Thus, an isolation transformer has $n = 1$.

As a third example, consider measuring the voltage across 13.2-kV lines. It is obviously not safe to connect a voltmeter directly to such high-voltage lines. A transformer can be used both to electrically isolate the line power from the voltmeter and to step down the voltage to a safe level, as shown in Fig. 13.61. Once the voltmeter is used to measure the

secondary voltage, the turns ratio is used to determine the line voltage on the primary side.

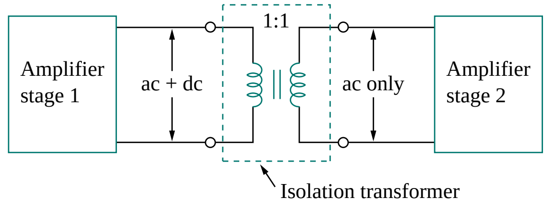


Figure 13.60 A transformer providing dc isolation between two amplifier stages.

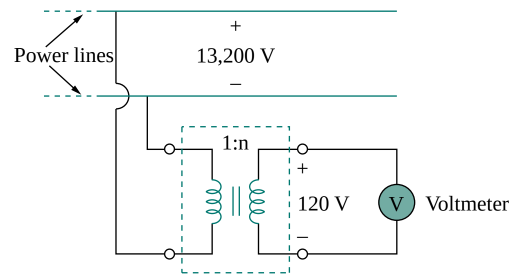


Figure 13.61 A transformer providing isolation between the power lines and the voltmeter.

EXAMPLE 13.15

Determine the voltage across the load in Fig. 13.62.

Solution:

We can apply the superposition principle to find the load voltage. Let $v_L = v_{L1} + v_{L2}$, where v_{L1} is due to the dc source and v_{L2} is due to the ac source. We consider the dc and ac sources separately, as shown in Fig. 13.63. The load voltage due to the dc source is zero, because a time-varying voltage is necessary in the primary circuit to induce a voltage in the secondary circuit. Thus, $v_{L1} = 0$. For the ac source,

$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{\mathbf{V}_2}{120} = \frac{1}{3} \qquad \text{or} \qquad \mathbf{V}_2 = \frac{120}{3} = 40 \text{ V}$$

Hence, $\mathbf{V}_{L2} = 40$ V ac or $v_{L2} = 40 \cos \omega t$; that is, only the ac voltage is passed to the load by the transformer. This example shows how the transformer provides dc isolation.

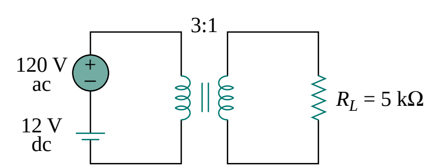


Figure 13.62 For Example 13.15.

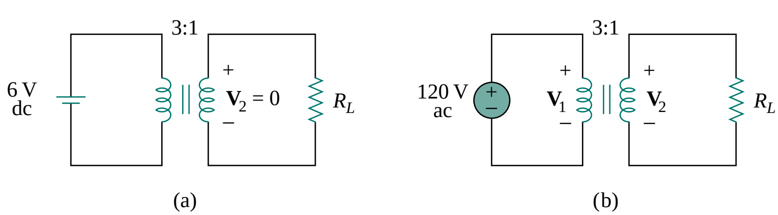


Figure 13.63 For Example 13.15: (a) dc source, (b) ac source.

PRACTICE PROBLEM 13.15

Refer to Fig. 13.61. Calculate the turns ratio required to step down the 13.2-kV line voltage to a safe level of 120 V.

Answer: 1/110.

13.9.2 Transformer as a Matching Device

We recall that for maximum power transfer, the load resistance R_L must be matched with the source resistance R_s. In most cases, the two resistances are not matched; both are fixed and cannot be altered. However, an iron-core transformer can be used to match the load resistance to the source resistance. This is called *impedance matching*. For example, to connect a loudspeaker to an audio power amplifier requires a transformer, because the speaker's resistance is only a few ohms while the internal resistance of the amplifier is several thousand ohms.

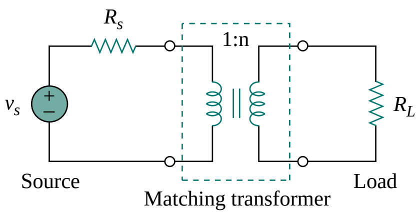


Figure 13.64 Transformer used as a matching device.

Consider the circuit shown in Fig. 13.64. We recall from Eq. (13.60) that the ideal transformer reflects its load back to the primary with a scaling factor of n^2. To match this reflected load R_L/n^2 with the source resistance R_s, we set them equal,

$$R_s = \frac{R_L}{n^2} \tag{13.73}$$

Equation (13.73) can be satisfied by proper selection of the turns ratio n. From Eq. (13.73), we notice that a step-down transformer ($n < 1$) is needed as the matching device when $R_s > R_L$, and a step-up ($n > 1$) is required when $R_s < R_L$.

EXAMPLE 13.16

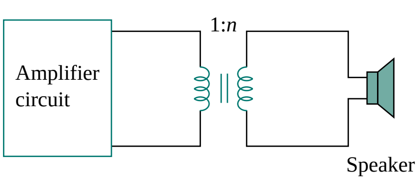


Figure 13.65 Using an ideal transformer to match the speaker to the amplifier; for Example 13.16.

The ideal transformer in Fig. 13.65 is used to match the amplifier circuit to the loudspeaker to achieve maximum power transfer. The Thevenin (or output) impedance of the amplifier is 192 Ω, and the internal impedance of the speaker is 12 Ω. Determine the required turns ratio.

Solution:

We replace the amplifier circuit with the Thevenin equivalent and reflect the impedance $\mathbf{Z}_L = 12\ \Omega$ of the speaker to the primary side of the ideal transformer. Figure 13.66 shows the result. For maximum power transfer,

$$\mathbf{Z}_{\text{Th}} = \frac{\mathbf{Z}_L}{n^2} \qquad \text{or} \qquad n^2 = \frac{\mathbf{Z}_L}{\mathbf{Z}_{\text{Th}}} = \frac{12}{192} = \frac{1}{16}$$

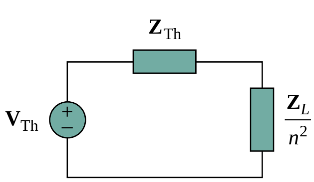


Figure 13.66 Equivalent circuit of the circuit in Fig. 13.65, for Example 13.16.

Thus, the turns ratio is $n = 1/4 = 0.25$.

Using $P = I^2R$, we can show that indeed the power delivered to the speaker is much larger than without the ideal transformer. Without the ideal transformer, the amplifier is directly connected to the speaker. The power delivered to the speaker is

$$P_L = \left(\frac{\mathbf{V}_{\text{Th}}}{\mathbf{Z}_{\text{Th}} + \mathbf{Z}_L}\right)^2 \mathbf{Z}_L = 288\ \mathbf{V}_{\text{Th}}^2\ \mu\text{W}$$

With the transformer in place, the primary and secondary currents are

$$I_p = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{Z}_{\text{Th}} + \mathbf{Z}_L/n^2}, \qquad I_s = \frac{I_p}{n}$$

Hence,

$$P_L = I_s^2 Z_L = \left(\frac{\mathbf{V}_{\mathrm{Th}}/n}{\mathbf{Z}_{\mathrm{Th}} + \mathbf{Z}_L/n^2}\right)^2 \mathbf{Z}_L$$

$$= \left(\frac{n\mathbf{V}_{\mathrm{Th}}}{n^2\mathbf{Z}_{\mathrm{Th}} + \mathbf{Z}_L}\right)^2 \mathbf{Z}_L = 1302\mathbf{V}_{\mathrm{Th}}^2\ \mu\mathrm{W}$$

confirming what was said earlier.

PRACTICE PROBLEM 13.16

Calculate the turns ratio of an ideal transformer required to match a 100-Ω load to a source with internal impedance of 2.5 kΩ. Find the load voltage when the source voltage is 30 V.

Answer: 0.2, 3 V.

13.9.3 Power Distribution

A power system basically consists of three components: generation, transmission, and distribution. The local electric company operates a plant that generates several hundreds of megavolt-amperes (MVA), typically at about 18 kV. As Fig. 13.67 illustrates, three-phase step-up transformers are used to feed the generated power to the transmission line. Why do we need the transformer? Suppose we need to transmit 100,000 VA over a distance of 50 km. Since $S = VI$, using a line voltage of 1000 V implies that the transmission line must carry 100 A and this requires a transmission line of a large diameter. If, on the other hand, we use a line voltage of 10,000 V, the current is only 10 A. The smaller current reduces the required conductor size, producing considerable savings as well as minimizing transmission line I^2R losses. To minimize losses requires a step-up transformer. Without the transformer, the majority of the power generated would be lost on the transmission line. The ability

One may ask, How would increasing the voltage not increase the current, thereby increasing I^2R losses? Keep in mind that $I = V_\ell/R$, where V_ℓ is the potential difference between the sending and receiving ends of the line. The voltage that is stepped up is the sending end voltage V, not V_ℓ. If the receiving end is V_R, then $V_\ell = V - V_R$. Since V and V_R are close to each other, V_ℓ is small even when V is stepped up.

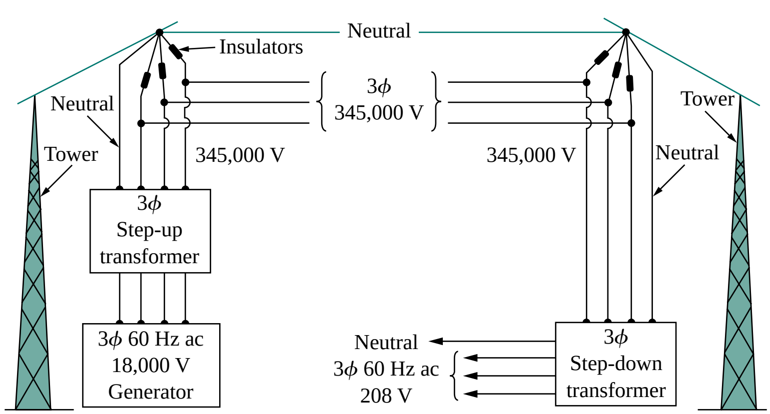


Figure 13.67 A typical power distribution system.
(Source: A. Marcus and C. M. Thomson, Electricity for Technicians, 2nd ed. [Englewood Cliffs, NJ: Prentice Hall, 1975], p. 337.)

of the transformer to step up or step down voltage and distribute power economically is one of the major reasons for generating ac rather than dc. Thus, for a given power, the larger the voltage, the better. Today, 1 MV is the largest voltage in use; the level may increase as a result of research and experiments.

Beyond the generation plant, the power is transmitted for hundreds of miles through an electric network called the *power grid*. The three-phase power in the power grid is conveyed by transmission lines hung overhead from steel towers which come in a variety of sizes and shapes. The (aluminum-conductor, steel-reinforced) lines typically have overall diameters up to about 40 mm and can carry current of up to 1380 A.

At the substations, distribution transformers are used to step down the voltage. The step-down process is usually carried out in stages. Power may be distributed throughout a locality by means of either overhead or underground cables. The substations distribute the power to residential, commercial, and industrial customers. At the receiving end, a residential customer is eventually supplied with 120/240 V, while industrial or commercial customers are fed with higher voltages such as 460/208 V. Residential customers are usually supplied by distribution transformers often mounted on the poles of the electric utility company. When direct current is needed, the alternating current is converted to dc electronically.

EXAMPLE 13.17

A distribution transformer is used to supply a household as in Fig. 13.68. The load consists of eight 100-W bulbs, a 350-W TV, and a 15-kW kitchen range. If the secondary side of the transformer has 72 turns, calculate: (a) the number of turns of the primary winding, and (b) the current I_p in the primary winding.

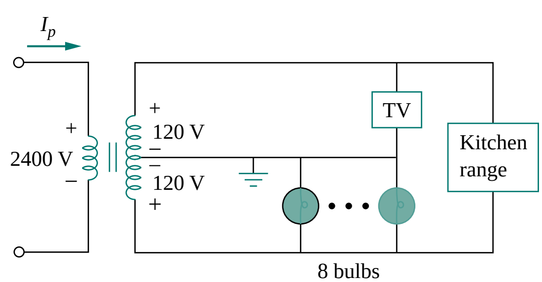


Figure 13.68 For Example 13.17.

Solution:

(a) The dot locations on the winding are not important, since we are only interested in the magnitudes of the variables involved. Since

$$\frac{N_p}{N_s} = \frac{V_p}{V_s}$$

we get

$$N_p = N_s \frac{V_p}{V_s} = 72\frac{2400}{240} = 720 \text{ turns}$$

(b) The total power absorbed by the load is

$$S = 8 \times 100 + 350 + 15{,}000 = 16.15 \text{ kW}$$

But $S = V_p I_p = V_s I_s$, so that

$$I_p = \frac{S}{V_p} = \frac{16{,}150}{2400} = 6.729 \text{ A}$$

PRACTICE PROBLEM 13.17

In Example 13.17, if the eight 100-W bulbs are replaced by twelve 60-W bulbs and the kitchen range is replaced by a 4.5-kW air-conditioner, find: (a) the total power supplied, (b) the current I_p in the primary winding.

Answer: (a) 5.57 kW, (b) 2.321 A.

13.10 SUMMARY

1. Two coils are said to be mutually coupled if the magnetic flux ϕ emanating from one passes through the other. The mutual inductance between the two coils is given by

$$M = k\sqrt{L_1 L_2}$$

where k is the coupling coefficient, $0 < k < 1$.

2. If v_1 and i_1 are the voltage and current in coil 1, while v_2 and i_2 are the voltage and current in coil 2, then

$$v_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt} \qquad \text{and} \qquad v_2 = L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$$

Thus, the voltage induced in a coupled coil consists of self-induced voltage and mutual voltage.

3. The polarity of the mutually induced voltage is expressed in the schematic by the dot convention.

4. The energy stored in two coupled coils is

$$\frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 \pm M i_1 i_2$$

5. A transformer is a four-terminal device containing two or more magnetically coupled coils. It is used in changing the current, voltage, or impedance level in a circuit.

6. A linear (or loosely coupled) transformer has its coils wound on a magnetically linear material. It can be replaced by an equivalent T or Π network for the purposes of analysis.

7. An ideal (or iron-core) transformer is a lossless ($R_1 = R_2 = 0$) transformer with unity coupling coefficient ($k = 1$) and infinite inductances ($L_1, L_2, M \to \infty$).

8. For an ideal transformer,

$$\mathbf{V}_2 = n\mathbf{V}_1, \qquad \mathbf{I}_2 = \frac{\mathbf{I}_1}{n}, \qquad \mathbf{S}_1 = \mathbf{S}_2, \qquad \mathbf{Z}_R = \frac{\mathbf{Z}_L}{n^2}$$

where $n = N_2/N_1$ is the turns ratio. N_1 is the number of turns of the primary winding and N_2 is the number of turns of the secondary winding. The transformer steps up the primary voltage when $n > 1$, steps it down when $n < 1$, or serves as a matching device when $n = 1$.

9. An autotransformer is a transformer with a single winding common to both the primary and the secondary circuits.

10. *PSpice* is a useful tool for analyzing magnetically coupled circuits.

11. Transformers are necessary in all stages of power distribution systems. Three-phase voltages may be stepped up or down by three-phase transformers.

12. Important uses of transformers in electronics applications are as electrical isolation devices and impedance-matching devices.

REVIEW QUESTIONS

13.1 Refer to the two magnetically coupled coils of Fig. 13.69(a). The polarity of the mutual voltage is:
(a) Positive (b) Negative

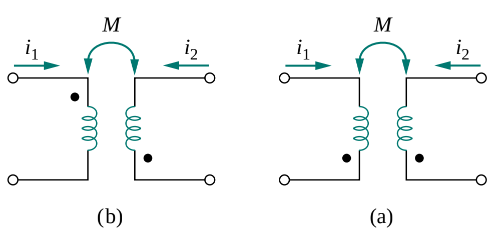


Figure 13.69 For Review Questions 13.1 and 13.2.

13.2 For the two magnetically coupled coils of Fig. 13.69(b), the polarity of the mutual voltage is:
(a) Positive (b) Negative

13.3 The coefficient of coupling for two coils having $L_1 = 2$ H, $L_2 = 8$ H, $M = 3$ H is:
(a) 0.1875 (b) 0.75
(c) 1.333 (d) 5.333

13.4 A transformer is used in stepping down or stepping up:
(a) dc voltages (b) ac voltages
(c) both dc and ac voltages

13.5 The ideal transformer in Fig. 13.70(a) has $N_2/N_1 = 10$. The ratio V_2/V_1 is:
(a) 10 (b) 0.1 (c) -0.1 (d) -10

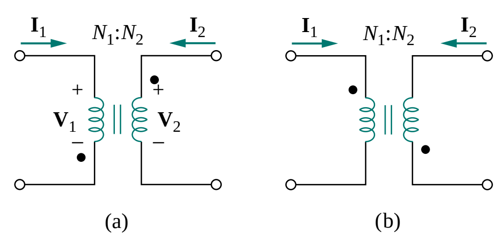


Figure 13.70 For Review Questions 13.5 and 13.6.

13.6 For the ideal transformer in Fig. 13.70(b), $N_2/N_1 = 10$. The ratio I_2/I_1 is:
(a) 10 (b) 0.1 (c) -0.1 (d) -10

13.7 A three-winding transformer is connected as portrayed in Fig. 13.71(a). The value of the output voltage V_o is:
(a) 10 (b) 6 (c) -6 (d) -10

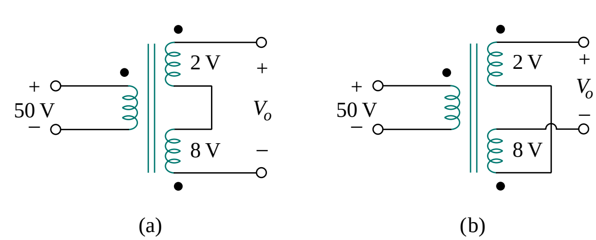


Figure 13.71 For Review Questions 13.7 and 13.8.

13.8 If the three-winding transformer is connected as in Fig. 13.71(b), the value of the output voltage V_o is:
(a) 10 (b) 6 (c) -6 (d) -10

13.9 In order to match a source with internal impedance of 500 Ω to a 15-Ω load, what is needed is:

(a) step-up linear transformer
(b) step-down linear transformer
(c) step-up ideal transformer
(d) step-down ideal transformer
(e) autotransformer

13.10 Which of these transformers can be used as an isolation device?

(a) linear transformer (b) ideal transformer
(c) autotransformer (d) all of the above

Answers: 13.1b, 13.2a, 13.3b, 13.4b, 13.5d, 13.6b, 13.7c, 13.8a, 13.9d, 13.10b.

PROBLEMS

Section 13.2 Mutual Inductance

13.1 For the three coupled coils in Fig. 13.72, calculate the total inductance.

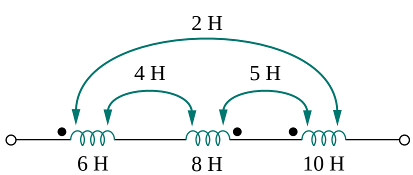


Figure 13.72 For Prob. 13.1.

13.2 Determine the inductance of the three series-connected inductors of Fig. 13.73.

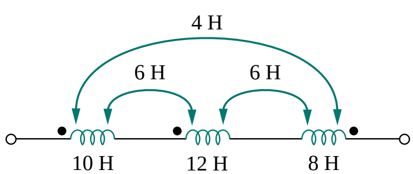


Figure 13.73 For Prob. 13.2.

13.3 Two coils connected in series-aiding fashion have a total inductance of 250 mH. When connected in a series-opposing configuration, the coils have a total inductance of 150 mH. If the inductance of one coil (L_1) is three times the other, find L_1, L_2, and M. What is the coupling coefficient?

13.4 (a) For the coupled coils in Fig. 13.74(a), show that

$$L_{eq} = L_1 + L_2 + 2M$$

(b) For the coupled coils in Fig. 13.74(b), show that

$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 L_2 - 2M^2}$$

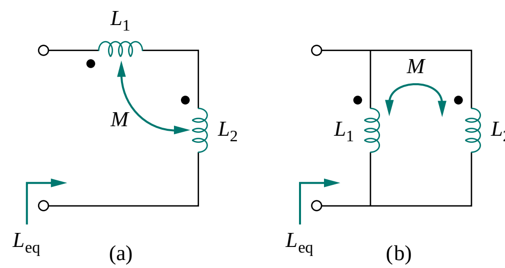


Figure 13.74 For Prob. 13.4.

13.5 Determine $\mathbf{V}_1$ and $\mathbf{V}_2$ in terms of $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit in Fig. 13.75.

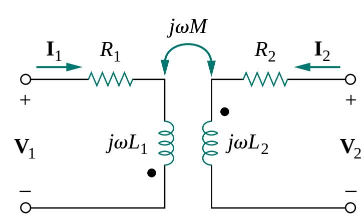


Figure 13.75 For Prob. 13.5.

13.6 Find $\mathbf{V}_o$ in the circuit of Fig. 13.76.

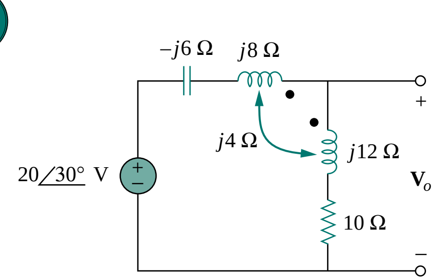


Figure 13.76 For Prob. 13.6.

13.7 Obtain $\mathbf{V}_o$ in the circuit of Fig. 13.77.

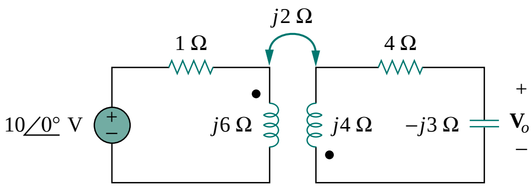


Figure 13.77 For Prob. 13.7.

13.8 Find $\mathbf{V}_x$ in the network shown in Fig. 13.78.

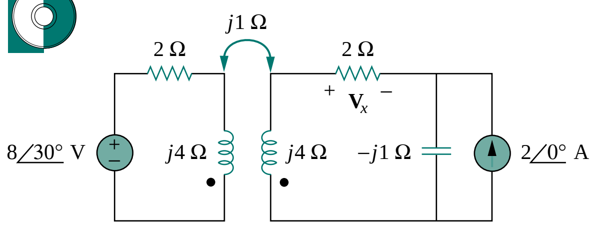


Figure 13.78 For Prob. 13.8.

13.9 Find $\mathbf{I}_o$ in the circuit of Fig. 13.79.

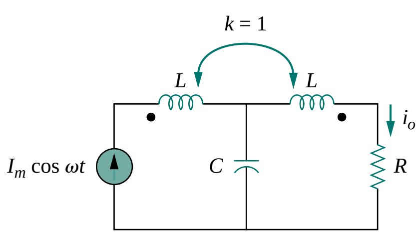


Figure 13.79 For Prob. 13.9.

13.10 Obtain the mesh equations for the circuit in Fig. 13.80.

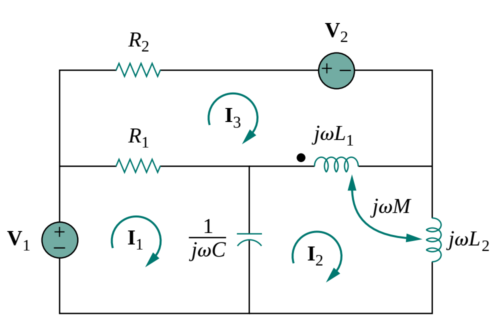


Figure 13.80 For Prob. 13.10.

13.11 Obtain the Thevenin equivalent circuit for the circuit in Fig. 13.81 at terminals a-b.

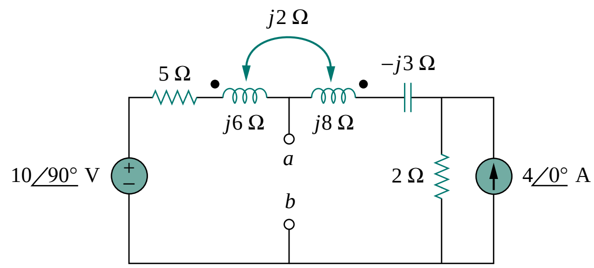


Figure 13.81 For Prob. 13.11.

13.12 Find the Norton equivalent for the circuit in Fig. 13.82 at terminals a-b.

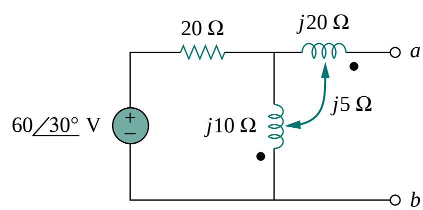


Figure 13.82 For Prob. 13.12.

Section 13.3 Energy in a Coupled Circuit

13.13 Determine currents $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ in the circuit of Fig. 13.83. Find the energy stored in the coupled coils at $t = 2$ ms. Take $\omega = 1000$ rad/s.

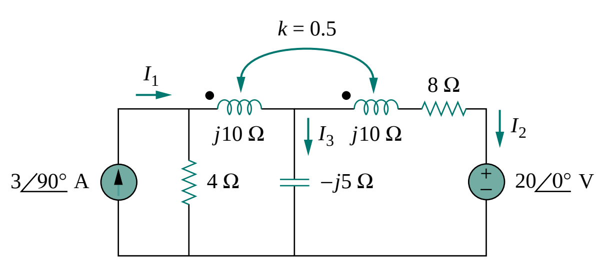


Figure 13.83 For Prob. 13.13.

13.14 Find $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit of Fig. 13.84. Calculate the power absorbed by the 4-Ω resistor.

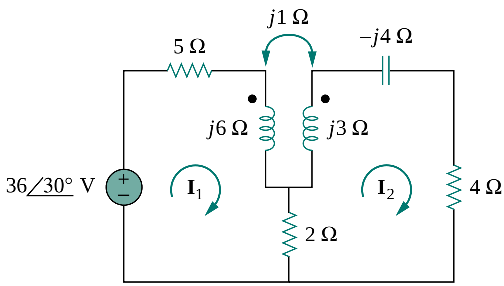


Figure 13.84 For Prob. 13.14.

*13.15 Find current $\mathbf{I}_o$ in the circuit of Fig. 13.85.

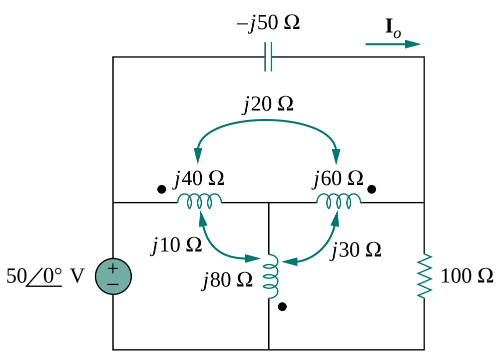


Figure 13.85 For Prob. 13.15.

13.16 If $M = 0.2$ H and $v_s = 12\cos 10t$ V in the circuit of Fig. 13.86, find i_1 and i_2. Calculate the energy stored in the coupled coils at $t = 15$ ms.

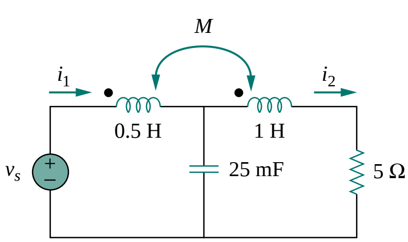


Figure 13.86 For Prob. 13.16.

13.17 In the circuit of Fig. 13.87,
(a) find the coupling coefficient,
(b) calculate v_o,
(c) determine the energy stored in the coupled inductors at $t = 2$ s.

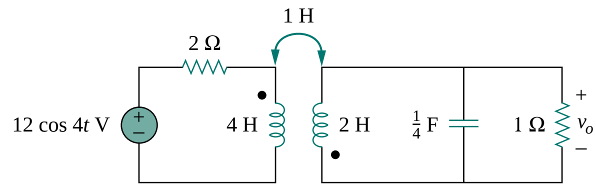


Figure 13.87 For Prob. 13.17.

13.18 For the network in Fig. 13.88, find $\mathbf{Z}_{ab}$ and $\mathbf{I}_o$.

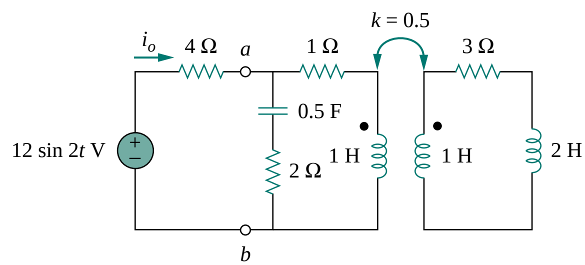


Figure 13.88 For Prob. 13.18.

13.19 Find $\mathbf{I}_o$ in the circuit of Fig. 13.89. Switch the dot on the winding on the right and calculate $\mathbf{I}_o$ again.

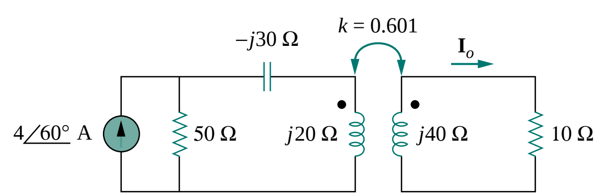


Figure 13.89 For Prob. 13.19.

13.20 Rework Example 13.1 using the concept of reflected impedance.

Section 13.4 Linear Transformers

13.21 In the circuit of Fig. 13.90, find the value of the coupling coefficient k that will make the 10-Ω resistor dissipate 320 W. For this value of k, find the energy stored in the coupled coils at $t = 1.5$ s.

*An asterisk indicates a challenging problem.

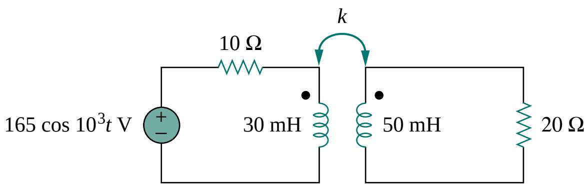


Figure 13.90 For Prob. 13.21.

13.22 (a) Find the input impedance of the circuit in Fig. 13.91 using the concept of reflected impedance.

(b) Obtain the input impedance by replacing the linear transformer by its T equivalent.

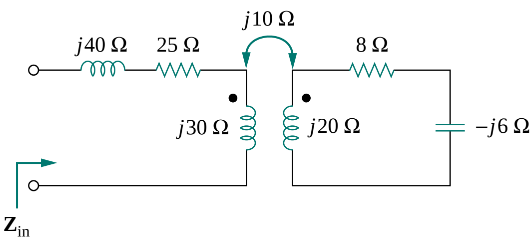


Figure 13.91 For Prob. 13.22.

13.23 For the circuit in Fig. 13.92, find:

(a) the T-equivalent circuit,

(b) the Π-equivalent circuit.

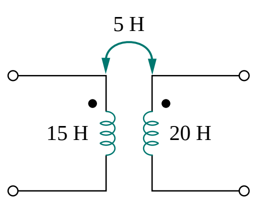


Figure 13.92 For Prob. 13.23.

***13.24** Two linear transformers are cascaded as shown in Fig. 13.93. Show that

$$\mathbf{Z}_{\text{in}} = \frac{\omega^2 R(L_a^2 + L_a L_b - M_a^2) + j\omega^3(L_a^2 L_b + L_a L_b^2 - L_a M_b^2 - L_b M_a^2)}{\omega^2(L_a L_b + L_b^2 - M_b^2) - j\omega R(L_a + L_b)}$$

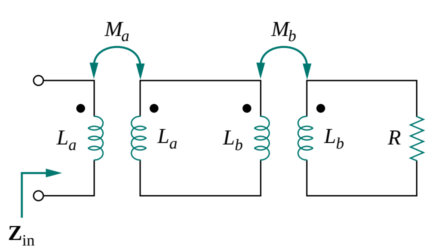


Figure 13.93 For Prob. 13.24.

13.25 Determine the input impedance of the air-core transformer circuit of Fig. 13.94.

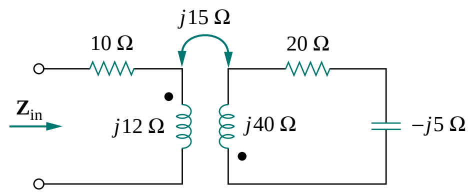


Figure 13.94 For Prob. 13.25.

Section 13.5 Ideal Transformers

13.26 As done in Fig. 13.32, obtain the relationships between terminal voltages and currents for each of the ideal transformers in Fig. 13.95.

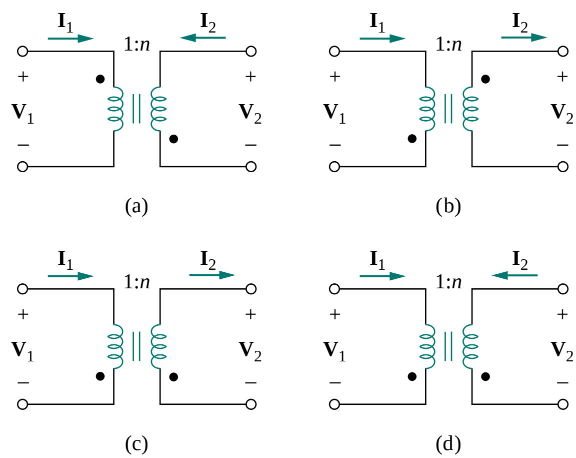


Figure 13.95 For Prob. 13.26.

13.27 A 4-kVA, 2300/230-V rms transformer has an equivalent impedance of $2\angle 10^\circ\ \Omega$ on the primary side. If the transformer is connected to a load with 0.6 power factor leading, calculate the input impedance.

13.28 A 1200/240-V rms transformer has impedance $60\angle -30^\circ\ \Omega$ on the high-voltage side. If the transformer is connected to a $0.8\angle 10^\circ$-Ω load on the low-voltage side, determine the primary and secondary currents.

13.29 Determine $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit of Fig. 13.96.

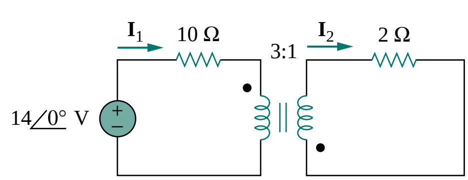


Figure 13.96 For Prob. 13.29.

13.30 Obtain $\mathbf{V}_1$ and $\mathbf{V}_2$ in the ideal transformer circuit of Fig. 13.97.

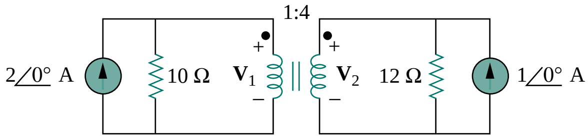


Figure 13.97 For Prob. 13.30.

13.31 In the ideal transformer circuit of Fig. 13.98, find $i_1(t)$ and $i_2(t)$.

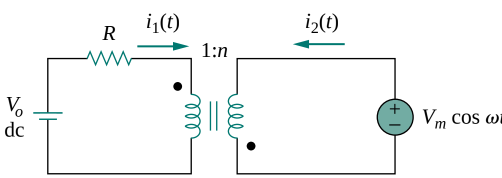


Figure 13.98 For Prob. 13.31.

13.32 (a) Find $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit of Fig. 13.99 below.
(b) Switch the dot on one of the windings. Find $\mathbf{I}_1$ and $\mathbf{I}_2$ again.

13.33 For the circuit in Fig. 13.100, find $\mathbf{V}_o$. Switch the dot on the secondary side and find $\mathbf{V}_o$ again.

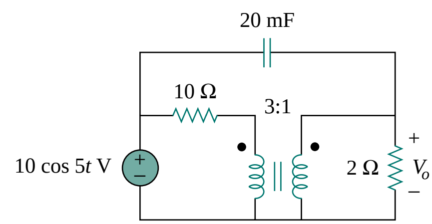


Figure 13.100 For Prob. 13.33.

13.34 Calculate the input impedance for the network in Fig. 13.101 below.

13.35 Use the concept of reflected impedance to find the input impedance and current $\mathbf{I}_1$ in Fig. 13.102 below.

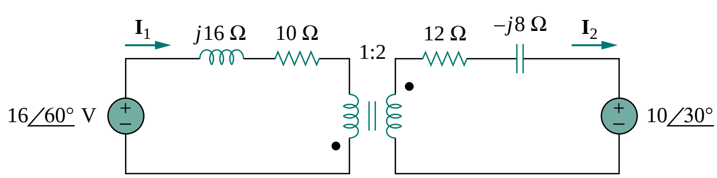


Figure 13.99 For Prob. 13.32.

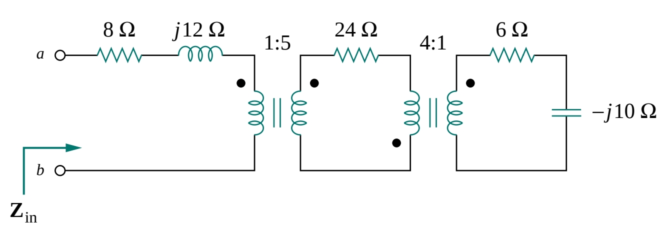


Figure 13.101 For Prob. 13.34.

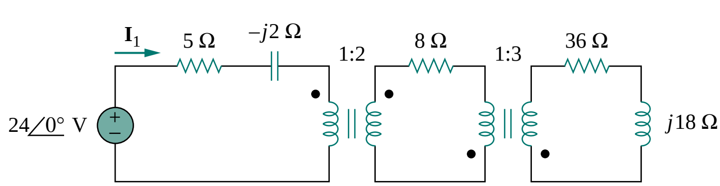


Figure 13.102 For Prob. 13.35.

13.36 For the circuit in Fig. 13.103, determine the turns ratio n that will cause maximum average power transfer to the load. Calculate that maximum average power.

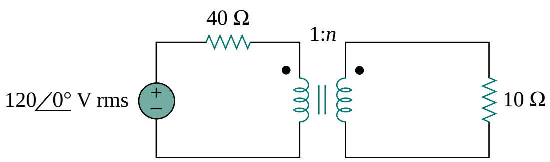


Figure 13.103 For Prob. 13.36.

13.37 Refer to the network in Fig. 13.104.

(a) Find n for maximum power supplied to the 200-Ω load.

(b) Determine the power in the 200-Ω load if $n = 10$.

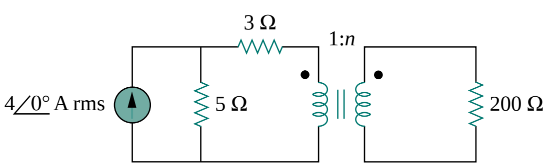


Figure 13.104 For Prob. 13.37.

13.38 A transformer is used to match an amplifier with an 8-Ω load as shown in Fig. 13.105. The Thevenin equivalent of the amplifier is: $V_{\mathrm{Th}} = 10$ V, $Z_{\mathrm{Th}} = 128\ \Omega$.

(a) Find the required turns ratio for maximum energy power transfer.

(b) Determine the primary and secondary currents.

(c) Calculate the primary and secondary voltages.

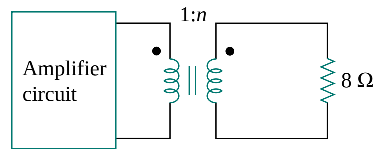


Figure 13.105 For Prob. 13.38.

13.39 In Fig. 13.106 below, determine the average power delivered to $\mathbf{Z}_s$.

13.40 Find the power absorbed by the 10-Ω resistor in the ideal transformer circuit of Fig. 13.107.

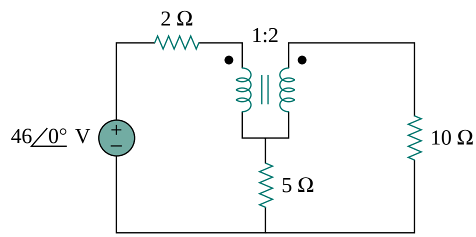


Figure 13.107 For Prob. 13.40.

13.41 For the ideal transformer circuit of Fig. 13.108 below, find:

(a) $\mathbf{I}_1$ and $\mathbf{I}_2$,

(b) $\mathbf{V}_1$, $\mathbf{V}_2$, and $\mathbf{V}_o$,

(c) the complex power supplied by the source.

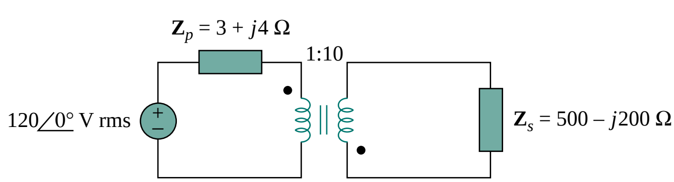


Figure 13.106 For Prob. 13.39.

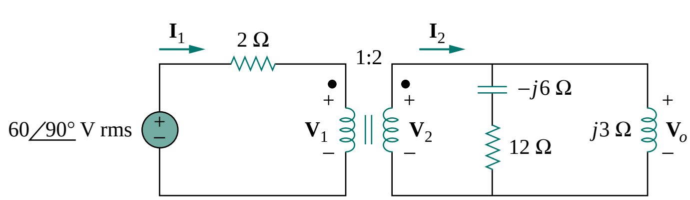


Figure 13.108 For Prob. 13.41.

13.42 Determine the average power absorbed by each resistor in the circuit of Fig. 13.109.

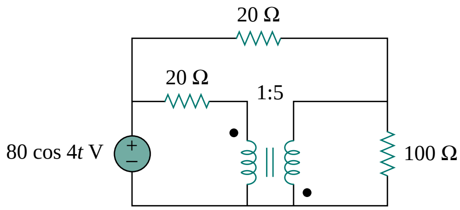


Figure 13.109 For Prob. 13.42.

13.43 Find the average power delivered to each resistor in the circuit of Fig. 13.110.

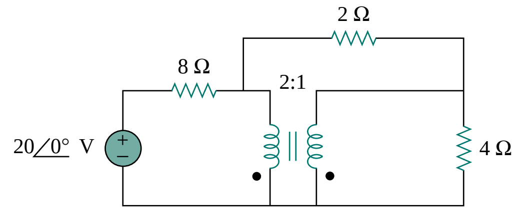


Figure 13.110 For Prob. 13.43.

13.44 Refer to the circuit in Fig. 13.111 below.
(a) Find currents $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$.
(b) Find the power dissipated in the 40-Ω resistor.

*__13.45__ For the circuit in Fig. 13.112 below, find $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{V}_o$.

13.46 For the network in Fig. 13.113 below, find
(a) the complex power supplied by the source,
(b) the average power delivered to the 18-Ω resistor.

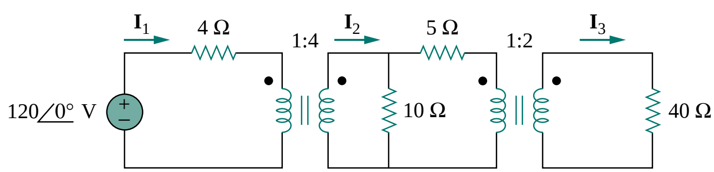


Figure 13.111 For Prob. 13.44.

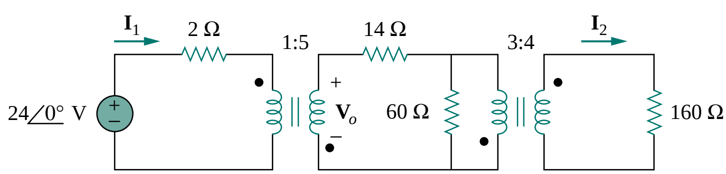


Figure 13.112 For Prob. 13.45.

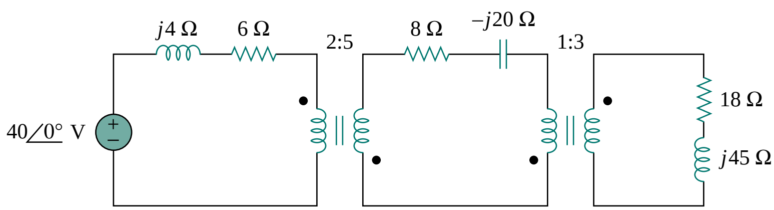


Figure 13.113 For Prob. 13.46.

13.47 Find the mesh currents in the circuit of Fig. 13.114 below.

Section 13.6 Ideal Autotransformers

13.48 An ideal autotransformer with a 1:4 step-up turns ratio has its secondary connected to a 120-Ω load and the primary to a 420-V source. Determine the primary current.

13.49 In the ideal autotransformer of Fig. 13.115, calculate $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_o$. Find the average power delivered to the load.

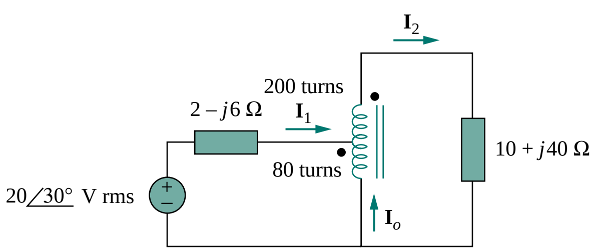


Figure 13.115 For Prob. 13.49.

*__13.50__ In the circuit of Fig. 13.116, $\mathbf{Z}_L$ is adjusted until maximum average power is delivered to $\mathbf{Z}_L$. Find $\mathbf{Z}_L$ and the maximum average power transferred to it. Take $N_1 = 600$ turns and $N_2 = 200$ turns.

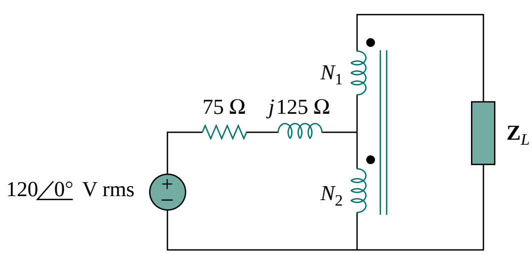


Figure 13.116 For Prob. 13.50.

13.51 In the ideal transformer circuit shown in Fig. 13.117, determine the average power delivered to the load.

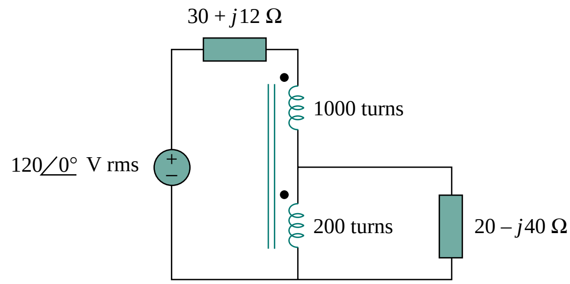


Figure 13.117 For Prob. 13.51.

13.52 In the autotransformer circuit in Fig. 13.118, show that

$$\mathbf{Z}_{\text{in}} = \left(1 + \frac{N_1}{N_2}\right)^2 \mathbf{Z}_L$$

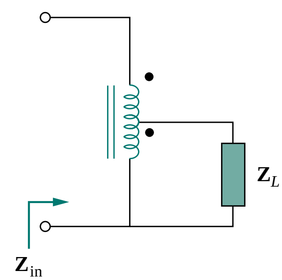


Figure 13.118 For Prob. 13.52.

Section 13.7 Three-Phase Transformers

13.53 In order to meet an emergency, three single-phase transformers with 12,470/7200 V rms are connected in Δ-Y to form a three-phase transformer which is fed by a 12,470-V transmission line. If the transformer supplies 60 MVA to a load, find:

(a) the turns ratio for each transformer,

(b) the currents in the primary and secondary windings of the transformer,

(c) the incoming and outgoing transmission line currents.

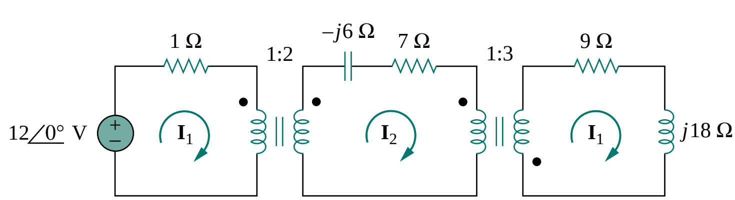


Figure 13.114 For Prob. 13.47.

13.54 Figure 13.119 below shows a three-phase transformer that supplies a Y-connected load.
(a) Identify the transformer connection.
(b) Calculate currents $\mathbf{I}_2$ and $\mathbf{I}_c$.
(c) Find the average power absorbed by the load.

13.55 Consider the three-phase transformer shown in Fig. 13.120. The primary is fed by a three-phase source with line voltage of 2.4 kV rms, while the secondary supplies a three-phase 120-kW balanced load at pf of 0.8. Determine:
(a) the type of transformer connections,
(b) the values of I_{LS} and I_{PS},
(c) the values of I_{LP} and I_{PP},
(d) the kVA rating of each phase of the transformer.

13.56 A balanced three-phase transformer bank with the Δ-Y connection depicted in Fig. 13.121 below is used to step down line voltages from 4500 V rms to 900 V rms. If the transformer feeds a 120-kVA load, find:
(a) the turns ratio for the transformer,
(b) the line currents at the primary and secondary sides.

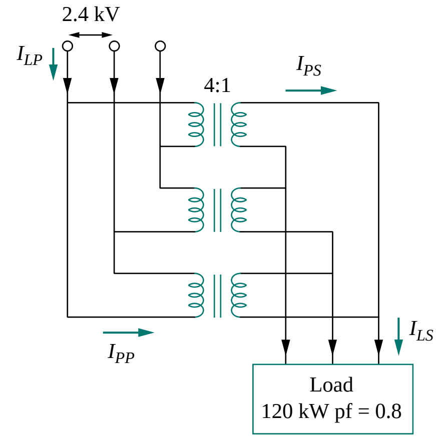


Figure 13.120 For Prob. 13.55.

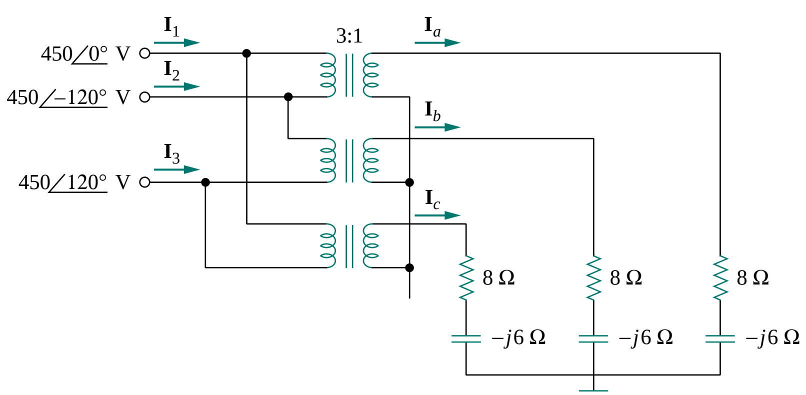


Figure 13.119 For Prob. 13.54.

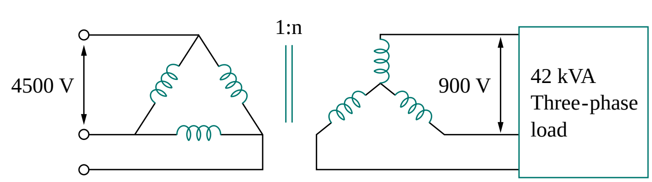


Figure 13.121 For Prob. 13.56.

13.57 A Y-Δ three-phase transformer is connected to a 60-kVA load with 0.85 power factor (leading) through a feeder whose impedance is $0.05 + j0.1\ \Omega$ per phase, as shown in Fig. 13.122 below. Find the magnitude of:

(a) the line current at the load,

(b) the line voltage at the secondary side of the transformer,

(c) the line current at the primary side of the transformer.

13.58 The three-phase system of a town distributes power with a line voltage of 13.2 kV. A pole transformer connected to single wire and ground steps down the high-voltage wire to 120 V rms and serves a house as shown in Fig. 13.123.

(a) Calculate the turns ratio of the pole transformer to get 120 V.

(b) Determine how much current a 100-W lamp connected to the 120-V hot line draws from the high-voltage line.

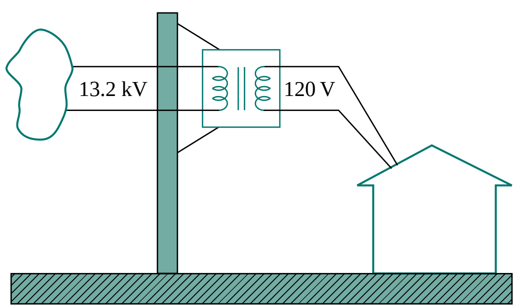


Figure 13.123 For Prob. 13.58.

Section 13.8 *PSpice* Analysis of Magnetically Coupled Circuits

13.59 Rework Prob. 13.14 using *PSpice*.

13.60 Use *PSpice* to find $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ in the circuit of Fig. 13.124.

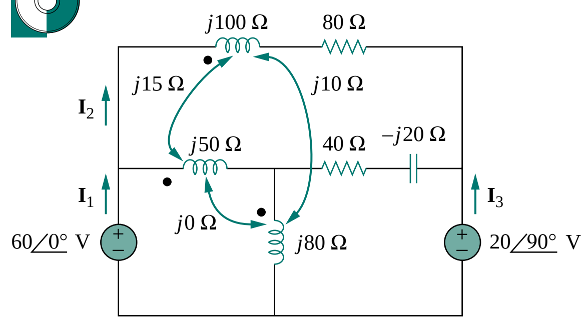


Figure 13.124 For Prob. 13.60.

13.61 Rework Prob. 13.15 using *PSpice*.

13.62 Use *PSpice* to find $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ in the circuit of Fig. 13.125.

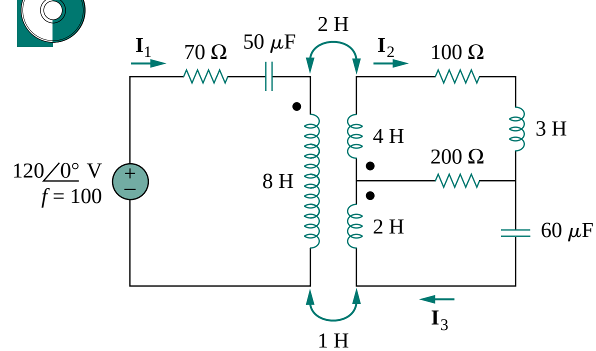


Figure 13.125 For Prob. 13.62.

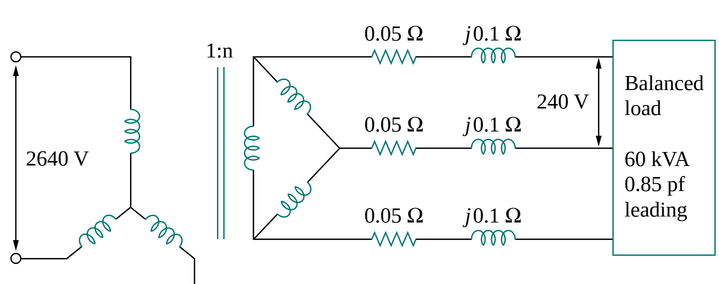


Figure 13.122 For Prob. 13.57.

13.63 Use *PSpice* to find $\mathbf{V}_1$, $\mathbf{V}_2$, and $\mathbf{I}_o$ in the circuit of Fig. 13.126.

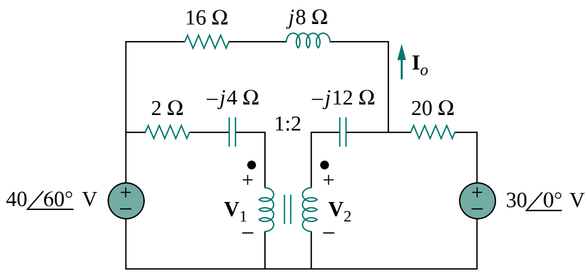


Figure 13.126 For Prob. 13.63.

13.64 Find $\mathbf{I}_x$ and $\mathbf{V}_x$ in the circuit of Fig. 13.127 below using *PSpice*.

13.65 Determine $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ in the ideal transformer circuit of Fig. 13.128 using *PSpice*.

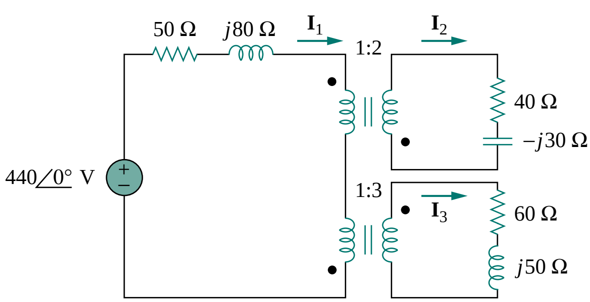


Figure 13.128 For Prob. 13.65.

Section 13.9 Applications

13.66 A stereo amplifier circuit with an output impedance of 7.2 kΩ is to be matched to a speaker with an input impedance of 8 Ω by a transformer whose primary side has 3000 turns. Calculate the number of turns required on the secondary side.

13.67 A transformer having 2400 turns on the primary and 48 turns on the secondary is used as an impedance-matching device. What is the reflected value of a 3-Ω load connected to the secondary?

13.68 A radio receiver has an input resistance of 300 Ω. When it is connected directly to an antenna system with a characteristic impedance of 75 Ω, an impedance mismatch occurs. By inserting an impedance-matching transformer ahead of the receiver, maximum power can be realized. Calculate the required turns ratio.

13.69 A step-down power transformer with a turns ratio of $n = 0.1$ supplies 12.6 V rms to a resistive load. If the primary current is 2.5 A rms, how much power is delivered to the load?

13.70 A 240/120-V rms power transformer is rated at 10 kVA. Determine the turns ratio, the primary current, and the secondary current.

13.71 A 4-kVA, 2400/240-V rms transformer has 250 turns on the primary side. Calculate:

(a) the turns ratio,

(b) the number of turns on the secondary side,

(c) the primary and secondary currents.

13.72 A 25,000/240-V rms distribution transformer has a primary current rating of 75 A.

(a) Find the transformer kVA rating.

(b) Calculate the secondary current.

13.73 A 4800-V rms transmission line feeds a distribution transformer with 1200 turns on the primary and 28 turns on the secondary. When a 10-Ω load is connected across the secondary, find:

(a) the secondary voltage,

(b) the primary and secondary currents,

(c) the power supplied to the load.

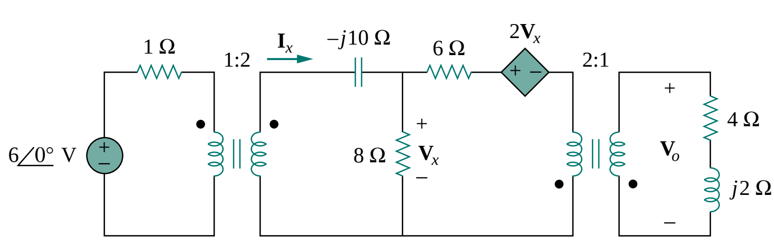


Figure 13.127 For Prob. 13.64.

COMPREHENSIVE PROBLEMS

13.74 A four-winding transformer (Fig. 13.129) is often used in equipment (e.g., PCs, VCRs) that may be operated from either 110 V or 220 V. This makes the equipment suitable for both domestic and foreign use. Show which connections are necessary to provide:

(a) an output of 12 V with an input of 110 V,

(b) an output of 50 V with an input of 220 V.

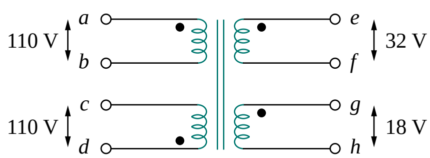


Figure 13.129 For Prob. 13.74.

***13.75** A 440/110-V ideal transformer can be connected to become a 550/440-V ideal autotransformer. There are four possible connections, two of which are wrong. Find the output voltage of:

(a) a wrong connection,

(b) the right connection.

13.76 Ten bulbs in parallel are supplied by a 7200/120-V transformer as shown in Fig. 13.130, where the bulbs are modeled by the 144-Ω resistors. Find:

(a) the turns ratio n,

(b) the current through the primary winding.

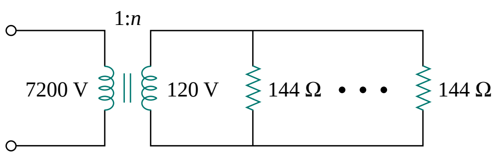


Figure 13.130 For Prob. 13.76.