1

Fundamental Circuit Concepts

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1.1 The Electrical Circuit

An **electrical circuit** or **electrical network** is an array of interconnected elements wired so as to be capable of conducting current. As discussed earlier, the fundamental **two-terminal elements** of an electrical circuit are the **resistor**, the **capacitor**, the **inductor**, the **voltage source**, and the **current source**. The circuit schematic symbols of these elements, together with the algebraic symbols used to denote their respective general values, appear in Figure 1.1.

As suggested in Figure 1.1, the value of a resistor is known as its *resistance*, R, and its dimensional units are *ohms*. The case of a wire used to interconnect the terminals of two electrical elements corresponds to the special case of a resistor whose resistance is ideally zero ohms; that is, $R = 0$. For the capacitor in Figure 1.1(b), the *capacitance*, C, has units of *farads*, and from Figure 1.1(c), the value of an inductor is its *inductance*, L, the dimensions of which are *henries*. In the case of the voltage sources depicted in Figure 1.1(d), a constant, time invariant source of voltage, or *battery*, is distinguished from a voltage source that varies with time. The latter type of voltage source is often referred to as a **time varying signal** or simply, a **signal**. In either case, the value of the battery voltage, E, and the time varying signal, $v(t)$, is in units of *volts*. Finally, the current source of Figure 1.1(e) has a value, I, in units of *amperes*, which is typically abbreviated as amps.

Elements having three, four, or more than four terminals can also appear in practical electrical networks. The discrete component **bipolar junction transistor** (BJT), which is schematically portrayed in Figure 1.2(a), is an example of a three-terminal element, in which the three terminals are the collector, the base, and the emitter. On the other hand, the monolithic *metal-oxide-semiconductor field-effect transistor* (MOSFET) depicted in Figure 1.2(b) has four terminals: the drain, the gate, the source, and the bulk substrate.

Multiterminal elements appearing in circuits identified for systematic mathematical analyses are routinely represented, or *modeled*, by equivalent subcircuits formed of only interconnected two-terminal elements. Such a representation is always possible, provided that the list of two-terminal elements itemized in Figure 1.1 is appended by an additional type of two-terminal element known as the **controlled source**, or **dependent generator**. Two of the four types of controlled sources are voltage sources and two are current sources. In Figure 1.3(a), the dependent generator is a *voltage-controlled voltage source* (VCVS) in that the voltage, $v_0(t)$, developed from terminal 3 to terminal 4 is a function of, and is therefore

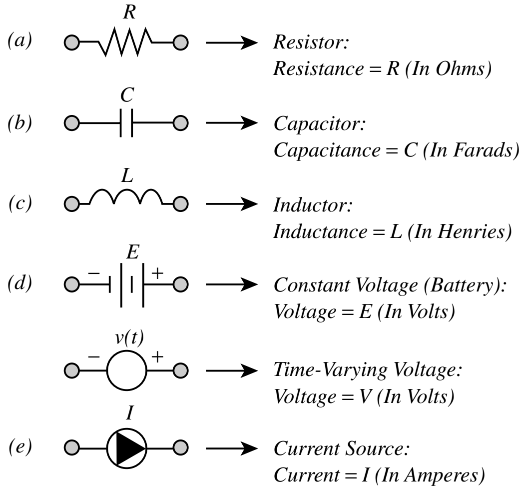

FIGURE 1.1 Circuit schematic symbol and corresponding value notation for (a) resistor, (b) capacitor, (c) inductor, (d) voltage source, and (e) current source. Note that a constant voltage source, or battery, is distinguished from a voltage source that varies with time.

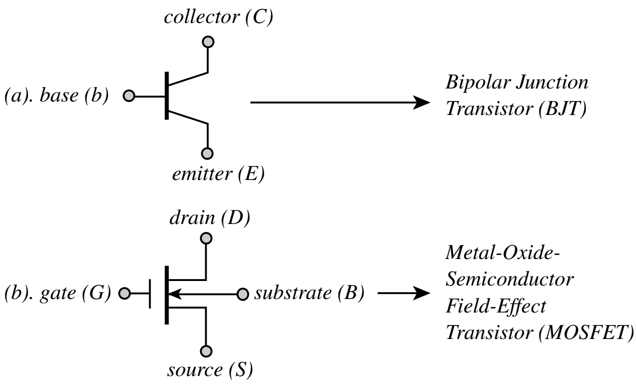

FIGURE 1.2 Circuit schematic symbol for (a) discrete component bipolar junction transistor (BJT) and (b) monolithic metal-oxide-semiconductor field-effect transistor (MOSFET).

dependent on, the voltage, $v_i(t)$, established elsewhere in the considered network from terminal 1 to terminal 2. The *controlled voltage*, $v_0(t)$, as well as the *controlling voltage*, $v_i(t)$, can be constant or time varying. Regardless of the time-domain nature of these two voltage, the value of $v_0(t)$ is not an independent number. Instead, its value is determined by $v_i(t)$ in accordance with a prescribed functional relationship, e.g.,

$$v_0(t) = f\left[v_i(t)\right] \tag{1.1}$$

If the function, $f(\cdot)$, is linearly related to its argument, (1.1) collapses to the form

$$v_0(t) = f_\mu v_i(t) \tag{1.2}$$

where $f\mu$ is a constant, independent of either $v_0(t)$ or $v_i(t)$. When the function on the right-hand side of (1.1) is linear, the subject VCVS becomes known as a *linear voltage-controlled voltage source.*

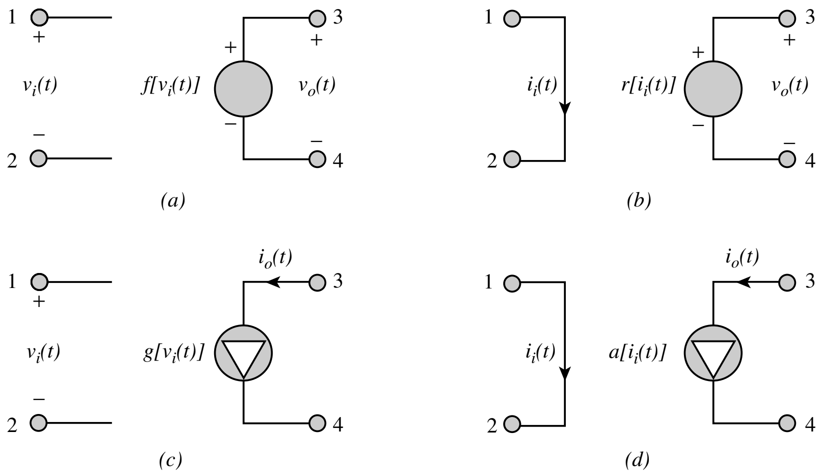

FIGURE 1.3 Circuit schematic symbol for (a) voltage-controlled voltage source (VCVS), (b) current-controlled voltage source (CCVS), (c) voltage-controlled current source (VCCS), and (d) current-controlled current source (CCCS).

The second type of controlled voltage source is the *current-controlled voltage source* (CCVS) depicted in Figure 1.3(b). In this dependent generator, the controlled voltage, $v_0(t)$, developed from terminal 3 to terminal 4 is a function of the *controlling current*, $i_i(t)$, flowing elsewhere in the network between terminals 1 and 2, as indicated. In this case, the generalized functional dependence of $v_0(t)$ on $i_i(t)$ is expressible as

$$v_0(t) = r[i_i(t)] \tag{1.3}$$

which reduces to

$$v_0(t) = r_m i_i(t) \tag{1.4}$$

when r(·) is a linear function of its argument.

The two types of dependent current sources are diagrammed symbolically in Figures 1.3(c) and (d). Figure 1.3(c) depicts a *voltage-controlled current source* (VCCS), for which the controlled current $i_0(t)$, flowing in the electrical path from terminal 3 to terminal 4, is determined by the controlling voltage, $v_i(t)$, established across terminals 1 and 2. Therefore, the controlled current can be written as

$$i_0(t) = g[v_i(t)] \tag{1.5}$$

In the *current-controlled current source* (CCCS) of Figure 1.3(d),

$$i_0(t) = a[i_i(t)] \tag{1.6}$$

where the controlled current, $i_0(t)$, flowing from terminal 3 to terminal 4 is a function of the controlling current, $i_i(t)$, flowing elsewhere in the circuit from terminal 1 to terminal 2. As is the case with the two controlled voltage sources studied earlier, the preceding two equations collapse to the linear relationships

$$i_0(t) = g_m v_i(t) \tag{1.7}$$

and

$$i_0(t) = a_\alpha i_i(t) \tag{1.8}$$

when $g(\cdot)$ and $a(\cdot)$, respectively, are linear functions of their arguments.

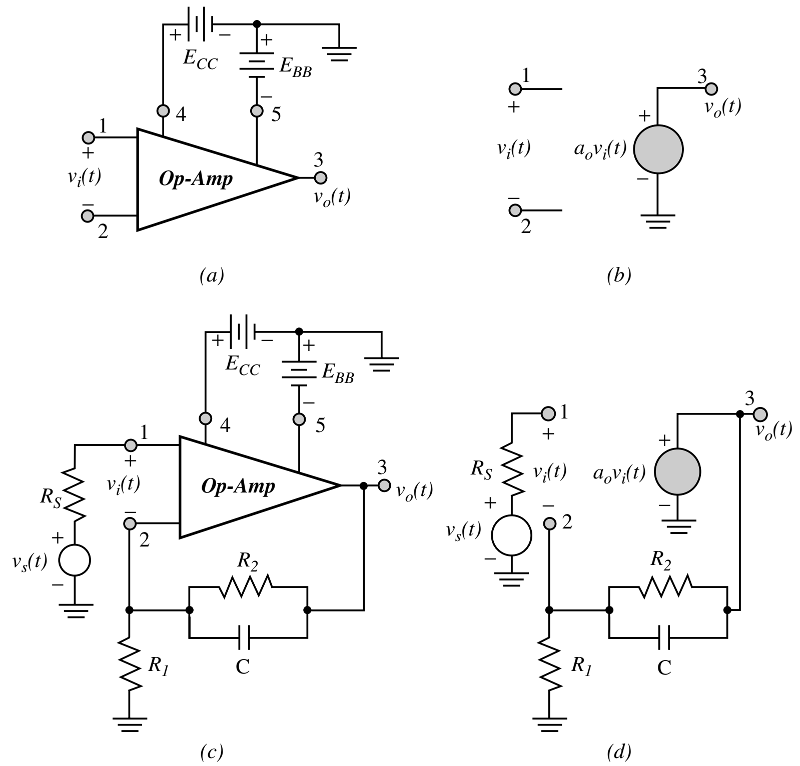

FIGURE 1.4 (a) Circuit schematic symbol for a voltage mode operational amplifier. (b) First-order linear model of the op-amp. (c) A voltage amplifier realized with the op-amp functioning as the gain element. (d) Equivalent circuit of the voltage amplifier in (c).

The immediate implication of the controlled source concept is that the definition for an electrical circuit given at the beginning of this subsection can be revised to read "an electrical circuit or electrical network is an array of *interconnected two-terminal elements* wired in such a way as to be capable of conducting current". Implicit in this revised definition is the understanding that the two-terminal elements allowed in an electrical circuit are the resistor, the capacitor, the inductor, the voltage source, the current source, and any of the four possible types of dependent generators.

In, an attempt to reinforce the engineering utility of the foregoing definition, consider the voltage mode **operational amplifier**, or **op-amp**, whose circuit schematic symbol is submitted in Figure 1.4(a). Observe that the op-amp is a five-terminal element. Two terminals, labeled 1 and 2, are provided to receive input signals that derive either from external signal sources or from the output terminals of subcircuits that feed back a designable fraction of the output signal established between terminal 3 and the system ground. Battery voltages, identified as E_{CC} and E_{BB} in the figure, are applied to the remaining two op-amp terminals (terminals 4 and 5) with respect to ground to *bias* or *activate* the op-amp for its intended application. When E_{CC} and E_{BB} are selected to ensure that the subject op-amp behaves as a linear circuit element, the voltages, E_{CC} and E_{BB}, along with the corresponding terminals at which they are incident, are inconsequential. In this event the op-amp of Figure 1.4(a) can be modeled by the electrical circuit appearing in Figure 1.4(b), which exploits a linear VCVS. Thus, the voltage amplifier of Figure 1.4(c), which interconnects two batteries, a signal source voltage, three resistors, a capacitor, and an op-amp, can be represented by the network given in Figure 1.4(d). Note that the latter configuration uses only two terminal elements, one of which is a VCVS.

Current and Current Polarity

The concept of an electrical *current* is implicit to the definition of an electrical circuit in that a circuit is said to be an array of two-terminal elements that are connected in such a way as to permit the *condition of current*. Current flow through an element that is capable of current conduction requires that the net *charge* observed at any elemental cross-section change with time. Equivalently, a net nonzero charge, $q(t)$, must be transferred over finite time across any cross-sectional area of the element. The current, $i(t)$, that actually flows is the time rate of change of this transferred charge;

$$i(t) = \frac{dq(t)}{dt} \tag{1.9}$$

where the MKS unit of charge is the *coulomb*, time t is measured in seconds, and the resultant current is measured in units of amperes. Note that zero current does not necessarily imply a lack of charge at a given cross-section of a conductive element. Instead, zero current implies only that the subject charge is not changing with time; that is, the charge is not moving through the elemental cross-section.

Electrical charge can be negative, as in the case of *electrons* transported through a cross-section of a conductive element such as aluminum or copper. A single electron has a charge of $-(1.6021 \times 10^{-19})$ coulomb. Thus, (1.9) implies a need to transport an average of (6.242×10^{18}) electrons in 1 second through a cross-section of aluminum if the aluminum element is to conduct a constant current of 1 amp. Charge can also be positive, as in the case of *holes* transported through a cross-section of a semiconductor such as germanium or silicon. Hole transport in a semiconductor is actually electron transport at an energy level that is smaller than the energy required to effect electron transport in that semiconductor. To first order, therefore, the electrical charge of a hole is the negative of the charge of an electron, which implies that the charge of a hole is $+(1.602 \times 10^{-19})$ coulomb.

A positive charge, $q(t)$, transported from the left of the cross-section to the right of the cross-section in the element abstracted in Figure 1.5(a) gives rise to a positive current, $i(t)$, which also flows from left to right across the indicated cross-section. Assume that, prior to the transport of such charge, the volumes to the left and to the right of the cross-section are electrically neutral; that is, these volumes have zero initial net charge. Then, the transport of a positive charge, q_0, from the left side to the right side of the element charges the right side to $+1q_0$ and the left side to $-1q_0$.

Alternatively, suppose a negative charge in the amount of $-q_0$ is transported from the right side of the element to its left side, as suggested in Figure 1.5(b). Then, the left side charges to $-q_0$, and the right side charges to $+q_0$, which is identical to the electrostatic condition incurred by the transport of a positive charge in the amount of q_0 from left- to right-hand sides. As a result, the transport of a net negative charge from right to left produces a positive current, $i(t)$, flowing from left to right, just as positive charge transported from left- to right-hand sides induces a current flow from left to right.

Assume, as portrayed in Figure 1.5(c), that a positive or a negative charge, say, $q_1(t)$, is transported from the left side of the indicated cross-section to the right side. Simultaneously, a positive or a negative charge in the amount of $q_2(t)$ is directed through the cross-section from right to left. If $i_1(t)$ is the current arising from the transport of the charge $q_1(t)$, and if $i_2(t)$ denotes the current corresponding to the transport of the charge, $q_2(t)$, the net effective current $i_e(t)$, flowing from the left side of the cross-section to the right side of the cross-section is

$$i_e(t) = \frac{d}{dt}\left[q_1(t) - q_2(t)\right] = i_1(t) - i_2(t) \tag{1.10}$$

where the charge difference, $[q_1(t) - q_2(t)]$, represents the net charge transported from left to right. Observe that if $q_1(t) \equiv q_2(t)$, the net effective current is zero, even though conceivably large numbers of charges are transported back and forth across the junction.

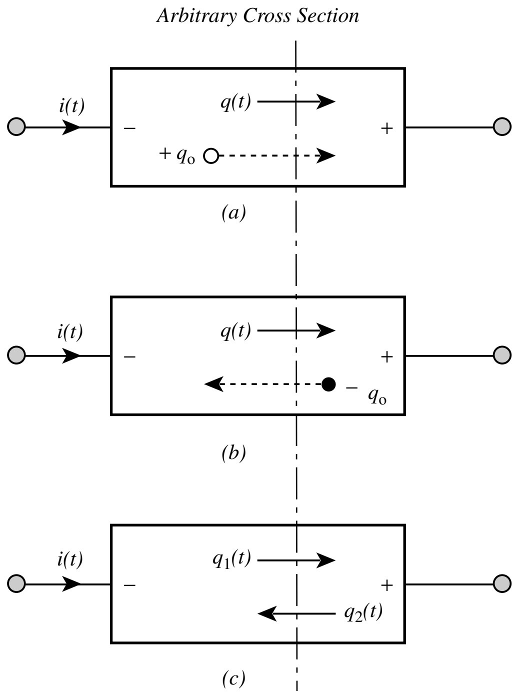

FIGURE 1.5 (a) Transport of a positive charge from the left-hand side to the right-hand side of an arbitrary cross-section of a conductive element. (b) Transport of a negative charge from the right-hand side to the left-hand side of an arbitrary cross-section of a conductive element. (c) Transport of positive or negative charges from either side to the other side of an arbitrary cross-section of a conductive element.

Energy and Voltage

The preceding section highlights the fundamental physical fact that the flow of current through a conductive electrical element mandates that a net charge be transported over finite time across any arbitrary cross-section of that element. The electrical effect of this charge transport is a net positive charge induced on one side of the element in question and a net negative charge (equal in magnitude to the aforementioned positive charge) mirrored on the other side of the element. This ramification conflicts with the observable electrical properties of an element in equilibrium. In particular, an element sitting in free space, without any electrical connection to a source of *energy*, is necessarily in equilibrium in the sense that the net positive charge in any volume of the element is precisely counteracted by an equal amount of charge of opposite sign in said volume. Thus, if none of the elements abstracted in Figure 1.5 is connected to an external source of energy, it is physically impossible to achieve the indicated electrical charge differential that materializes across an arbitrary cross-section of the element when charge is transferred from one side of the cross-section to the other.

The energy commensurate with sustaining current flow through an electrical element derives from the application of a *voltage*, $v(t)$, across the element in question. Equivalently, the application of electrical energy to an element manifests itself as a voltage developed across the terminals of an element to which energy is supplied. The amount of applied voltage, $v(t)$, required to sustain the flow of current, $i(t)$, as diagrammed in Figure 1.6(a), is precisely the voltage required to offset the electrostatic implications of the differential charge induced across the element through which $i(t)$ flows. This is to say that without the connection of the voltage, $v(t)$, to the element in Figure 1.6(a), the element cannot be in equilibrium. With $v(t)$ connected, equilibrium for the entire system comprised of element and voltage source is reestablished by allowing for the conduction of the current, $i(t)$.

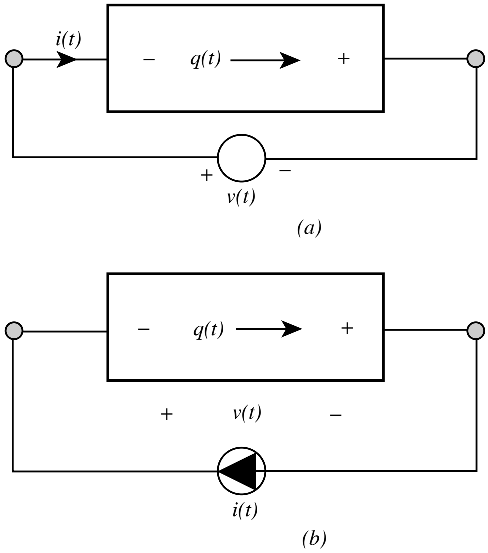

FIGURE 1.6 (a) The application of energy in the form of a voltage applied to an element that is made to conduct a specified current. The applied voltage, $v(t)$, causes the current, $i(t)$, to flow. (b) The application of energy in the form of a current applied to an element that is made to establish a specified terminal voltage. The applied current, $i(t)$, causes the voltage, $v(t)$, to be developed across the terminals of the electrical element.

Instead of viewing the delivery of energy to an electrical element as the ramification of a voltage source applied to the element, the energy delivery may be interpreted as the upshot of a current source used to excite the element, as depicted in Figure 1.6(b). This interpretation follows from the fact that energy must be applied in an amount that effects charge transport at a desired time rate of change. It follows that the application of a current source in the amount of the desired current is necessarily in one-to-one correspondence with the voltage required to offset the charge differential manifested by the charge transport that yields the subject current. To be sure, a voltage source is a physical entity, while current source is not; but the mathematical modeling of energy delivery to an electrical element can nonetheless be accomplished through either a voltage source or a current source.

In Figure 1.6, the terminal voltage, $v(t)$, corresponding to the energy, $w(t)$, required to transfer an amount of charge, $q(t)$, across an arbitrary cross-section of the element is

$$v(t) = \frac{dw(t)}{dq(t)} \tag{1.11}$$

where $v(t)$ is in units of volts when $q(t)$ is expressed in coulombs, and $w(t)$ is specified in joules. Thus, if 1 joule of applied energy results in the transport of 1 coulomb of charge through an element, the elemental terminal voltage manifested by the 1 joule of applied energy is 1 volt.

It should be understood that the derivative on the right-hand side of (1.11), and thus the terminal voltage demanded of an element that is transporting a certain amount of charge through its cross-section, is a function of the properties of the type of material from which the element undergoing study is fabricated. For example, an insulator such as paper, air, or silicon dioxide is ideally incapable of current conduction and hence, intrinsic charge transport. Thus, $q(t)$ is essentially zero in an insulator and by (1.11), an infinitely large terminal voltage is required for even the smallest possible current. In a conductor such as aluminum, iron, or copper, large amounts of charge can be transported for very small applied energies. Accordingly, the requisite terminal voltage for even very large currents approaches zero in ideal conductors. The electrical properties of semiconductors such as germanium, silicon, and gallium arsenide lie between the extremes of those for an insulator and a conductor. In particular, semiconductor elements behave as insulators when their terminals are subjected to small voltages, while progressively larger terminal voltages render the electrical behavior of semiconductors akin to conductors. This conditional conductive property of a semiconductor explains why semiconductor devices and circuits generally must be biased to appropriate voltage levels before these devices and circuits can function in accordance with their requirements.

Power

The foregoing material underscores the fact that the flow of current through a two-terminal element, or more generally, through any two terminals of an electrical network, requires that charge be transported over time across any cross-section of that element or network. In turn, such charge transport requires that energy be supplied to the network, usually through the application of an external voltage source. The time rate of change of this applied energy is the *power* delivered by the external voltage or current source to the network in question. If $p(t)$ denotes this power in units of watts

$$p(t) = \frac{dw(t)}{dq} \tag{1.12}$$

where, of course, $w(t)$ is the energy supplied to the network in joules. By rewriting (1.12) in the form

$$p(t) = \frac{dw(t)}{dq(t)} \frac{dq(t)}{dt} \tag{1.13}$$

and applying (1.9) and (1.11), the power supplied to the two terminals of an element or a network becomes the more expedient relationship

$$p(t) = v(t)i(t) \tag{1.14}$$

Equation (1.14) expresses the power delivered to an element as a simple product of the voltage applied across the terminals of the element and the resultant current conducted by that element. However, care must be exercised with respect to relative voltage and current polarity, when applying (1.14) to practical circuits.

To the foregoing end, it is useful to revisit the simple abstraction of Figure 1.6(a), which is redrawn as the slightly modified form in Figure 1.7. In this circuit, a signal source voltage, $v_s(t)$, is applied across the two terminals, 1 and 2, of an element, which responds by conducting a current $i(t)$, from terminal 1 to terminal 2 and developing a corresponding terminal voltage $v(t)$, as illustrated. If the wires (zero resistance conductors, as might be approximated by either aluminum or copper interconnects) that connect the signal source to the element are ideal, the voltage, $v(t)$, is identical to $v_s(t)$. Moreover, because the current is manifested by the application of the signal source, which thereby establishes a closed electrical path for current conduction, the element current, $i(t)$, is necessarily the same as the current, $i_s(t)$, that flows through $v_s(t)$.

If attention is focused on only the element in Figure 1.7, it is natural to presume that the current conducted by the element actually flows from terminal 1 to terminal 2 when (as shown) the voltage developed across the element is positive at terminal 1 with respect to terminal 2. This assertion may be rationalized qualitatively by noting that the positive voltage nature at terminal 1 acts to repel positive charges from terminal 1 to terminal 2, where the negative nature of the developed voltage, $v(t)$, tends to attract the repulsed positive charges. Similarly, the positive nature of the voltage at terminal 1 serves to attract negative charges from terminal 2, where the negative nature of $v(t)$ tends to repel such negative charges. Because current flows in the direction of transported positive charge and opposite to the direction of transported negative charge, either interpretation gives rise to an elemental current, $i(t)$, which flows

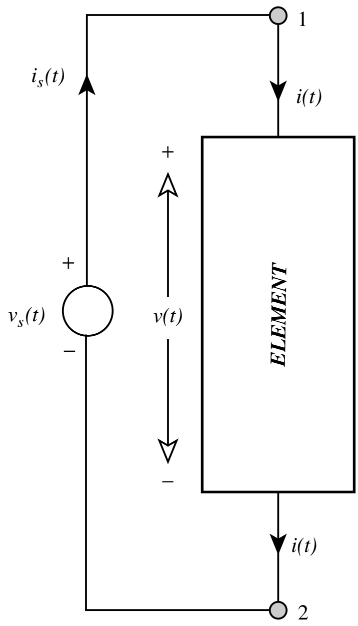

FIGURE 1.7 Circuit used to illustrate power calculations and the associated reference polarity convention.

from terminal 1 to terminal 2. In general, if current is indicated as flowing from the "high" (+) voltage terminal to the "low" (–) voltage terminal of an element, the current conducted by the element and the voltage developed across the element to cause this flow of current are said to be in **associated reference polarity**. When the element current, $i(t)$, and the corresponding element voltage, $v(t)$, as exploited in the defining power relationship of (1.14), are in associated reference polarity, the resulting computed power is a positive number and is said to represent the *power delivered to* the element. In contrast, $v(t)$ and $i(t)$ are said to be in **disassociated reference polarity** when $i(t)$ flows from the "low" voltage terminal of the element to its "high" voltage terminal. In this case the voltage-current product in (1.14) is a negative number. Instead of stating that the resulting negative power is delivered to the element, it is more meaningful to assert that the computed negative power is a *positive power that is generated by* the element in question.

At first glance, it may appear as though the latter polarity disassociation between element voltage and current variables is an impossible circumstance. Not only is polarity disassociation possible, it is absolutely necessary if electrical circuits are to subscribe to the fundamental principle of **conservation of power**. This principle states that the net power dissipated by a circuit must be identical to the net power supplied to that circuit. A confirmation of this basic principle derives from a further consideration of the topology in Figure 1.7. The electrical variables, $v(t)$ and $i(t)$, pertinent to the element delineated in this circuit, are in associated reference polarity. Accordingly, the power, $p_e(t)$, dissipated by this element is positive and given by (1.14):

$$p_e(t) = v(t)i(t) \tag{1.15}$$

However, the voltage and current variables, $v_s(t)$ and $i_s(t)$, relative to the signal source voltage are in disassociated polarity. It follows that the power, $p_s(t)$, *delivered to the signal source* is

$$p_s(t) = -v_s(t)i_s(t) \tag{1.16}$$

Because, as stated previously, $v_s(t) = v(t)$ and $i_s(t) = i(t)$, for the circuit at hand, (1.16) can be written as

$$p_s(t) = -v(t)i(t) \tag{1.17}$$

The last result implies that the

$$\text{power } \textbf{delivered by} \text{ the signal source} = +v(t)i(t) \equiv p_e(t) \tag{1.18}$$

that is, the power delivered to the element by the signal source is equal to the power dissipated by the element.

An alternative statement to conservation of power, as applied to the circuit in Figure 1.7 derives from combining (1.15) and (1.17) to arrive at

$$p_s(t) + p_e(t) = 0 \tag{1.19}$$

The foregoing result may be generalized to the case of a more complex circuit comprised of an electrical interconnection of N elements, some of which may be voltage and current sources. Let the voltage across the kth element by $v_k(t)$, and let the current flowing through this kth element, in associated reference polarity with $v_k(t)$, be $i_k(t)$. Then, the power, $p_k(t)$, delivered to the kth electrical element is $v_k(t)\ i_k(t)$. By conservation of power,

$$\sum_{k=1}^{N} p_k(t) = \sum_{k=1}^{N} v_k(t) = i_k(t) = 0 \tag{1.20}$$

The satisfaction of the expression requires that at least one of the $p_k(t)$ be negative, or equivalently, at least one of the N elements embedded in the circuit at hand must be a source of energy.

1.2 Circuit Classifications

It was pointed out earlier that the relationship between the current that is made to flow through an electrical element and the applied energy, and thus voltage, that is required to sustain such current flow is dictated by the material from which the subject element is fabricated. The element material and the associated manufacturing methods exploited to realize a particular type of circuit element determine the mathematical nature between the voltage applied across the terminals of the element and the resultant current flowing through the element. To this end, electrical elements and circuits in which they are embedded are generally codified as linear or nonlinear, active or passive, time varying or time invariant, and lumped or distributed.

Linear vs. Nonlinear

A **linear two-terminal circuit element** is one for which the voltage developed across, and the current flowing through, are related to one another by a linear algebraic or a linear integro-differential equation. If the relationship between terminal voltage and corresponding current is nonlinear, the element is said to be *nonlinear*. A linear circuit contains only linear circuit elements, while a circuit is said to be nonlinear if a least one of its embedded electrical elements is nonlinear.

All practical circuit elements, and thus all practical electrical networks, are inherently nonlinear. However, over suitably restricted ranges of applied voltages and corresponding currents, the volt-ampere characteristics of these elements and networks emulate idealized linear relationships. In the design of an electronic linear signal processor, such as an amplifier, an implicit engineering task is the implementation of biasing subcircuits that constrain the voltages and currents of internal semiconductor elements to ranges that ensure linear elemental behavior over all possible operating conditions.

The voltage–current relationship for the linear resistor offered in Figure 1.8(a) is

$$v(t) = Ri(t) \tag{1.21}$$

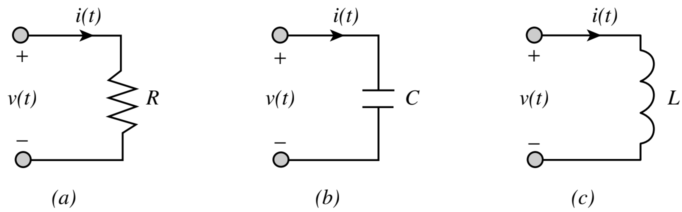

FIGURE 1.8 Circuit schematic symbol and corresponding voltage and current notation for (a) a linear resistor, (b) a linear capacitor, and (c) a linear inductor.

where the voltage, $v(t)$, appearing across the terminals of the resistor and the resultant current, $i(t)$, conducted by the resistor are in associated reference polarity. The resistance, R, is independent of either $v(t)$ or $i(t)$. From (1.14), the dissipated resistor power, which is mainfested in the form of heat, is

$$p_r(t) = i^2(t)R = \frac{v^2(t)}{R} \tag{1.22}$$

The linear capacitor and the linear inductor, with schematic symbols that appear, respectively, in Figures 1.8(b) and (c), store energy as opposed to dissipating power. Their volt-ampere equations are the linear relationships

$$i(t) = C\frac{dv(t)}{dt} \tag{1.23}$$

for the capacitor, whereas for the inductor in Figure 1.8(c),

$$v(t) = L\frac{di(t)}{dt} \tag{1.24}$$

Observe from (1.23) and (1.14) that the power, $p_c(t)$, delivered to the linear capacitor is

$$p_c(t) = v(t)i(t) = Cv(t)\frac{dv(t)}{dt} \tag{1.25}$$

From (1.12), this power is related to the energy, e.g., $w_c(t)$, stored in the form of charge deposited on the plates of the capacitor by

$$Cv(t)dv(t) = dw_c(t) \tag{1.26}$$

It follows that the energy delivered to, and hence stored in, the capacitor from time $t = 0$ to time t is

$$w_c(t) = \frac{1}{2}Cv^2(t) \tag{1.27}$$

It should be noted that this stored energy, like the energy associated with a signal source or a battery voltage, is available to supply power to other elements in the network in which the capacitor is embedded. For example, if very little current is conducted by the capacitor in question, (1.23) implies that the voltage across the capacitor is essentially constant. However, an element whose terminal voltage is a constant and in which energy is stored and therefore available for use behaves as a battery.

If the preceding analysis is repeated for the inductor of Figure 1.8(c), it can be shown that the energy, $w_l(t)$, stored in the inductive element form time $t = 0$ to time t is

$$w_l(t) = \frac{1}{2} L i^2(t) \tag{1.28}$$

Although an energized capacitor conducting almost zero current functions as a voltage source, an energized inductor supporting almost zero terminal voltage emulates a constant current source.

Active vs. Passive

An electrical element or network is said to be *passive* if the power delivered to it, defined in accordance with (1.14), is positive. This definition exploits the requirement that the terminal voltage, $v(t)$, and the element current $i(t)$, appearing in (1.14) be in associated reference polarity. In constrast, an element or network to which the delivered power is negative is said to be *active;* that is, an active element or network generates power instead of dissipating it.

Conventional two-terminal resistors, capacitors, and inductors are **passive elements**. It follows that networks formed of interconnected two-terminal resistors, capacitors, and inductors are **passive networks**. Two-terminal voltage and current sources generally behave as active elements. However, when more than one source of externally applied energy is present in an electrical network, it is possible for one more of these sources to behave as passive elements. Comments similar to those made in conjunction with two-terminal voltage and current sources apply equally well to each of the four possible dependent generators. Accordingly, multiterminal configurations, whose models exploit dependent sources, can behave as either passive or active networks.

Time Varying vs. Time Invariant

The elements of a circuit are defined electrically by an identifying parameter, such as resistance, capacitance, inductance, and the gain factors associated with dependent voltage or current sources. An element whose indentifying parameter changes as a function of time is said to be a **time varying** element. If said parameter is a constant over time, the element in question is **time invariant**. A network containing at least one time varying electrical element it is said to be a time varying network. Otherwise, the network is time invariant. Excluded from the list of elements whose electrical character establishes the time variance or time invariance of a considered network are externally applied voltage and current sources. Thus, for example, a network with internal elements that are exclusively time-invariant resistors, capacitors, inductors, and dependent sources, but which is excited by a sinusoidal signal source, is nonetheless a time-invariant network.

Although some circuits, and particularly electromechanical networks, are purposely designed to exhibit time varying volt–ampere characteristics, **parametric time variance** is generally viewed as a parasitic phenomena in the majority of practical circuits. Unfortunately, a degree of parametric time variance is unavoidable in even those circuits that are specifically designed to achieve input–output response properties that closely approximate time-invariant characteristics. For example, the best of network elements exhibit a slow aging phenomenon that shifts the values of its intrinsic physical parameters. The upshot of these shifts is electrical circuits where overall performance deterioriates with time.

Lumped vs. Distributed

Electrons in conventional conductive elements are not transported instantaneously across elemental cross sections, but their transport velocities are very high. In fact, these velocities approach the speed of light, say c, which is (3×10^8) m/s or about 982 ft/μsec. Electrons and holes in semiconductors are transported at somewhat slower speeds, but generally no less than an order of magnitude or so smaller than the speed of light. The time required to transport charge from one terminal of a two-terminal electrical element to its other terminal, compared with the time required to propagate energy uniformly through the element, determines whether an element is lumped or distributed. In particular, if the time required to transport charge through an element is significantly smaller than the time required to propagate the energy through the element that is required to incur such charge transport, the element in question is said to be lumped. On the other hand, if the charge transport time is comparable to the energy propagation time, the element is said to be **distributed**.

The concept of a lumped, as opposed to a distributed, circuit element can be qualitatively understood through a reconsideration of the circuit provided in Figure 1.7. As argued, the indicated element current, $i(t)$, is identical to the indicated source current, $i_s(t)$. This equality implies that $i(t)$, is effectively circulating around the loop that is electrically formed by the interconnection of the signal source voltage, $v_s(t)$, to the element. Equivalently, the subject equality implies that $i(t)$ is entering terminal 1 of the element and simultaneously is exiting at terminal 2, as illustrated. Assuming that the element at hand is not a semiconductor, the current, $i(t)$, arises from the transport of electrons through the element in a direction opposite to that of the indicated polarity of $i(t)$. Specifically, electrons must be transported from terminal 2, at the bottom of the element, to terminal 1, at the top of the element, and in turn the requisite amount of energy must be applied in the immediate neighborhoods of both terminals. The implication of presuming that the element at hand is lumped is that $i(t)$ is entering terminal 1 at precisely the same time that it is leaving terminal 2. Such a situation is clearly impossible, for it mandates that electrons be transported through the entire length of the element in zero time. However, given that electrons are transported at a nominal velocity of 982 ft/μsec, a very small physical elemental length renders the approximation of zero electron transport time reasonable. For example, if the element is 1/2 inch long (a typical size for an off-the-shelf resistor), the average transport time for electrons in this unit is only about 42.4 psec. As long as the period of the applied excitation, $v_s(t)$, is significantly larger than 42.4 psec, the electron transport time is significantly smaller than the time commensurate with the propagation of this energy through the entire element. A period of 42.4 psec corresponds to a signal whose frequency of approximately 23.6 GHz. Thus, a 1/2-in resistive element excited by a signal whose frequency is significantly smaller than 23.6 GHz can be viewed as a lumped circuit element.

In the vast majority of electrical and electronic networks it is difficult not to satisfy the lumped circuit approximation. Nevertheless, several practical electrical systems cannot be viewed as lumped entities. For example, consider the lead-in wire that connects the antenna input terminals of a frequency modulated (FM) radio receiver to an antenna, as diagrammed in Figure 1.9. Let the signal voltage, $v_a(t)$, across the lead-in wires at point "a" be the sinusoid,

$$v_a(t) = V_M \cos(\omega t) \tag{1.29}$$

where V_M represent the amplitude of the signal, and ω is its frequency in units of radians per second. Consider the case in which $\omega = 2\pi(103.5 \times 10^6)$ rad/s, which is a carrier frequency lying within the commercial FM broadcast band. This high signal frequency makes the length of antenna lead-in wire critically important for proper signal reception.

In an attempt to verify the preceding contention, let the voltage developed across the lead-in lines at point "b" in Figure (1.9) be denoted as $v_b(t)$, and let point "b" be 1 foot displaced from point "a"; that is, $L_{ab} = 1$ foot. The time, π_{ab} required to transport electrons over the indicated length, L_{ab}, is

$$\tau_{ab} = \frac{L_{ab}}{c} = 1.018 \text{ ns} \tag{1.30}$$

Thus, assuming an idealized line in the sense of zero effective resistance, capacitance, and inductance, the signal, $v_b(t)$, at point "b" is seen as the signal appearing at "a", delayed by approximately 1.02 ns. It follows that

$$v_b(t) = V_M \cos\left[\omega\left(t - \tau_{ab}\right)\right] = V_M \cos(\omega t - 0.662) \tag{1.31}$$

where the phase angle associated with $v_b(t)$ is 0.662 radian, or almost 38°. Obviously, the signal established at point "b" is a significantly phase-shifted version of the signal presumed at point "a".

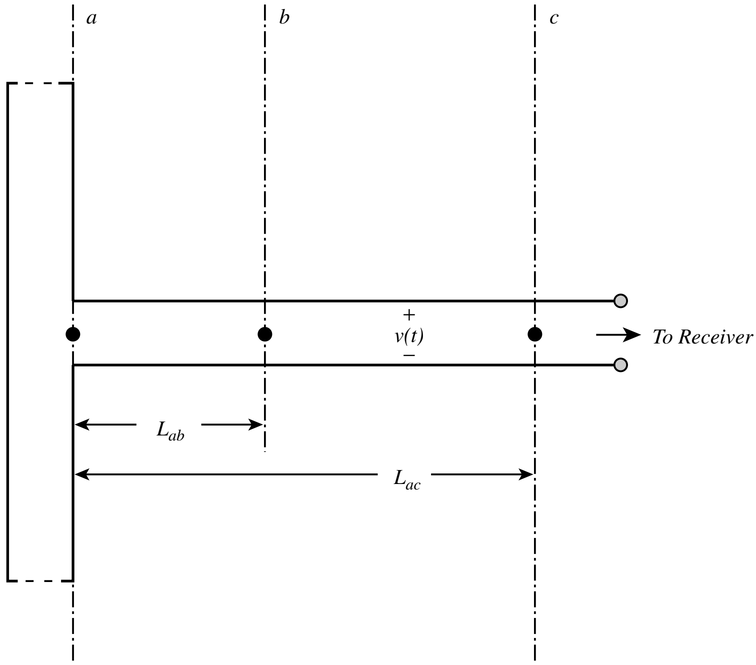

FIGURE 1.9 Schematic abstraction of a dipole antenna for an FM receiver application.

An FM receiver can effectively retrieve the signal voltage, $v_a(t)$, by detecting a phase-inverted version of $v_a(t)$ at its input terminals. To this end, it is of interest to determine the length, L_{ac}, such that the signal, $v_c(t)$, established at point "c" in Figure 1.9 is

$$v_c(t) = V_M \cos(\omega t - \pi) \tag{1.32}$$

The required phase shift of 180°, or π radians, corresponds to a time delay, τ_{ac}, of

$$\tau_{ac} = \frac{\pi}{\omega} = 4.831 \text{ ns} \tag{1.33}$$

In turn, a time delay of τ_{ac} implies a required line length, L_{ac} of

$$L_{ac} = c\tau_{ac} = 4.744 \text{ ft} \tag{1.34}$$

A parenthetically important point is the observation that the carrier frequency of 103.5 MHz corresponds to a wavelength, λ, of

$$\lambda = \frac{2\pi c}{\omega} = 9.489 \text{ ft} \tag{1.35}$$

Accordingly, the lead-in length computed in (1.34) is λ/2; that is, a half-wavelength.