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Measurement of Return Difference[1]

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The zeros of the network determinant are called the **natural frequencies**. Their locations in the complex-frequency plane are extremely important in that they determine the stability of the network. A network is said to be **stable** if all of its natural frequencies are restricted to the open left-half side (LHS) of the complex-frequency plane. If a network determinant is known, its roots can readily be computed explicitly with the aid of a computer if necessary, and the stability problem can then be settled directly. However, for a physical network there remains the difficulty of getting an accurate formulation of the network determinant itself, because every equivalent network is, to a greater or lesser extent, an idealization of the physical reality. As frequency is increased, parasitic effects of the physical elements must be taken into account. What is really needed is some kind of experimental verification that the network is stable and will remain so under certain prescribed conditions. The measurement of the return difference provides an elegant solution to this problem.

The return difference with respect to an element x in a feedback amplifier is defined by

$$F(x) = \frac{Y_{uv}(x)}{Y_{uv}(0)} \tag{15.1}$$

Because $Y_{uv}(x)$ denotes the nodal determinant, the zeros of the return difference are exactly the same as the zeros of the nodal determinant provided that there is no cancellation of common factors between $Y_{uv}(x)$ and $Y_{uv}(0)$. Therefore, if $Y_{uv}(0)$ is known to have no zeros in the closed right-half side (RHS) of the complex-frequency plane, which is usually the case in a single-loop feedback amplifier when x is set to zero, $F(x)$ gives precisely the same information about the stability of a feedback amplifier as does the nodal determinant itself. The difficulty inherent in the measurement of the return difference with respect to the controlling parameter of a controlled source is that, in a physical system, the controlling branch and the controlled source both form part of a single device such as a transistor, and cannot be physically separated. In the following, we present a scheme that does not require the physical decomposition of a device.

Let a device of interest be brought out as a two-port network connected to a general four-port network as shown in Figure 15.1. For our purposes, assume that this device is characterized by its y parameters, and represented by its y-parameter equivalent two-port network as indicated in Figure 15.2, in which

[1]References for this chapter can be found on page **16**-17.

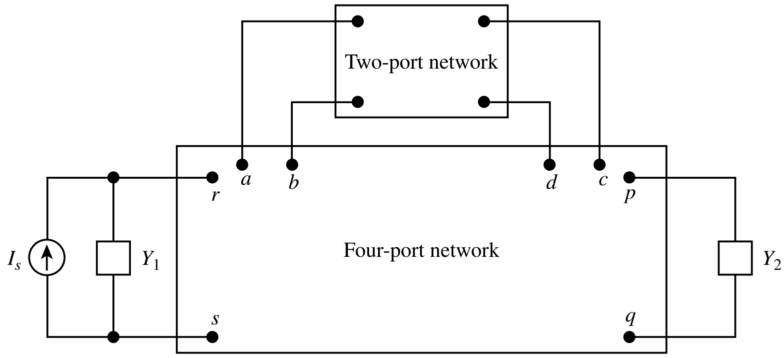

FIGURE 15.1 The general configuration of a feedback amplifier with a two-port device.

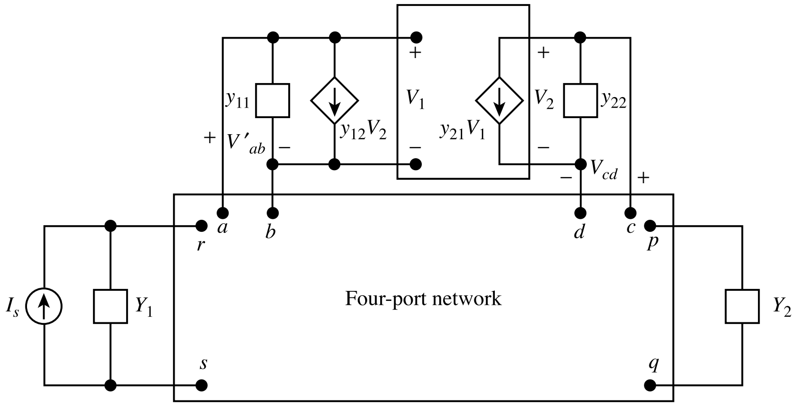

FIGURE 15.2 The representation of a two-port device in Figure 15.1 by its y parameters.

the parameter y_{21} controls signal transmission in the forward direction through the device, whereas y_{12} gives the reverse transmission, accounting for the internal feedback within the device. Our objective is to measure the return difference with respect to the forward short circuit transfer admittance y_{21}.

15.1 Blecher's Procedure [1]

Let the two-port device be a transistor operated in the common-emitter configuration with terminals a, $b = d$, and c representing, respectively, the base, emitter, and collector terminals. To simplify our notation, let $a = 1$, $b = d = 3$ and $c = 2$, as exhibited explicitly in Figure 15.3.

To measure $F(y_{21})$, we break the base terminal of the transistor and apply a 1-V excitation at its input as exhibited in Figure 15.3. To ensure that the controlled current source $y_{21}V_{13}$ drives a replica of what it sees during normal operation, we connect an active one-port network composed of a parallel combination of the admittance y_{11} and a controlled current source $y_{12}V_{23}$ at terminals 1 and 3. The returned voltage V_{13} is precisely the negative of the return ratio with respect to the element y_{21}. If, in the frequency band of interest, the externally applied feedback is large compared with the internal feedback of the transistor, the controlled source $y_{12}V_{23}$ can be ignored. If, however, we find that this internal feedback cannot be ignored, we can simulate it by using an additional transistor, connected as shown in Figure 15.4. This additional transistor must be matched as closely as possible to the one in question. The one-port admittance y_o denotes the admittance presented to the output port of the transistor under consideration as indicated in Figures 15.3 and 15.4. For a common-emitter state, it is perfectly reasonable to assume that $|y_o| \gg |y_{12}|$ and $|y_{11}| \gg |y_{12}|$. Under these assumptions, it is straightforward to show that the Norton equivalent network looking into the two-port network at terminals 1 and 3 of Figure 15.4 can be approximated by

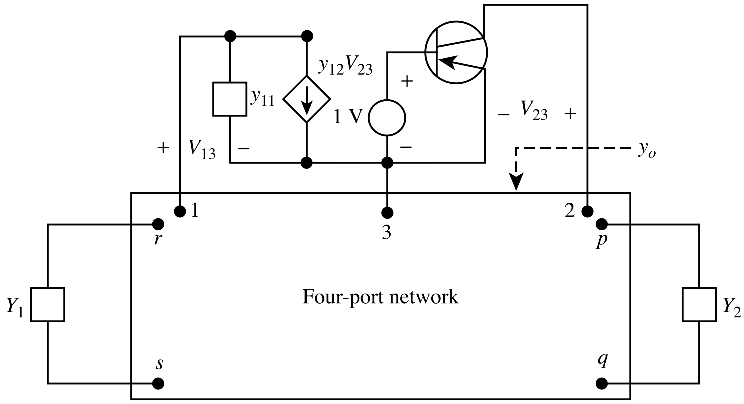

FIGURE 15.3 A physical interpretation of the return difference $F(y_{21})$ for a transistor operated in the common-emitter configuration and represented by its y parameters y_{ij}.

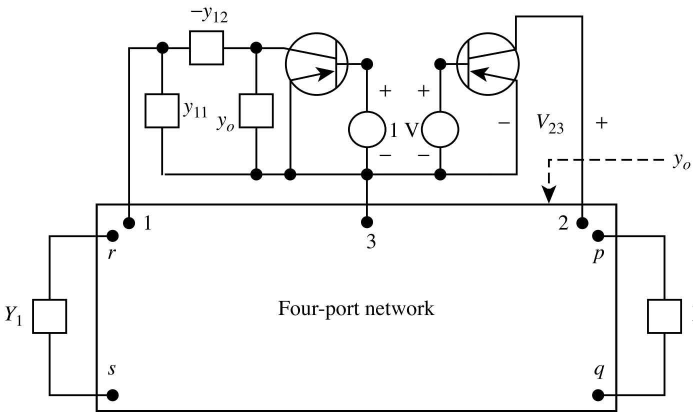

FIGURE 15.4 The measurement of return difference $F(y_{21})$ for a transistor operated in the common-emitter configuration and represented by its y parameters y_{ij}.

the parallel combination of y_{11} and $y_{12}V_{23}$, as indicated in Figure 15.3. In Figure 15.4, if the voltage sources have very low internal impedances, we can join together the two base terminals of the transistors and feed them both from a single voltage source of very low internal impedance. In this way, we avoid the need of using two separate sources. For the procedure to be feasible, we must demonstrate the admittances y_{11} and $-y_{12}$ can be realized as the input admittances of one-port RC networks.

Consider the hybrid-pi equivalent network of a common-emitter transistor of Figure 15.5, the short circuit admittance matrix of which is found to be

$$\mathbf{Y}_{sc} = \frac{1}{g_x + g_\pi + sC_\pi + sC_\mu}\begin{bmatrix} g_x(g_\pi + sC_\pi + sC_\mu) & -g_x sC_\mu \\ g_x(g_m - sC_\mu) & sC_\mu(g_x + g_\pi + sC_\pi + g_m) \end{bmatrix} \tag{15.2}$$

It is easy to confirm that the admittance y_{11} and $-y_{12}$ can be realized by the one-port networks of Figure 15.6.

15.2 Impedance Measurements

In this section, we show that the return difference can be evaluated by measuring two driving-point impedances at a convenient port in the feedback amplifier [8].

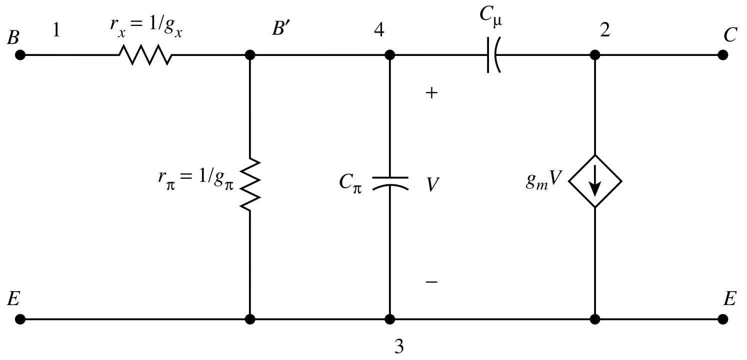

FIGURE 15.5 The hybrid-pi equivalent network of a common-emitter transistor.

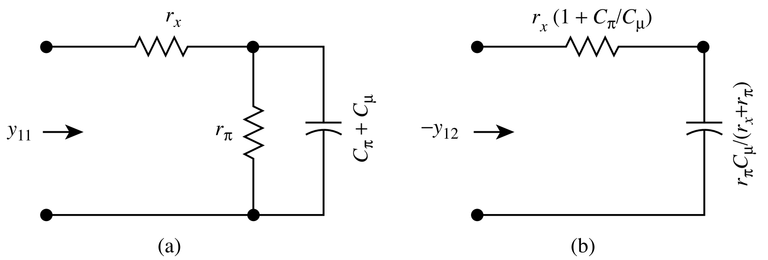

FIGURE 15.6 (a) The realization of y_{11} and (b) the realization of $-y_{12}$.

Refer again to the general feedback configuration of Figure 15.2. Suppose that we wish to evaluate the return difference with respect to the forward short circuit transfer admittance y_{21}. The controlling parameters y_{12} and y_{21} enter the indefinite-admittance matrix **Y** in the rectangular patterns as shown next:

$$\mathbf{Y}(x) = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} & & y_{12} & -y_{12} \\ & & -y_{12} & y_{12} \\ y_{21} & -y_{21} & & \\ -y_{21} & y_{21} & & \end{bmatrix} \end{array} \tag{15.3}$$

To emphasize the importance of y_{12} and y_{21}, we again write $Y_{uv}(x)$ as $Y_{uv}(y_{12}, y_{21})$ and $z_{aa,bb}(x)$ as $z_{aa,bb}(y_{12}, y_{21})$. By appealing to formula (13.25), the impedance looking into terminals a and b of Figure 15.2 is:

$$z_{aa,bb}(y_{12}, y_{21}) = \frac{Y_{aa,bb}(y_{12}, y_{21})}{Y_{dd}(y_{12}, y_{21})} \tag{15.4}$$

The return difference with respect to y_{21} is given by

$$F(y_{21}) = \frac{Y_{dd}(y_{12}, y_{21})}{Y_{dd}(y_{12}, 0)} \tag{15.5}$$

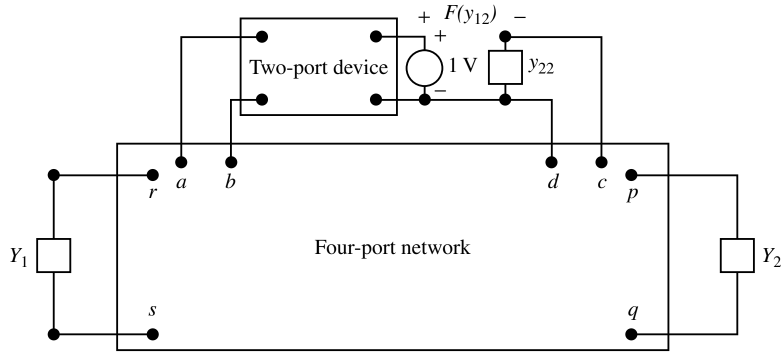

FIGURE 15.7 The measurement of the return difference $F(y_{12})$ with y_{21} set to zero.

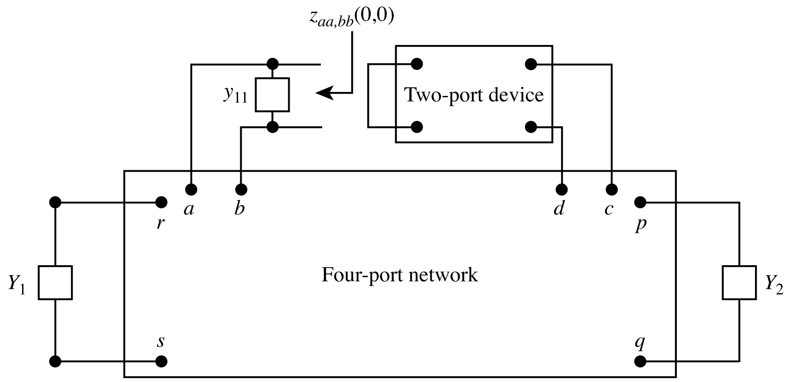

FIGURE 15.8 The measurement of the driving-point impedance $z_{aa,bb}(0, 0)$.

Combining these yields

$$\begin{aligned} F(y_{21})z_{aa,bb}(y_{12}, y_{21}) &= \frac{Y_{aa,bb}(y_{12}, y_{21})}{Y_{dd}(y_{12}, 0)} = \frac{Y_{aa,bb}(0,0)}{Y_{dd}(y_{12}, 0)} \\ &= \frac{Y_{aa,bb}(0,0)}{Y_{dd}(0,0)} \frac{Y_{dd}(0,0)}{Y_{dd}(y_{12}, 0)} = \frac{z_{aa,bb}(0,0)}{F(y_{12})\big|_{y_{21}=0}} \end{aligned} \tag{15.6}$$

obtaining a relation

$$F(y_{12})\big|_{y_{21}=0} F(y_{21}) = \frac{z_{aa,bb}(0,0)}{z_{aa,bb}(y_{12}, y_{21})} \tag{15.7}$$

among the return differences and the driving-point impedances. $F(y_{12})|_{y_{21}=0}$ is the return difference with respect to y_{12} when y_{21} is set to zero. This quantity can be measured by the arrangement of Figure 15.7. $z_{aa,bb}(y_{12}, y_{21})$ is the driving-point impedance looking into terminals a and b of the network of Figure 15.2. Finally, $z_{aa,bb}(0, 0)$ is the impedance to which $z_{aa,bb}(y_{12}, y_{21})$ reduces when the controlling parameters y_{12} and y_{21} are both set to zero. This impedance can be measured by the arrangement of Figure 15.8. Note that, in all three measurements, the independent current source I_s is removed.

Suppose that we wish to measure the return difference $F(y_{21})$ with respect to the forward transfer admittance y_{21} of a common-emitter transistor shown in Figure 15.2. Then, the return difference $F(y_{12})$

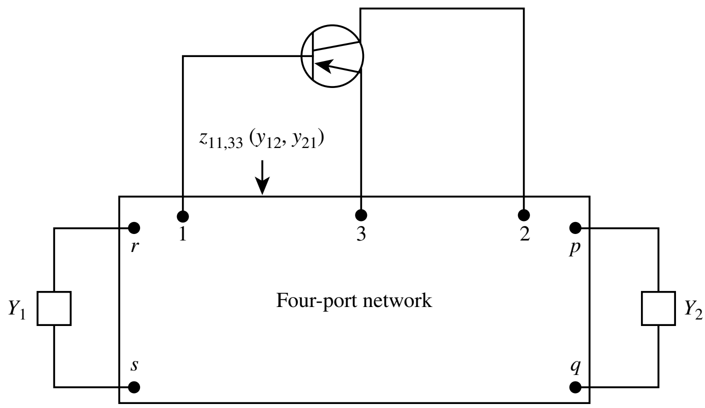

FIGURE 15.9 The measurement of the driving-point impedance $z_{11,33}(y_{12}, y_{21})$.

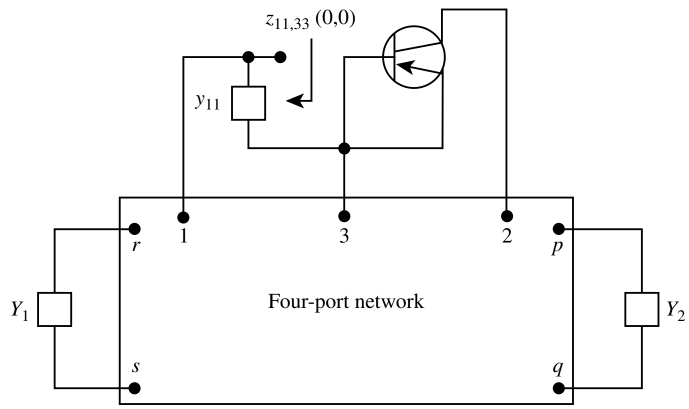

FIGURE 15.10 The measurement of the driving-point impedance $z_{11,33}(0, 0)$.

when y_{21} is set to zero, for all practical purposes, is indistinguishable from unity. Therefore, (15.7) reduces to the following simpler form:

$$F(y_{21}) \approx \frac{z_{11,33}(0,0)}{z_{11,33}(y_{12}, y_{21})} \tag{15.8}$$

showing that the return difference $F(y_{21})$ effectively equals the ratio of two functional values assumed by the driving-point impedance looking into terminals 1 and 3 of Figure 15.2 under the condition that the controlling parameters y_{12} and y_{21} are both set to zero and the condition that they assume their nominal values. These two impedances can be measured by the network arrangements of Figures 15.9 and 15.10.