13

General Feedback Theory[1]

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13.1 Introduction

In Chapter 11.2, we used the ideal feedback model to study the properties of feedback amplifiers. The model is useful only if we can separate a feedback amplifier into the basic amplifier $\mu(s)$ and the feedback network $\beta(s)$. The procedure is difficult and sometimes virtually impossible, because the forward path may no be strictly unilateral, the feedback path is usually bilateral, and the input and output coupling networks are often complicated. Thus, the ideal feedback model is not an adequate representation of a practical amplifier. In the remainder of this section, we shall develop Bode's feedback theory, which is applicable to the general network configuration and avoids the necessity of identifying the transfer functions $\mu(s)$ and $\beta(s)$.

Bode's feedback theory [2] is based on the concept of return difference, which is defined in terms of network determinants. We show that the return difference is a generalization of the concept of the feedback factor of the ideal feedback model, and can be measured physically from the amplifier itself. We then introduce the notion of null return difference and discuss its physical significance. Because the feedback theory will be formulated in terms of the first- and second-order cofactors of the elements of the indefinite-admittance matrix of a feedback circuit, we first review briefly the formulation of the indefinite-admittance matrix.

13.2 The Indefinite-Admittance Matrix

Figure 13.1 is an n-terminal network N composed of an arbitrary number of active and passive network elements connected in any way whatsoever. Let $V_1, V_2, \ldots, V_n$ be the Laplace-transformed potentials measured between terminals $1, 2, \ldots, n$ and some arbitrary but unspecified reference point, and let $I_1, I_2, \ldots, I_n$ be the Laplace-transformed currents entering the terminals $1, 2, \ldots, n$ from outside the network. The network N together with its load is linear, so the terminal current and voltages are related by the equation

[1]References for this chapter can be found on page **16**-17.

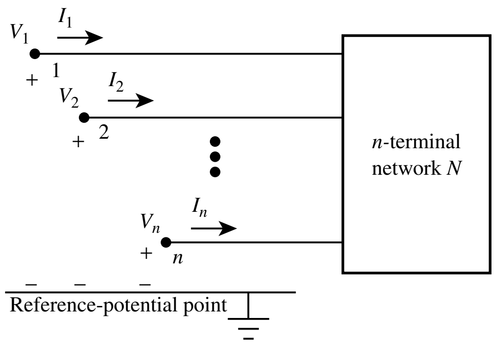

FIGURE 13.1 The general symbolic representation of an n-terminal network.

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1n} \\ y_{21} & y_{22} & \cdots & y_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ y_{n1} & y_{n2} & \cdots & y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} + \begin{bmatrix} J_1 \\ J_2 \\ \vdots \\ J_n \end{bmatrix} \tag{13.1}$$

or more succinctly as

$$\mathbf{I}(s) = \mathbf{Y}(s)\mathbf{V}(s) + \mathbf{J}(s) \tag{13.2}$$

where J_k ($k = 1,2, \ldots, n$) denotes the current flowing into the kth terminal when all terminals of N are grounded to the reference point. The coefficient matrix $\mathbf{Y}(s)$ is called the **indefinite-admittance matrix** because the reference point for the potentials is some arbitrary but unspecified point outside the network. Notice that the symbol $\mathbf{Y}(s)$ is used to denote either the admittance matrix or the indefinite-admittance matrix. This should not create any confusion because the context will tell. In the remainder of this section, we shall deal exclusively with the indefinite-admittance matrix.

We remark that the short-circuit currents J_k result from the independent sources and/or initial conditions in the interior of N. For our purposes, we shall consider all independent sources outside the network and set all initial conditions to zero. Hence, $\mathbf{J}(s)$ is considered to be zero, and (13.2) becomes

$$\mathbf{I}(s) = \mathbf{Y}(s)\mathbf{V}(s) \tag{13.3}$$

where the elements y_{ij} of $\mathbf{Y}(s)$ can be obtained as

$$y_{ij} = \left.\frac{I_i}{V_j}\right|_{v_x=0,\, x\neq j} \tag{13.4}$$

As an illustration, consider a small-signal equivalent model of a transistor in Figure 13.2. Its indefinite-admittance matrix is found to be

$$\mathbf{Y}(s) = \begin{bmatrix} g_1 + sC_1 + sC_2 & -sC_2 & -g_1 - sC_1 \\ g_m - sC_2 & g_2 + sC_2 & -g_2 - g_m \\ -g_1 - sC_1 - g_m & -g_2 & g_1 + g_2 + g_m + sC_1 \end{bmatrix} \tag{13.5}$$

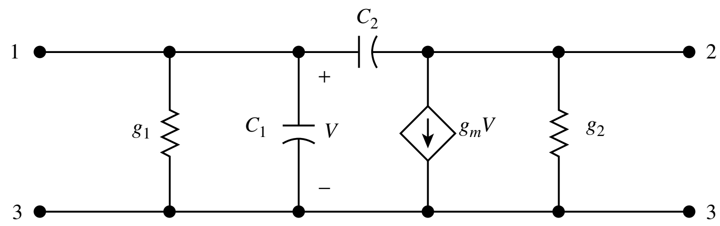

FIGURE 13.2 A small-signal equivalent network of a transistor.

Observe that the sum of elements of each row or column is equal to zero. The fact that these properties are valid in general for the indefinite-admittance matrix will now be demonstrated.

To see that the sum of the elements in each column of **Y**(s) equals zero, we add all n equations of (13.1) to yield

$$\sum_{i=1}^{n}\sum_{j=1}^{n} y_{ji}V_i = \sum_{m=1}^{n} I_m - \sum_{m=1}^{n} J_m = 0 \qquad (13.6)$$

The last equation is obtained by appealing to Kirchhoff's current law (KCL) for the node corresponding to the reference point. Setting all the terminal voltages to zero except the kth one, which is nonzero, gives

$$V_k \sum_{j=1}^{n} y_{jk} = 0 \qquad (13.7)$$

Because $V_k \neq 0$, it follows that the sum of the elements of each column of **Y**(s) equals zero. Thus, the indefinite-admittance matrix is always singular.

To demonstrate that each row sum of **Y**(s) is also zero, we recognize that because the point of zero potential may be chosen arbitrarily, the currents J_k and I_k remain invariant when all the terminal voltages V_k are changed by the same but arbitrary constant amount. Thus, if V_0 is an n-vector, each element of which is $v_0 \neq 0$, then

$$\mathbf{I}(s) - \mathbf{J}(s) = \mathbf{Y}(s)\left[\mathbf{V}(s) + \mathbf{V}_0\right] = \mathbf{Y}(s)\mathbf{V}(s) + \mathbf{Y}(s)\mathbf{V}_0 \qquad (13.8)$$

which after invoking (13.2) yields that

$$\mathbf{Y}(s)\,\mathbf{V}_0 = \mathbf{0} \qquad (13.9)$$

or

$$\sum_{j=1}^{n} y_{ij} = 0, \qquad i = 1, 2, \cdots, n \qquad (13.10)$$

showing that each row sum of **Y**(s) equals zero.

Thus, if $\mathbf{Y}_{uv}$ denotes the submatrix obtained from an indefinite-admittance matrix **Y**(s) by deleting the uth row and vth column, then the **(first-order) cofactor**, denoted by the symbol Y_{uv}, of the element y_{uv} of **Y**(s), is defined by

$$Y_{uv} = (-1)^{u+v} \det \mathbf{Y}_{uv} \qquad (13.11)$$

As a consequence of the zero-row-sum and zero-column-sum properties, all the cofactors of the elements of the indefinite-admittance matrix are equal. Such a matrix is also referred to as the **equicofactor matrix**. If Y_{uv} and Y_{ij} are any two cofactors of the elements of **Y**(s), then

$$Y_{uv} = Y_{ij} \tag{13.12}$$

for all u, v, i and j. For the indefinite-admittance matrix $\mathbf{Y}(s)$ of (13.5) it is straightforward to verify that all of its nine cofactors are equal to

$$Y_{uv} = s^2 C_1 C_2 + s(C_1 g_2 + C_2 g_1 + C_2 g_2 + g_m C_2) + g_1 g_2 \tag{13.13}$$

for u, v = 1, 2, 3.

Denote by $\mathbf{Y}_{rp,sq}$ the submatrix obtained from $\mathbf{Y}(s)$ by striking out rows r and s and columns p and q. Then the **second-order cofactor**, denoted by the symbol $Y_{rp,sq}$ of the elements y_{rp}, and y_{sq} of $\mathbf{Y}(s)$ is a scalar quantity defined by the relation

$$Y_{rp,sq} = \operatorname{sgn}(r-s)\operatorname{sgn}(p-q)(-1)^{r+p+s+q}\det \mathbf{Y}_{rp,sq} \tag{13.14}$$

where $r \neq s$ and $p \neq q$, and

$$\operatorname{sgn}\ u = +1 \quad \text{if } u > 0 \tag{13.15a}$$

$$\operatorname{sgn}\ u = -1 \quad \text{if } u < 0 \tag{13.15b}$$

The symbols $\mathbf{Y}_{uv}$ and Y_{uv} or $\mathbf{Y}_{rp,sq}$ and $Y_{rp,sq}$ should not create any confusion because one is in boldface whereas the other is italic. Also, for our purposes, it is convenient to define

$$Y_{rp,sq} = 0, \qquad r = s \quad \text{or} \quad p = q \tag{13.16a}$$

or

$$\operatorname{sgn} 0 = 0 \tag{13.16b}$$

This convention will be followed throughout the remainder of this section.

As an example, consider the hybrid-pi equivalent network of a transistor in Figure 13.3. Assume that each node is an accessible terminal of a four-terminal network. Its indefinite-admittance matrix is:

$$\mathbf{Y}(s) = \begin{bmatrix} 0.02 & 0 & -0.02 & 0 \\ 0 & 5\times10^{-12}s & 0.2 - 5\times10^{-12}s & -0.2 \\ -0.02 & -5\times10^{-12}s & 0.024 + 105\times10^{-12}s & -0.004 - 10^{-10}s \\ 0 & 0 & -0.204 - 10^{-10}s & 0.204 + 10^{-10}s \end{bmatrix} \tag{13.17}$$

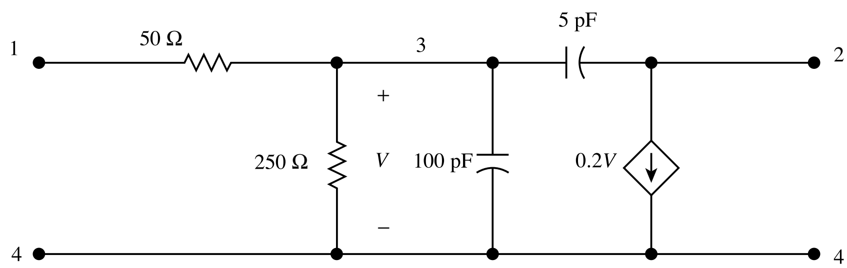

FIGURE 13.3 The hybrid-pi equivalent network of a transistor.

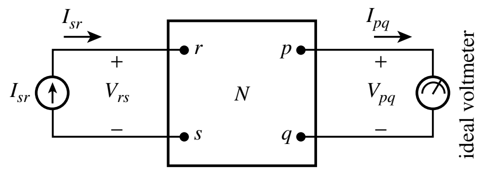

FIGURE 13.4 The symbolic representation for the measurement of the transfer impedance.

The second-order cofactor $Y_{31,42}$ and $Y_{11,34}$ of the elements of $\mathbf{Y}(s)$ of (13.17) are computed as follows:

$$Y_{31,42} = \text{sgn}(3-4)\,\text{sgn}(1-2)(-1)^{3+1+4+2} \det\begin{bmatrix} -0.02 & 0 \\ 0.2 - 5\times 10^{-12}s & -0.2 \end{bmatrix} \tag{13.18a}$$
$$= 0.004$$

$$Y_{11,34} = \text{sgn}(1-3)\,\text{sgn}(1-4)(-1)^{1+1+3+4} \det\begin{bmatrix} 5\times 10^{-12}s & 0.2 - 5\times 10^{-12}s \\ 0 & -0.204 - 10^{-10}s \end{bmatrix} \tag{13.18b}$$
$$= 5\times 10^{-12}s\left(0.204 + 10^{-10}s\right)$$

The usefulness of the indefinite-admittance matrix lies in the fact that it facilitates the computation of the driving-point or transfer functions between any pair of nodes or from any pair of nodes to any other pair. In the following, we present elegant, compact, and explicit formulas that express the network functions in terms of the ratios of the first- and/or second-order cofactors of the elements of the indefinite-admittance matrix.

Assume that a current source is connected between any two nodes r and s so that a current I_{sr} is injected into the rth node and at the same time is extracted from the sth node. Suppose that an ideal voltmeter is connected from node p to node q so that it indicates the potential rise from q to p, as depicted symbolically in Figure 13.4. Then the **transfer impedance**, denoted by the symbol $z_{rp,sq}$, between the node pairs rs and pq of the network of Figure 13. 4 is defined by the relation

$$z_{rp,sq} = \frac{V_{pq}}{I_{sr}} \tag{13.19}$$

with all initial conditions and independent sources inside N set to zero. The representation is, of course, quite general. When $r = p$ and $s = q$, the transfer impedance $z_{rp,sq}$, becomes the *driving-point impedance* $z_{rr,ss}$ between the terminal pair rs.

In Figure 13.4, set all initial conditions and independent sources in N to zero and choose terminal q to be the reference-potential point for all other terminals. In terms of (13.1), these operations are equivalent to setting $\mathbf{J} = \mathbf{0}$, $V_q = 0$, $I_x = 0$ for $x \neq r, s$ and $I_r = -I_s = I_{sr}$. Because $\mathbf{Y}(s)$ is an equicofactor matrix, the equations of (13.1) are not linearly independent and one of them is superfluous. Let us suppress the sth equation from (13.1), which then reduces to

$$\mathbf{I}_{-s} = \mathbf{Y}_{sq}\mathbf{V}_{-q} \tag{13.20}$$

where $\mathbf{I}_{-s}$ and $\mathbf{V}_{-q}$ denote the subvectors obtained from $\mathbf{I}$ and $\mathbf{V}$ of (13.3) by deleting the sth row and qth row, respectively. Applying Cramer's rule to solve for V_p yields

$$V_p = \frac{\det \tilde{\mathbf{Y}}_{sq}}{\det \mathbf{Y}_{sq}} \tag{13.21}$$

where $\tilde{\mathbf{Y}}_{sq}$ is the matrix derived from $\mathbf{Y}_{sq}$ by replacing the column corresponding to V_p by $\mathbf{I}_{-s}$. We recognize that $\mathbf{I}_{-s}$ is in the pth column if $p < q$ but in the $(p - 1)$th column if $p > q$. Furthermore, the row in which I_{sr} appears is the rth row if $r < s$, but is the $(r - 1)$th row if $r > s$. Thus, we obtain

$$(-1)^{s+q} \det \tilde{\mathbf{Y}}_{sq} = I_{sr} Y_{rp,sq} \tag{13.22}$$

In addition, we have

$$\det \mathbf{Y}_{sq} = (-1)^{s+q} Y_{sq} \tag{13.23}$$

Substituting these in (13.21) in conjunction with (13.19), we obtain

$$z_{rp,sq} = \frac{Y_{rp,sq}}{Y_{uv}} \tag{13.24}$$

$$z_{rr,ss} = \frac{Y_{rr,ss}}{Y_{uv}} \tag{13.25}$$

in which we have invoked the fact that $Y_{sq} = Y_{uv}$.

The *voltage gain*, denoted by $g_{rp,sq}$, between the node pairs rs and pq of the network of Figure 13.4 is defined by

$$g_{rp,sq} = \frac{V_{pq}}{V_{rs}} \tag{13.26}$$

again with all initial conditions and independent sources in N being set to zero. Thus, from (13.24) and (13.25) we obtain

$$g_{rp,sq} = \frac{z_{rp,sq}}{z_{rr,ss}} = \frac{Y_{rp,sq}}{Y_{rr,ss}} \tag{13.27}$$

The symbols have been chosen to help us remember. In the numerators of (13.24), (13.25), and (13.27), the order of the subscripts is as follows: r, the current injecting node; p, the voltage measurement node; s, the current extracting node; and q the voltage reference node. Nodes r and p designate the input and output transfer measurement, and nodes s and q form a sort of double datum.

As an illustration, we consider the hybrid-pi transistor equivalent network of Figure 13.3. For this transistor, suppose that we connect a 100-Ω load resistor between nodes 2 and 4, and excite the resulting circuit by a voltage source V_{14}, as depicted in Figure 13.5. To simplify our notation, let $p = 10^{-9} s$. The indefinite-admittance matrix of the amplifier is:

$$\mathbf{Y}(s) = \begin{bmatrix} 0.02 & 0 & -0.02 & 0 \\ 0 & 0.01 + 0.005p & 0.2 - 0.005p & -0.21 \\ -0.02 & -0.005p & 0.024 + 0.105p & -0.004 - 0.1p \\ 0 & -0.01 & -0.204 - 0.1p & 0.214 + 0.1p \end{bmatrix} \tag{13.28}$$

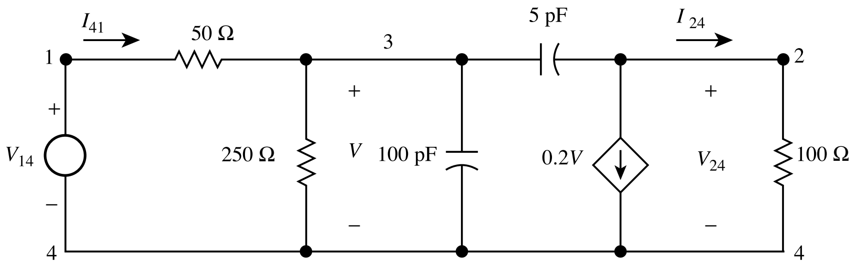

FIGURE 13.5 A transistor amplifier used to illustrate the computation of $g_{rp,sq}$.

To compute the voltage gain $g_{12,44}$, we appeal to (13.27) and obtain

$$g_{12,44} = \frac{V_{24}}{V_{14}} = \frac{Y_{12,44}}{Y_{11,44}} = \frac{p-40}{5p^2+21.7p+2.4} \tag{13.29}$$

The input impedance facing the voltage source V_{14} is determined by

$$z_{11,44} = \frac{V_{14}}{I_{41}} = \frac{Y_{11,44}}{Y_{uv}} = \frac{Y_{11,44}}{Y_{44}} = \frac{50p^2+217p+24}{p^2+4.14p+0.08} \tag{13.30}$$

To compute the current gain defined as the ratio of the current I_{24} in the 100-Ω resistor to the input current I_{41}, we apply (13.24) and obtain

$$\frac{I_{24}}{I_{41}} = 0.01\frac{V_{24}}{I_{41}} = 0.01 z_{12,44} = 0.01\frac{Y_{12,44}}{Y_{44}} = \frac{0.1p-4}{p^2+4.14p+0.08} \tag{13.31}$$

Finally, to compute the transfer admittance defined as the ratio of the load current I_{24} to the input voltage V_{14}, we appeal to (13.27) and obtain

$$\frac{I_{24}}{V_{14}} = 0.01\frac{V_{24}}{V_{14}} = 0.01 g_{12,44} = 0.01\frac{Y_{12,44}}{Y_{11,44}} = \frac{p-40}{500p^2+2170p+240} \tag{13.32}$$

13.3 The Return Difference

In the study of feedback amplifier response, we are usually interested in how a particular element of the amplifier affects that response. This element is either crucial in terms of its effect on the entire system or of primary concern to the designer. It may be the transfer function of an active device, the gain of an amplifier, or the immittance of a one-port network. For our purposes, we assume that this element x is the controlling parameter of a voltage-controlled current source defined by the equation

$$I = xV \tag{13.33}$$

To focus our attention on the element x, Figure 13.6 is the general configuration of a feedback amplifier in which the controlled source is brought out as a two-port network connected to a general four-port network, along with the input source combination of I_s and admittance Y_1 and the load admittance Y_2.

We remark that the two-port representation of a controlled source (13.33) is quite general. It includes the special situation where a one-port element is characterized by its immittance. In this case, the controlling voltage V is the terminal voltage of the controlled current source I, and x become the one-port admittance.

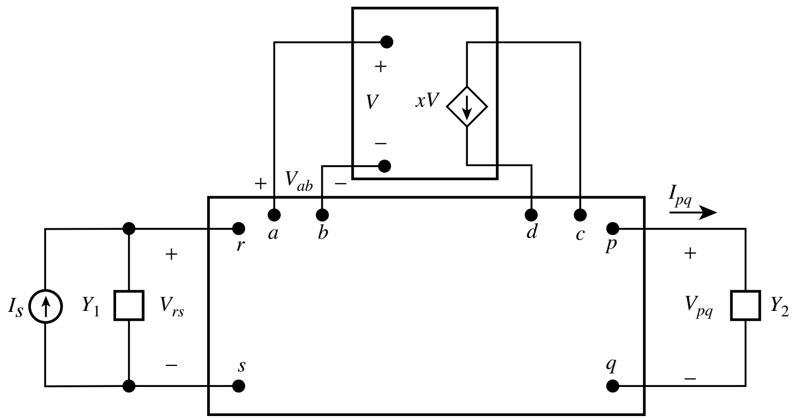

FIGURE 13.6 The general configuration of a feedback amplifier.

The *return difference* $F(x)$ of a feedback amplifier with respect to an element x is defined as the ratio of the two functional values assumed by the first-order cofactor of an element of its indefinite-admittance matrix under the condition that the element x assumes its nominal value and the condition that the element x assumes the value zero. To emphasize the importance of the feedback element x, we express the indefinite-admittance matrix $\mathbf{Y}$ of the amplifier as a function of x, even though it is also a function of the complex-frequency variable s, and write $\mathbf{Y} = \mathbf{Y}(x)$. Then, we have [3]

$$F(x) \equiv \frac{Y_{uv}(x)}{Y_{uv}(0)} \tag{13.34}$$

where

$$Y_{uv}(0) = Y_{uv}(x)\Big|_{x=0} \tag{13.35}$$

The physical significance of the return difference will now be considered. In the network of Figure 13.6, the input, the output, the controlling branch, and the controlled source are labeled as indicated. Then, the element x enters the indefinite-admittance matrix $\mathbf{Y}(x)$ in a rectangular pattern as shown next:

$$\mathbf{Y}(x) = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array}\!\!\begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} & & & \\ & & & \\ x & -x & & \\ -x & x & & \end{bmatrix} \end{array} \tag{13.36}$$

If in Figure 13.6 we replace the controlled current source xV by an independent current source of x A and set the excitation current source I_s to zero, the indefinite-admittance matrix of the resulting network is simply $\mathbf{Y}(0)$. By appealing to (13.24), the new voltage V'_{ab} appearing at terminals a and b of the controlling branch is:

$$V'_{ab} = x\frac{Y_{da,cb}(0)}{Y_{uv}(0)} = -x\frac{Y_{ca,db}(0)}{Y_{uv}(0)} \tag{13.37}$$

Notice that the current injecting point is terminal d, not c.

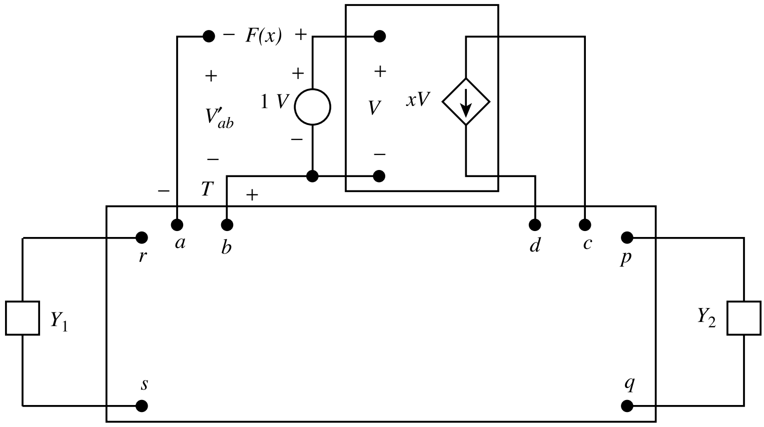

FIGURE 13.7 The physical interpretation of the return difference with respect to the controlling parameter of a voltage-controlled current source.

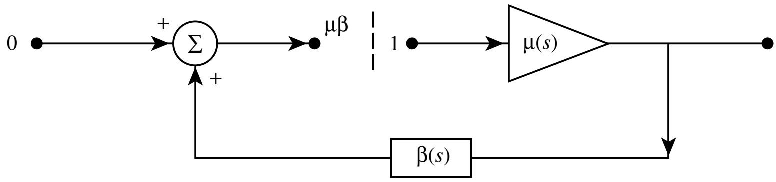

FIGURE 13.8 The physical interpretation of the loop transmission.

The preceding operation of replacing the controlled current source by an independent current source and setting the excitation I_s to zero can be represented symbolically as in Figure 13.7. Observe that the controlling branch is broken off as marked and a 1-V voltage source is applied to the right of the breaking mark. This 1-V sinusoidal voltage of a fixed angular frequency produces a current of x A at the controlled current source. The voltage appearing at the left of the breaking mark caused by this 1-V excitation is then V'_{ab} as indicated. This returned voltage V'_{ab} has the same physical significance as the loop transmission μβ defined for the ideal feedback model in Chapter 11. To see this, we set the input excitation to the ideal feedback model to zero, break the forward path, and apply a unit input to the right of the break, as depicted in Figure 13.8. The signal appearing at the left of the break is precisely the loop transmission.

For this reason, we introduce the concept of **return ratio** T, which is defined as the negative of the voltage appearing at the controlling branch when the controlled current source is replaced by an independent current source of x A and the input excitation is set to zero. Thus, the return ratio T is simply the negative of the returned voltage V'_{ab}, or $T = -V'_{ab}$. With this in mind, we next compute the difference between the 1-V excitation and the returned voltage V'_{ab} obtaining

$$\begin{aligned}1 - V'_{ab} &= 1 + x\frac{Y_{ca,db}}{Y_{uv}(0)} = \frac{Y_{uv}(0) + xY_{ca,db}}{Y_{uv}(0)} = \frac{Y_{db}(0) + xY_{ca,db}}{Y_{db}(0)} \\ &= \frac{Y_{db}(x)}{Y_{db}(0)} = \frac{Y_{uv}(x)}{Y_{uv}(0)} = F(x)\end{aligned} \tag{13.38}$$

in which we have invoked the indentities $Y_{uv} = Y_{ij}$ and

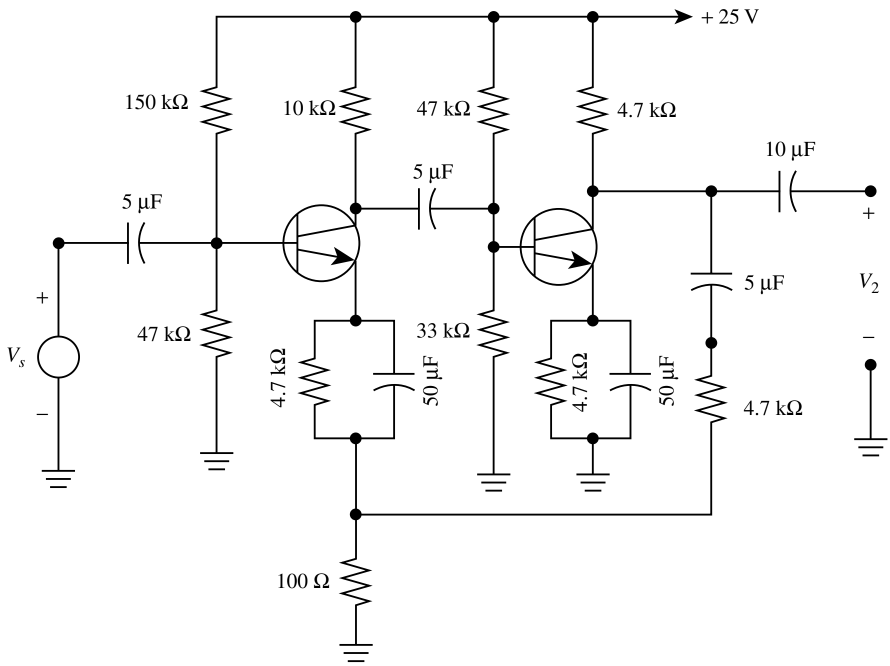

FIGURE 13.9 A voltage-series feedback amplifier together with its biasing and coupling circuitry.

$$Y_{db}(x) = Y_{db}(0) + xY_{ca,db} \tag{13.39}$$

We remark that we write $Y_{ca,db}(x)$ as $Y_{ca,db}$ because it is independent of x. In other words, the return difference $F(x)$ is simply the difference of the 1-V excitation and the returned voltage V'_{ab} as illustrated in Figure 13.7, and hence its name. Because

$$F(x) = 1 + T = 1 - \mu\beta \tag{13.40}$$

we conclude that the return difference has the same physical significance as the feedback factor of the ideal feedback model. The significance of the previous physical interpretations is that it permits us to determine the return ratio T or $-\mu\beta$ by measurement. Once the return ratio is measured, the other quantities such as return difference and loop transmission are known.

To illustrate, consider the voltage-series or the series-parallel feedback amplifier of Figure 13.9. Assume that the two transistors are identical with the following hybrid parameters:

$$h_{ie} = 1.1\ \text{k}\Omega, \qquad h_{fe} = 50, \qquad h_{re} = h_{oe} = 0 \tag{13.41}$$

After the biasing and coupling circuitry have been removed, the equivalent network is presented in Figure 13.10. The effective load of the first transistor is composed of the parallel combination of the 10, 33, 47, and 1.1-kΩ resistors. The effect of the 150- and 47-kΩ resistors can be ignored; they are included in the equivalent network to show their insignificance in the computation.

To simplify our notation, let

$$\tilde{\alpha}_k = \alpha_k \times 10^{-4} = \frac{h_{fe}}{h_{ie}} = 455 \times 10^{-4}, \qquad k = 1, 2 \tag{13.42}$$

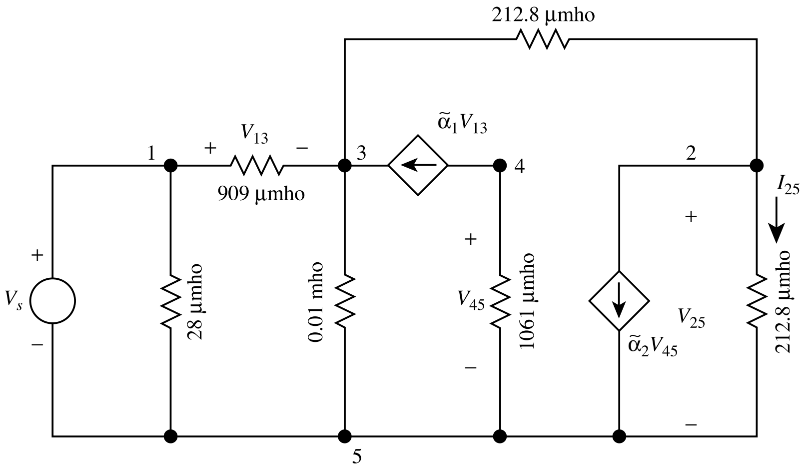

FIGURE 13.10 An equivalent network of the feedback amplifier of Figure 13.9.

The subscript k is used to distinguish the transconductances of the first and the second transistors. The indefinite-admittance matrix of the feedback amplifier of Figure 13.9 is:

$$\mathbf{Y} = 10^{-4}\begin{bmatrix} 9.37 & 0 & -9.09 & 0 & -0.28 \\ 0 & 4.256 & -2.128 & \alpha_2 & -2.128-\alpha_2 \\ -9.09-\alpha_1 & -2.128 & 111.218+\alpha_1 & 0 & -100 \\ \alpha_1 & 0 & -\alpha_1 & 10.61 & -10.61 \\ -0.28 & -2.128 & -100 & -10.61-\alpha_2 & 113.018+\alpha_2 \end{bmatrix} \tag{13.43}$$

By applying (13.27), the amplifier voltage gain is computed as

$$g_{12,25} = \frac{V_{25}}{V_s} = \frac{V_{12,25}}{V_{11,25}} = \frac{211.54\times 10^{-7}}{4.66\times 10^{-7}} = 45.39 \tag{13.44}$$

To calculate the return differences with respect to the transconductances $\tilde{\alpha}_k$ of the transistors, we short-circuit the voltage source V_s. The resulting indefinite-admittance matrix is obtained from (13.43) by adding the first row to the fifth row and the first column to the fifth column and then deleting the first row and column. Its first-order cofactor is simply $Y_{11,55}$. Thus, the return differences with respect to $\tilde{\alpha}_k$ are:

$$F(\tilde{\alpha}_1) = \frac{Y_{11,55}(\tilde{\alpha}_1)}{Y_{11,55}(0)} = \frac{466.1\times 10^{-9}}{4.97\times 10^{-9}} = 93.70 \tag{13.45a}$$

$$F(\tilde{\alpha}_2) = \frac{Y_{11,55}(\tilde{\alpha}_2)}{Y_{11,55}(0)} = \frac{466.1\times 10^{-9}}{25.52\times 10^{-9}} = 18.26 \tag{13.45b}$$

13.4 The Null Return Difference

In this section, we introduce the notion of null return difference, which is found to be very useful in measurement situations and in the computation of the sensitivity for the feedback amplifiers.

The **null return difference** $\hat{F}(x)$ of a feedback amplifier with respect to an element x is defined to be the ratio of the two functional values assumed by the second-order cofactor $Y_{rp,sq}$ of the elements of its indefinite-admittance matrix **Y** under the condition that the element x assumes its nominal value and the condition that the element x assumes the value zero where r and s are input terminals, and p and q are the output terminals of the amplifier, or

$$\hat{F}(x) = \frac{Y_{rp,sq}(x)}{Y_{rp,sq}(0)} \tag{13.46}$$

Likewise, the **null return ratio** $\hat{T}$, with respect to a voltage-controlled current source $I = xV$, is the negative of the voltage appearing at the controlling branch when the controlled current source is replaced by an independent current source of x A and when the input excitation is adjusted so that the output of the amplifier is identically zero.

Now, we demonstrate that the null return difference is simply the return difference in the network under the situation that the input excitation I_s has been adjusted so that the output is identically zero. In the network of Figure 13.6, suppose that we replace the controlled current source by an independent current source of x A. Then by applying formula (13.24) and the superposition principle, the output current I_{pq} at the load is:

$$I_{pq} = Y_2 \left[I_s \frac{Y_{rp,sq}(0)}{Y_{uv}(0)} + x \frac{Y_{dp,cq}(0)}{Y_{uv}(0)} \right] \tag{13.47}$$

Setting $I_{pq} = 0$ or $V_{pq} = 0$ yields

$$I_s \equiv I_0 = -x \left[\frac{Y_{dp,cq}(0)}{Y_{rp,sq}(0)} \right] \tag{13.48}$$

in which $Y_{dp,cq}$ is independent of x. This adjustment is possible only if a direct transmission occurs from the input to the output when x is set to zero. Thus, in the network of Figure 13.7, if we connect an independent current source of strength I_0 at its input port, the voltage V'_{ab} is the negative of the null return ratio $\hat{T}$. Using (13.24), we obtain [4]

$$\begin{aligned} \hat{T} = -V'_{ab} &= -x \frac{Y_{da,cb}(0)}{Y_{uv}(0)} - I_0 \frac{Y_{ra,sb}(0)}{Y_{uv}(0)} \\ &= -\frac{x\left[Y_{da,cb}(0)Y_{rp,sq}(0) - Y_{ra,sb}(0)Y_{dp,cq}(0)\right]}{Y_{uv}(0)Y_{rp,sq}(0)} \\ &= \frac{x\dot{Y}_{rp,sq}}{Y_{rp,sq}(0)} = \frac{Y_{rp,sq}(x)}{Y_{rp,sq}(0)} - 1 \end{aligned} \tag{13.49}$$

where

$$\dot{Y}_{rp,sq} \equiv \frac{dY_{rp,sq}(x)}{dx} \tag{13.50}$$

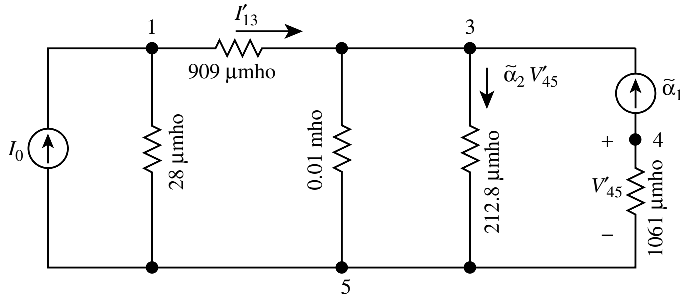

FIGURE 13.11 The network used to compute the null return difference $\hat{F}(\tilde{\alpha}_1)$ by its physical interpretation.

This leads to

$$\hat{F}(x) = 1 + \hat{T} = 1 - V'_{ab} \tag{13.51}$$

which demonstrates that the null return difference $\hat{F}(x)$ is simply the difference of the 1-V excitation applied to the right of the breaking mark of the broken controlling branch of the controlled source and the returned voltage V'_{ab} appearing at the left of the breaking mark under the situation that the input signal I_s is adjusted so that the output is identically zero.

As an illustration, consider the voltage-series feedback amplifier of Figure 13.9, an equivalent network of which is presented in Figure 13.10. Using the indefinite-admittance matrix of (13.43) in conjunction with (13.42), the null return differences with respect to $\hat{\alpha}_k$ are:

$$\hat{F}(\tilde{\alpha}_1) = \frac{Y_{12,55}(\tilde{\alpha}_1)}{Y_{12,55}(0)} = \frac{211.54 \times 10^{-7}}{205.24 \times 10^{-12}} = 103.07 \times 10^3 \tag{13.52a}$$

$$\hat{F}(\tilde{\alpha}_2) = \frac{Y_{12,55}(\tilde{\alpha}_2)}{Y_{12,55}(0)} = \frac{211.54 \times 10^{-7}}{104.79 \times 10^{-10}} = 2018.70 \tag{13.52b}$$

Alternatively, $\hat{F}(\tilde{\alpha}_1)$ can be computed by using its physical interpretation as follows. Replace the controlled source $\tilde{\alpha}_1 V_{13}$ in Figure 13.10 by an independent current source of $\tilde{\alpha}_1$ A. We then adjust the voltage source V_s so that the output current I_{25} is identically zero. Let I_0 be the input current resulting from this source. The corresponding network is presented in Figure 13.11. From this network, we obtain

$$\hat{F}(\tilde{\alpha}_1) = 1 + \hat{T} = 1 - V'_{13} = 1 - \frac{100V'_{35} + \alpha_2 V'_{45} - \alpha_1}{9.09} = 103.07 \times 10^3 \tag{13.53}$$

Likewise, we can use the same procedure to compute the return difference $\hat{F}(\tilde{\alpha}_2)$.