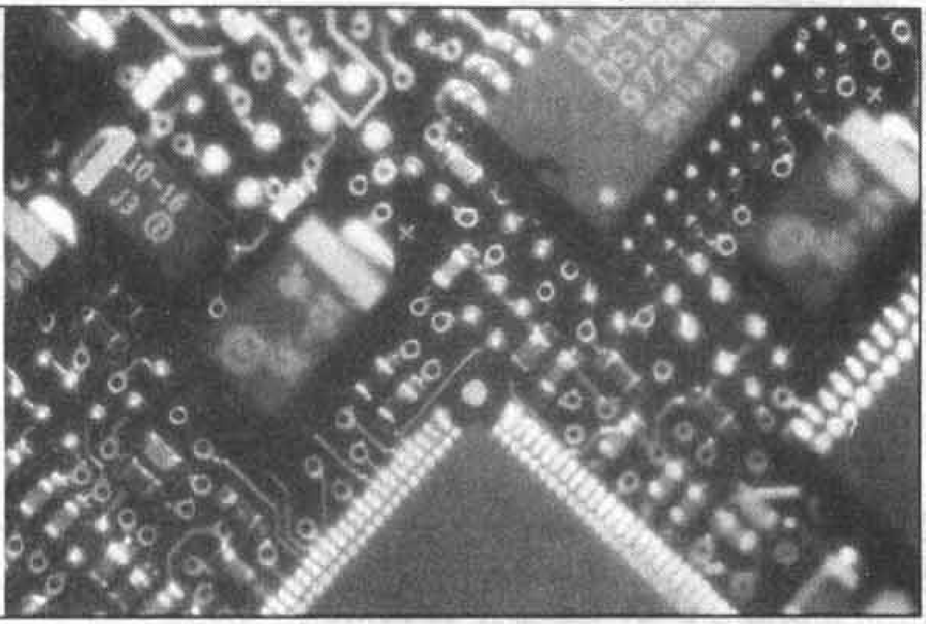
PART I



Semiconductor Devices and Basic Applications

In the first part of the text, we introduce the physical characteristics and operation of the major semiconductor devices and the basic circuits in which they are used, to illustrate how the device characteristics are utilized in switching, digital, and amplification applications

Chapter 1 briefly discusses semiconductor material characteristics and then introduces the semiconductor diode. Chapter 2 looks at various diode circuits that demonstrate how the nonlinear characteristics of the diode itself are used in switching and waveshaping applications. Chapter 3 introduces the bipolar transistor, presents the dc analysis of bipolar transistor circuits, and discusses basic applications of the transistor. In Chapter 4, we design and analyze fundamental bipolar transistor circuits, including amplifiers.

Chapter 5 introduces the field-effect transistor (FET), and FET circuits are analyzed and designed in Chapter 6. Chapter 7 considers the frequency response of both bipolar and field-effect transistor circuits. Finally, Chapter 8 discusses the designs and applications of these basic electronic circuits, including power amplifiers and various output stages.

CHAPTER

1

Semiconductor Materials and Diodes

1.0 PREVIEW

This text deals with the analysis and design of circuits containing electronic devices, such as diodes and transistors. These electronic devices are fabricated using semiconductor materials, so we begin Chapter 1 with a brief discussion of the properties and characteristics of semiconductors. The intent of this brief discussion is to become familiar with some of the semiconductor material terminology.

A basic electronic device is the pn junction diode. One of the more interesting characteristics of the diode is its nonlinear current–voltage properties. The resistor, for example, has a linear relation between the current through it and the voltage across the element. The diode is also a two-terminal device, but the i–v relationship is nonlinear. The current is an exponential function of voltage in one direction and is essentially zero in the other direction. As we will see, this nonlinear characteristic makes possible the generation of a dc voltage from an ac voltage source and the design of digital logic circuits, for example.

Since the diode is a nonlinear element, the analysis of circuits containing diodes is not as straightfoward as is the analysis of simple resistor circuits. A mathematical model of the diode, describing the nonlinear i–v properties, is developed. However, the circuit cannot be analyzed, in general, by direct mathematical calculations. In many engineering problems, approximate "back-of-the-envelope" solutions replace difficult complex solutions. We develop one such approximation technique using the piecewise linear model of the diode. In this case, we replace the nonlinear diode properties by linear characteristics that are approximately valid over a limited region of operation. This concept is used throughout the study of electronics.

Besides the pn junction diode, we consider five other types of diodes that are used in specialized electronic applications. These include the solar cell, photodiode, light-emitting diode, Schottky barrier diode, and the Zener diode.

The general properties of the diode are considered in this chapter. Simple diode circuits are analyzed with the intent of developing a basic understanding of analysis techniques and diode circuit characteristics. Chapter 2 then considers applications of diodes in circuits that perform various electronic functions.

1.1 SEMICONDUCTOR MATERIALS AND PROPERTIES

Most electronic devices are fabricated by using semiconductor materials along with conductors and insulators. To gain a better understanding of the behavior of the electronic devices in circuits, we must first understand a few of the characteristics of the semiconductor material. Silicon is by far the most common semiconductor material used for semiconductor devices and integrated circuits. Other semiconductor materials are used for specialized applications. For example, gallium arsenide and related compounds are used for very-high-speed devices and optical devices.

1.1.1 Intrinsic Semiconductors

An atom is composed of a nucleus, which contains positively charged protons and neutral neutrons, and negatively charged electrons that, in the classical sense, orbit the nucleus. The electrons are distributed in various "shells" at different distances from the nucleus, and electron energy increases as shell radius increases. Electrons in the outermost shell are called **valence electrons**, and the chemical activity of a material is determined primarily by the number of such electrons.

Elements in the period table can be grouped according to the number of valence electrons. Table 1.1 shows a portion of the periodic table in which the more common semiconductors are found. Silicon (Si) and germanium (Ge) are in group IV and are **elemental semiconductors**. In contrast, gallium arsenide is a group III–V **compound semiconductor**. We will show that the elements in group III and group V are also important in semiconductors.

Table 1.1 A portion of the periodic table

III	IV	V
B	C	
Al	Si	P
Ga	Ge	As

Figure 1.1(a) shows five noninteracting silicon atoms, with the four valence electrons of each atom shown as dashed lines emanating from the atom. As silicon atoms come into close proximity to each other, the valence electrons interact to form a crystal. The final crystal structure is a tetrahedral configuration in which each silicon atom has four nearest neighbors, as shown in Figure 1.1(b). The valence electrons are shared between atoms, forming what are called **covalent bonds**. Germanium, gallium arsenide, and many other semiconductor materials have the same tetrahedral configuration.

Figure 1.1(c) is a two-dimensional representation of the lattice formed by the five silicon atoms in Figure 1.1(a). An important property of such a lattice is that valence electrons are always available on the outer edge of the silicon crystal so that additional atoms can be added to form very large single-crystal structures.

A two-dimensional representation of a silicon single crystal is shown in Figure 1.2, for $T = 0\,°K$, where T = temperature. Each line between atoms

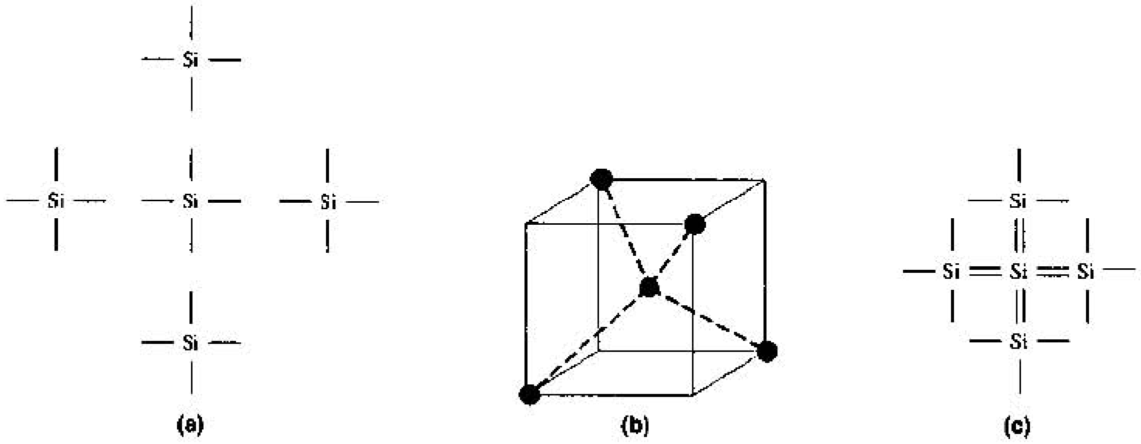


Figure 1.1 Silicon atoms in a crystal matrix: (a) five noninteracting silicon atoms, each with four valence electrons, (b) the tetrahedral configuration, (c) a two-dimensional representation showing the covalent bonding

represents a valence electron. At $T = 0\,^\circ$K, each electron is in its lowest possible energy state, so each covalent bonding position is filled. If a small electric field is applied to this material, the electrons will not move, because they will still be bound to their individual atoms. Therefore, at $T = 0\,^\circ$K, silicon is an **insulator**; that is, no charge flows through it.

If the temperature increases, the valence electrons will gain thermal energy. Any such electron may gain enough thermal energy to break the covalent bond and move away from its original position (Figure 1.3). The electron will then be free to move within the crystal.

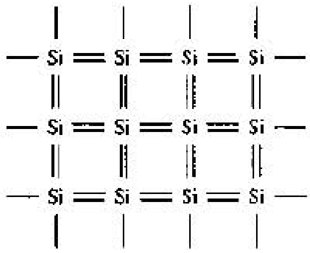


Figure 1.2 Two-dimensional representation of the silicon crystal at $T = 0\,^\circ$K

Since the net charge of the material is neutral, if a negatively charged electron breaks its covalent bond and moves away from its original position, a positively charged "empty state" is created at that position (Figure 1.3). As the temperature increases, more covalent bonds are broken and more free electrons and positive empty states are created.

In order to break the covalent bond, a valence electron must gain a minimum energy, E_g, called the **bandgap energy**. Materials that have large bandgap energies, in the range of 3 to 6 electron–volts[1] (eV), are insulators because, at room temperature, essentially no free electrons exist in these materials. In contrast, materials that contain very large numbers of free electrons at room temperature are **conductors**.

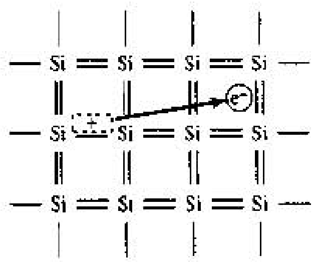


Figure 1.3 The breaking of a covalent bond for $T > 0\,^\circ$K

In a *semiconductor*, the bandgap energy is on the order of 1 eV. The net flow of free electrons in a semiconductor causes a current. In addition, a valence electron that has a certain thermal energy and is adjacent to an empty state may move into that position, as shown in Figure 1.4 making it appear as if a positive charge is moving through the semiconductor. This positively charged "particle" is called a **hole**. In semiconductors, then, two types of charged particles contribute to the current: the negatively charged free electron, and the positively charged hole. (This description of a hole is

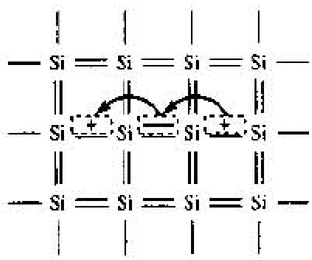


Figure 1.4 A two-dimensional representation of the silicon crystal showing the movement of the positively charged hole

[1]An electron-volt is the energy of an electron that has been accelerated through a potential difference of 1 volt, and $1\ \text{eV} = 1.6 \times 10^{-19}$ joules.

greatly oversimplified, and is meant only to convey the concept of the moving positive charge.)

The concentrations (#/cm^3) of electrons and holes are important parameters in the characteristics of a semiconductor material, because they directly influence the magnitude of the current. An **intrinsic semiconductor** is a single-crystal semiconductor material with no other types of atoms within the crystal. In an intrinsic semiconductor, the densities of electrons and holes are equal, since the thermally generated electrons and holes are the only source of such particles. Therefore, we use the notation n_i as the **intrinsic carrier concentration** for the concentration of the free electrons, as well as that of the holes. The equation for n_i is as follows:

$$n_i = BT^{3/2}e^{\left(\frac{-E_g}{2kT}\right)} \tag{1.1}$$

where B is a constant related to the specific semiconductor material, E_g is the bandgap energy (eV), T is the temperature (°K), and k is Boltzmann's constant (86×10^{-6} eV/°K). The values for B and E_g for several semiconductor materials are given in Table 1.2. The bandgap energy is not a strong function of temperature.

Table 1.2 Semiconductor constants

Material	E_g (eV)	B (cm^{-3} °K$^{-3/2}$)
Silicon (Si)	1.1	5.23×10^{15}
Gallium arsenide (GaAs)	1.4	2.10×10^{14}
Germanium (Ge)	0.66	1.66×10^{15}

Example 1.1 **Objective:** Calculate the intrinsic carrier concentration in silicon at $T = 300$ °K.

Solution: For silicon at $T = 300$ °K, we can write

$$n_i = BT^{3/2}e^{\left(\frac{-E_g}{2kT}\right)}$$

$$= (5.23 \times 10^{15})(300)^{3/2}e^{\left(\frac{-1.1}{2(86\times10^{-6})(300)}\right)}$$

or

$$n_i = 1.5 \times 10^{10}\ \text{cm}^{-3}$$

Comment: An intrinsic electron concentration of 1.5×10^{10} cm^{-3} may appear to be large, but it is relatively small compared to the concentration of silicon atoms, which is 5×10^{22} cm^{-3}.

The intrinsic concentration n_i is an important parameter that appears often in the current–voltage equations for semiconductor devices.

Test Your Understanding

1.1 Calculate the intrinsic carrier concentration in gallium arsenide and germanium at $T = 300°K$. (Ans. GaAs, $n_i = 1.80 \times 10^6\ cm^{-3}$; Ge, $n_i = 2.40 \times 10^{13}\ cm^{-3}$)

1.2 Determine the intrinsic carrier concentration in silicon, gallium arsenide, and germanium at $T = 400°K$. (Ans. Si, $n_i = 4.76 \times 10^{12}\ cm^{-3}$; GaAs, $n_i = 2.44 \times 10^9\ cm^{-3}$; Ge, $n_i = 9.06 \times 10^{14}\ cm^{-3}$)

1.1.2 Extrinsic Semiconductors

Because the electron and hole concentrations in an intrinsic semiconductor are relatively small, only very small currents are possible. However, these concentrations can be greatly increased by adding controlled amounts of certain impurities. A desirable impurity is one that enters the crystal lattice and replaces (i.e., substitutes for) one of the semiconductor atoms, even though the impurity atom does not have the same valence electron structure. For silicon, the desirable substitutional impurities are from the group III and V elements (see Table 1.1).

The most common group V elements used for this purpose are phosphorus and arsenic. For example, when a phosphorus atom substitutes for a silicon atom, as shown in Figure 1.5, four of its valence electrons are used to satisfy the covalent bond requirements. The fifth valence electron is more loosely bound to the phosphorus atom. At room temperature, this electron has enough thermal energy to break the bond, thus being free to move through the crystal and contribute to the electron current in the semiconductor.

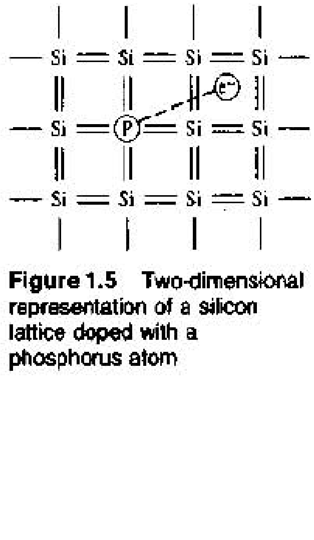


Figure 1.5 Two-dimensional representation of a silicon lattice doped with a phosphorus atom

The phosphorus atom is called a **donor impurity**, since it donates an electron that is free to move. Although the remaining phosphorus atom has a net positive charge, the atom is immobile in the crystal and cannot contribute to the current. Therefore, when a donor impurity is added to a semiconductor, free electrons are created without generating holes. This process is called **doping**, and it allows us to control the concentration of free electrons in a semiconductor.

A semiconductor that contains donor impurity atoms is called an **n-type semiconductor** (for the negatively charged electrons).

The most common group III element used for silicon doping is boron. When a boron atom replaces a silicon atom, its three valence electrons are used to satisfy the covalent bond requirements for three of the four nearest silicon atoms (Figure 1.6). This leaves one bond position open. At room temperature, adjacent silicon valence electrons have sufficient thermal energy to move into this position, thereby creating a hole. The boron atom then has a net negative charge, but cannot move, and a hole is created that can contribute to a hole current.

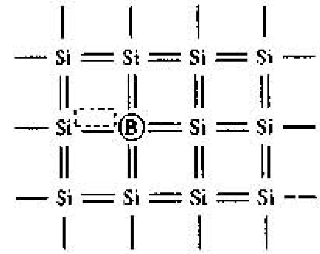


Figure 1.6 Two-dimensional representation of a silicon lattice doped with a boron atom

Because the boron atom has accepted a valence electron, the boron is therefore called an **acceptor impurity**. Acceptor atoms lead to the creation of holes without electrons being generated. This process, also called doping, can be used to control the concentration of holes in a semiconductor.

A semiconductor that contains acceptor impurity atoms is called a **p-type semiconductor** (for the positively charged holes created).

The materials containing impurity atoms are called **extrinsic semiconductors**, or **doped semiconductors**. The doping process, which allows us to control the concentrations of free electrons and holes, determines the conductivity and currents in the material.

A fundamental relationship between the electron and hole concentrations in a semiconductor *in thermal equilibrium* is given by

$$n_o p_o = n_i^2 \tag{1.2}$$

where n_o is the thermal equilibrium concentration of free electrons, p_o is the thermal equilibrium concentration of holes, and n_i is the intrinsic carrier concentration.

At room temperature ($T = 300\,^\circ\text{K}$), each donor atom donates a free electron to the semiconductor. If the donor concentration N_d is much larger than the intrinsic concentration, we can approximate

$$n_o \cong N_d \tag{1.3}$$

Then, from Equation (1.2), the hole concentration is

$$p_o = \frac{n_i^2}{N_d} \tag{1.4}$$

Similarly, at room temperature, each acceptor atom accepts a valence electron, creating a hole. If the acceptor concentration N_a is much larger than the intrinsic concentration, we can approximate

$$p_o \cong N_a \tag{1.5}$$

Then, from Equation (1.2), the electron concentration is

$$n_o = \frac{n_i^2}{N_a} \tag{1.6}$$

Example 1.2 **Objective:** Calculate the thermal equilibrium electron and hole concentrations.

Consider silicon at $T = 300\,^\circ\text{K}$ doped with phosphorus at a concentration of $N_d = 10^{16}\,\text{cm}^{-3}$. Recall from Example 1.1 that $n_i = 1.5 \times 10^{10}\,\text{cm}^{-3}$.

Solution: Since $N_d \gg n_i$, the electron concentration is

$$n_o \cong N_d = 10^{16}\,\text{cm}^{-3}$$

and the hole concentration is

$$p_o = \frac{n_i^2}{N_d} = \frac{(1.5 \times 10^{10})^2}{10^{16}} = 2.25 \times 10^4\,\text{cm}^{-3}$$

Comment: In an extrinsic semiconductor, the electron and hole concentrations normally differ by many orders of magnitude.

In an n-type semiconductor, electrons are called the **majority carrier** because they far outnumber the holes, which are termed the **minority carrier**. The results obtained in Example 1.2 clarify this definition. In contrast, in a p-type semiconductor, the holes are the majority carrier and the electrons are the minority carrier.

Test Your Understanding

1.3 Calculate the majority and minority carrier concentrations in silicon at $T = 300\,°K$ if (a) $N_a = 10^{17}\,cm^{-3}$, and (b) $N_d = 5 \times 10^{15}\,cm^{-3}$. (Ans. (a) $p_o = 10^{17}\,cm^{-3}$, $n_o = 2.25 \times 10^3\,cm^{-3}$, (b) $n_o = 5 \times 10^{15}\,cm^{-3}$, $p_o = 4.5 \times 10^4\,cm^{-3}$)

1.1.3 Drift and Diffusion Currents

The two basic processes which cause electrons and holes to move in a semiconductor are: (a) **drift**, which is the movement caused by electric fields; and (b) **diffusion**, which is the flow caused by variations in the concentration, that is, concentration gradients. Such gradients can be caused by a nonhomogeneous doping distribution, or by the injection of a quantity of electrons or holes into a region, using methods to be discussed later in this chapter.

To understand drift, assume an electric field is applied to a semiconductor. The field produces a force that acts on free electrons and holes, which then experience a net drift velocity and net movement. Consider an n-type semiconductor with a large number of free electrons (Figure 1.7(a)). An electric field E applied in one direction produces a force on the electrons in the *opposite* direction, because of the electrons' negative charge. The electrons acquire a drift velocity v_{dn} (in cm/s) which can be written as

$$v_{dn} = -\mu_n E \tag{1.7}$$

where μ_n is a constant called the electron mobility and has units of $cm^2/V\text{–}s$. For low-doped silicon, the value of μ_n is typically $1350\,cm^2/V\text{–}s$. The mobility can be thought of as a parameter indicating how well an electron can move in a semiconductor. The negative sign in Equation (1.7) indicates that the electron drift velocity is opposite to that of the applied electric field as shown in Figure 1.7(a). The electron drift produces a drift current density J_n (A/cm^2) given by

$$J_n = -env_{dn} = -en(-\mu_n E) = +en\mu_n E \tag{1.8}$$

where n is the electron concentration ($\#/cm^3$) and e is the magnitude of the electronic charge. The conventional drift current is in the opposite direction from the flow of negative charge, which means that the drift current in an n-type semiconductor is in the same direction as the applied electric field.

Next consider a p-type semiconductor with a large number of holes (Figure 1.7(b)). An electric field E applied in one direction produces a force on the holes in the *same* direction, because of the positive charge on the holes. The holes acquire a drift velocity v_{dp} (in cm/s) which can be written as

$$v_{dp} = +\mu_p E \tag{1.9}$$

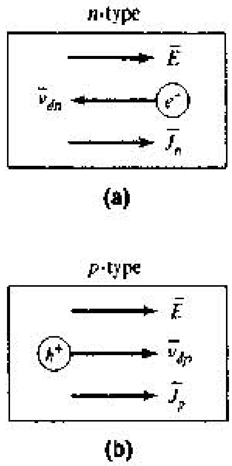


Figure 1.7 Applied electric field, carrier drift velocity, and drift current density in (a) an n-type semiconductor and (b) a p-type semiconductor

where μ_p is a constant called the hole mobility, and again has units of $cm^2/V\text{–}s$. For low-doped silicon, the value of μ_p is typically 480 $cm^2/V\text{–}s$, which is slightly less than half the value of the electron mobility. The positive sign in Equation (1.9) indicates that the hole drift velocity is in the same direction as the applied electric field as shown in Figure 1.7(b). The hole drift produces a drift current density J_p (A/cm^2) given by

$$J_p = +epv_{dp} = +ep(+\mu_p E) = +ep\mu_p E \tag{1.10}$$

where p is the hole concentration ($\#/cm^3$) and e is again the magnitude of the electronic charge. The conventional drift current is in the same direction as the flow of positive charge, which means that the drift current in a p-type material is also in the same direction as the applied electric field.

Since a semiconductor contains both electrons and holes, the total drift current density is the sum of the electron and hole components. The total drift current density is then written as

$$J = en\mu_n E + ep\mu_p E = \sigma E \tag{1.11(a)}$$

where

$$\sigma = en\mu_n + ep\mu_p \tag{1.11(b)}$$

and where σ is the **conductivity** of the semiconductor in $(\Omega\text{–cm})^{-1}$. The conductivity is related to the concentration of electrons and holes. If the electric field is the result of applying a voltage to the semiconductor, then Equation (1.11(a)) becomes a linear relationship between current and voltage and is one form of Ohm's law.

From Equation (1.11(b)), we see that the conductivity can be changed from strongly n-type, $n \gg p$, by donor impurity doping to strongly p-type, $p \gg n$, by acceptor impurity doping. **Being able to control the conductivity of a semiconductor by selective doping is what allows us to fabricate the variety of electronic devices that are available.**

With diffusion, particles flow from a region of high concentration to a region of lower concentration. This is a statistical phenomenon related to kinetic theory. To explain, the electrons and holes in a semiconductor are in continuous motion, with an average speed determined by the temperature, and with the directions randomized by interactions with the lattice atoms. Statistically, we can assume that, at any particular instant, approximately half of the particles in the high-concentration region are moving *away* from that region toward the lower-concentration region. We can also assume that, at the same time, approximately half of the particles in the lower-concentration region are moving *toward* the high-concentration region. However, by definition, there are fewer particles in the lower-concentration region than there are in the high-concentration region. Therefore, the net result is a flow of particles away from the high-concentration region and toward the lower-concentration region. This is the basic diffusion process.

For example, consider an electron concentration that varies as a function of distance x, as shown in Figure 1.8(a). The diffusion of electrons from a high-concentration region to a low-concentration region produces a flow of electrons in the negative x direction. Since electrons are negatively charged, the conventional current direction is in the positive x direction.

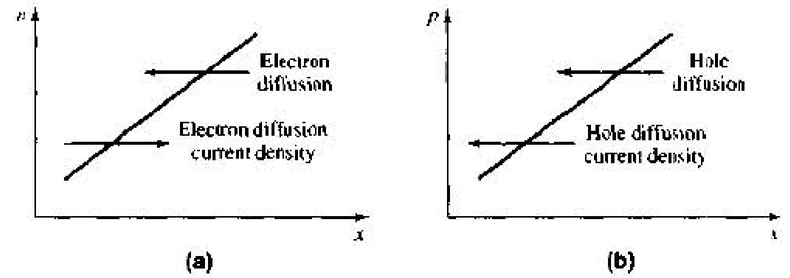


Figure 1.8 Current density caused by concentration gradients: (a) electron diffusion and corresponding current density and (b) hole diffusion and corresponding current density

In Figure 1.8(b), the hole concentration is a function of distance. The diffusion of holes from a high-concentration region to a low-concentration region produces a flow of holes in the negative x direction.

The *total* current density is the sum of the drift and diffusion components. Fortunately, in most cases only one component dominates the current at any one time in a given region of a semiconductor.

1.1.4 Excess Carriers

Up to this point, we have assumed that the semiconductor is in thermal equilibrium. In the discussion of drift and diffusion currents, we implicitly assumed that equilibrium was not significantly disturbed. Yet, when a voltage is applied to, or a current exists in, a semiconductor device, the semiconductor is really not in equilibrium. In this section, we will discuss the behavior of nonequilibrium electron and hole concentrations.

Valence electrons may acquire sufficient energy to break the covalent bond and become free electrons if they interact with high-energy photons incident on the semiconductor. When this occurs, both an electron and a hole are produced, thus generating an electron–hole pair. These additional electrons and holes are called **excess electrons** and **excess holes**.

When these excess electrons and holes are created, the concentrations of free electrons and holes increase above their thermal equilibrium values. This may be represented by

$$n = n_o + \delta n \qquad \textbf{(1.12(a))}$$

and

$$p = p_o + \delta p \qquad \textbf{(1.12(b))}$$

where n_o and p_o are the thermal equilibrium concentrations of electrons and holes, and δn and δp are the excess electron and hole concentrations.

If the semiconductor is in a steady-state condition, the creation of excess electrons and holes will not cause the carrier concentration to increase indefinitely, because a free electron may recombine with a hole, in a process called **electron–hole recombination**. Both the free electron and the hole disappear causing the excess concentration to reach a steady-state value. The mean time over which an excess electron and hole exist before recombination is called the **excess carrier lifetime**.

Test Your Understanding

1.4 Consider silicon at $T = 300°K$. Assume that $\mu_n = 1350\,cm^2/V\text{–}s$ and $\mu_p = 480\,cm^2/V\text{–}s$. Determine the conductivity if (a) $N_d = 5 \times 10^{16}\,cm^{-3}$ and (b) $N_a = 5 \times 10^{16}\,cm^{-3}$. (Ans. (a) $10.8\,(\Omega\text{–}cm)^{-1}$, (b) $3.84\,(\Omega\text{–}cm)^{-1}$.

1.5 A sample of silicon at $T = 300°K$ is doped to $N_d = 8 \times 10^{15}\,cm^{-3}$. (a) Calculate n_o and p_o. (b) If excess holes and electrons are generated such that their respective concentrations are $\delta p = \delta n = 10^{14}\,cm^{-3}$, determine the total concentrations of holes and electrons. (Ans. (a) $n_o = 8 \times 10^{15}\,cm^{-3}$, $p_o = 2.81 \times 10^{4}\,cm^{-3}$; (b) $n_o = 8.1 \times 10^{15}\,cm^{-3}$, $p_o \cong 10^{14}\,cm^{-3}$)

1.6 The conductivity of silicon is $\sigma = 10\,(\Omega\text{–}cm)^{-1}$. Determine the drift current density if an electric field of $E = 15\,V/cm$ is applied. (Ans. $J = 150\,A/cm^2$)

1.2 THE pn JUNCTION

In the preceding sections, we looked at characteristics of semiconductor materials. The real power of semiconductor electronics occurs when p- and n-regions are directly adjacent to each other, forming a **pn junction.** One important concept to remember is that in most integrated circuit applications, the entire semiconductor material is a single crystal, with one region doped to be p-type and the adjacent region doped to be n-type.

1.2.1 The Equilibrium pn Junction

Figure 1.9(a) is a simplified block diagram of a pn junction. Figure 1.9(b) shows the respective p-type and n-type doping concentrations, assuming uniform doping in each region, as well as the minority carrier concentrations in each region, assuming thermal equilibrium.

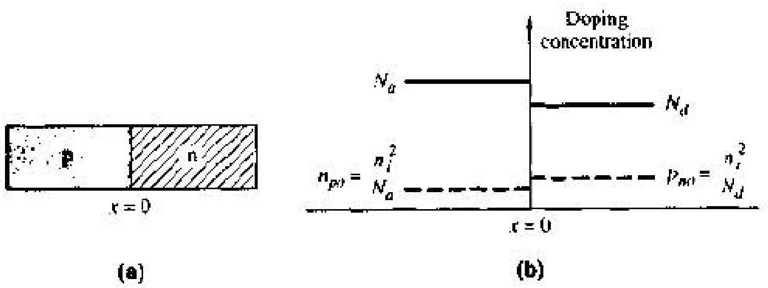


Figure 1.9 The pn junction: (a) simplified geometry of a pn junction and (b) doping profile of an ideal uniformly doped pn junction

The interface at $x = 0$ is called the **metallurgical junction.** A large density gradient in both the hole and electron concentrations occurs across this junction. Initially, then, there is a diffusion of holes from the p-region into the n-region, and a diffusion of electrons from the n-region into the p-region (Figure 1.10). The flow of holes from the p-region uncovers negatively charged acceptor ions, and the flow of electrons from the n-region uncovers positively

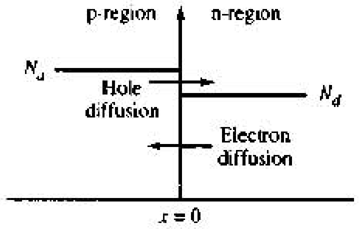


Figure 1.10 Initial diffusion of electrons and holes at the metallurgical junction, establishing thermal equilibrium

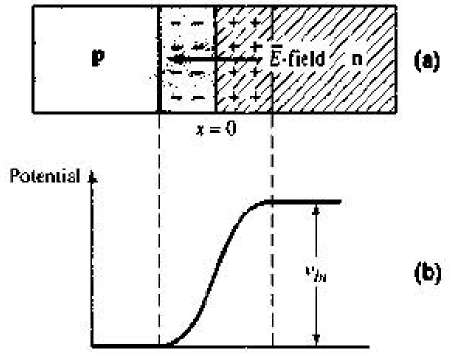


Figure 1.11 The pn junction in thermal equilibrium: (a) the space-charge region and electric field and (b) the potential through the junction

charged donor ions. This action creates a charge separation (Figure 1.11(a)), which sets up an electric field oriented in the direction from the positive charge to the negative charge.

If no voltage is applied to the pn junction, the diffusion of holes and electrons must eventually cease. The direction of the induced electric field will cause the resulting force to repel the diffusion of holes from the p-region and the diffusion of electrons from the n-region. Thermal equilibrium occurs when the force produced by the electric field and the "force" produced by the density gradient exactly balance.

The positively charged region and the negatively charged region comprise the **space-charge** region, or **depletion region**, of the pn junction, in which there are essentially no mobile electrons or holes. Because of the electric field in the space-charge region, there is a potential difference across that region (Figure 1.11(b)). This potential difference is called the **built-in potential barrier**, or built-in voltage, and is given by

$$V_{bi} = \frac{kT}{e}\ln\left(\frac{N_a N_d}{n_i^2}\right) = V_T \ln\left(\frac{N_a N_d}{n_i^2}\right) \tag{1.13}$$

where $V_T \equiv kT/e$, $k =$ Boltzmann's constant, $T =$ absolute temperature, $e =$ the magnitude of the electronic charge, and N_a and N_d are the net acceptor and donor concentrations in the p- and n-regions, respectively. The parameter V_T is called the **thermal voltage** and is approximately $V_T = 0.026$ V at room temperature, $T = 300\,°K$.

Example 1.3 **Objective:** Calculate the built-in potential barrier of a pn junction.

Consider a silicon pn junction at $T = 300\,°K$, doped at $N_a = 10^{16}\,cm^{-3}$ in the p-region and $N_d = 10^{17}\,cm^{-3}$ in the n-region.

Solution: From the results of Example 1.1, we have $n_i = 1.5 \times 10^{10}\,cm^{-3}$ for silicon at room temperature. We then find

$$V_{bi} = V_T \ln\left(\frac{N_a N_d}{n_i^2}\right) = (0.026)\ln\left[\frac{(10^{16})(10^{17})}{(1.5 \times 10^{10})^2}\right] = 0.757\text{ V}$$

Comment Because of the log function, the magnitude of V_{bi} is not a strong function of the doping concentrations. Therefore, the value of V_{bi} for silicon pn junctions is usually within 0.1 to 0.2 V of this calculated value.

The potential difference, or built-in potential barrier, across the space-charge region cannot be measured by a voltmeter because new potential barriers form between the probes of the voltmeter and the semiconductor, canceling the effects of V_{bi}. In essence, V_{bi} maintains equilibrium, so no current is produced by this voltage. However, the magnitude of V_{bi} becomes important when we apply a forward-bias voltage, as discussed later in this chapter.

Test Your Understanding

1.7 Determine V_{bi} for a silicon pn junction at $T = 300\,^\circ$K for (a) $N_a = 10^{15}\ \text{cm}^{-3}$, $N_d = 10^{17}\ \text{cm}^{-3}$, and for (b) $N_a = N_d = 10^{17}\ \text{cm}^{-3}$. (Ans. (a) $V_{bi} = 0.697\,\text{V}$, (b) $V_{bi} = 0.817\,\text{V}$)

1.8 Calculate V_{bi} for a GaAs pn junction at $T = 300\,^\circ$K for $N_a = 10^{16}\ \text{cm}^{-3}$ and $N_d = 10^{17}\ \text{cm}^{-3}$. (Ans. $V_{bi} = 1.23\,\text{V}$)

1.2.2 Reverse-Biased pn Junction

Assume a positive voltage is applied to the n-region of a pn junction, as shown in Figure 1.12. The applied voltage V_R induces an applied electric field, E_A, in the semiconductor. The direction of this applied field is the same as that of the E-field in the space-charge region. Since the electric fields in the areas outside the space-charge region are essentially zero, the magnitude of the electric field *in* the space-charge region increases above the thermal equilibrium value. This increased electric field holds back the holes in the p-region and the electrons in the n-region, so there is essentially no current across the pn junction. By definition, this applied voltage polarity is called **reverse bias**.

When the electric field in the space-charge region increases, the number of positive and negative charges also increases. If the doping concentrations are not changed, the increases in the charges can only occur if the width W of the

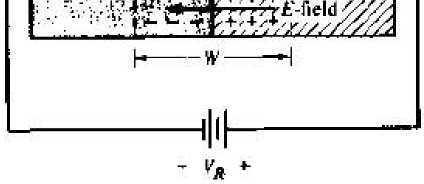


Figure 1.12 A pn junction with an applied reverse-bias voltage, showing the direction of the electric field induced by V_R and of the space-charge electric field

space-charge region increases. Therefore, with an increasing reverse-bias voltage V_R, space-charge width W also increases.

Because of the additional positive and negative charges in the space-charge region, a capacitance is associated with the pn junction when a reverse-bias voltage is applied. This **junction capacitance**, or depletion layer capacitance, can be written in the form

$$C_j = C_{jo}\left(1 + \frac{V_R}{V_{bi}}\right)^{-1/2} \quad \textbf{(1.14)}$$

where C_{jo} is the junction capacitance at zero applied voltage.

The capacitance–voltage characteristics make the pn junction useful for electrically tunable resonant circuits. Junctions fabricated specifically for this purpose are called **varactor diodes**. Varactor diodes can be used in electrically tunable oscillators, such as a Hartley oscillator, discussed in Chapter 15, or in tuned amplifiers, considered in Chapter 8.

Example 1.4 **Objective:** Calculate the junction capacitance of a pn junction.

Consider a silicon pn junction at $T = 300$ K, with doping concentrations of $N_a = 10^{16}\ \text{cm}^{-3}$ and $N_d = 10^{15}\ \text{cm}^{-3}$. Assume that $n_i = 1.5 \times 10^{10}\ \text{cm}^{-3}$ and let $C_{jo} = 0.5$ pF. Calculate the junction capacitance at $V_R = 1$ V and $V_R = 5$ V.

Solution: The built-in potential is determined by

$$V_{bi} = V_T \ln\left(\frac{N_a N_d}{n_i^2}\right) = (0.026)\ln\left[\frac{(10^{16})(10^{15})}{(1.5 \times 10^{10})^2}\right] = 0.637\ \text{V}$$

The junction capacitance for $V_R = 1$ V is then found to be

$$C_j = C_{jo}\left(1 + \frac{V_R}{V_{bi}}\right)^{-1/2} = (0.5)\left(1 + \frac{1}{0.637}\right)^{-1/2} = 0.312\ \text{pF}$$

For $V_R = 5$ V

$$C_j = (0.5)\left(1 + \frac{5}{0.637}\right)^{-1/2} = 0.168\ \text{pF}$$

Comment: The magnitude of the junction capacitance is usually at or below the picofarad range, and it decreases as the reverse-bias voltage increases.

As implied in the previous section, the magnitude of the electric field in the space-charge region increases as the reverse-bias voltage increases, and the maximum electric field occurs at the metallurgical junction. However, neither the electric field in the space-charge region nor the applied reverse-bias voltage can increase indefinitely because at some point, breakdown will occur and a large reverse bias current will be generated. This concept will be described in detail later in this chapter.

Test Your Understanding

1.9 A silicon pn junction at $T = 300$°K is doped at $N_d = 10^{16}\ cm^{-3}$ and $N_a = 10^{17}\ cm^{-3}$. The junction capacitance is to be $C_j = 0.8$ pF when a reverse-bias voltage of $V_R = 5$ V is applied. Find the zero-biased junction capacitance C_{jo}. (Ans. $C_{jo} = 2.21$ pF)

1.2.3 Forward-Biased pn Junction

To review briefly, the n-region contains many more free electrons than the p-region; similarly, the p-region contains many more holes than the n-region. With zero applied voltage, the built-in potential barrier prevents these majority carriers from diffusing across the space-charge region; thus, the barrier maintains equilibrium between the carrier distributions on either side of the pn junction.

If a positive voltage v_D is applied to the p-region, the potential barrier decreases (Figure 1.13). The electric fields in the space-charge region are very large compared to those in the remainder of the p- and n-regions, so essentially all of the applied voltage exists across the pn junction region. The applied electric field, E_A, induced by the applied voltage is in the opposite direction from that of the thermal equilibrium space-charge E-field. The net result is that the electric field in the space-charge region is lower than the equilibrium value. This upsets the delicate balance between diffusion and the E-field force. Majority carrier electrons from the n-region diffuse into the p-region, and majority carrier holes from the p-region diffuse into the n-region. The process continues as long as the voltage v_D is applied, thus creating a current in the pn junction. This process would be analogous to lowering a dam wall slightly. A slight drop in the wall height can send a large amount of water (current) over the barrier.

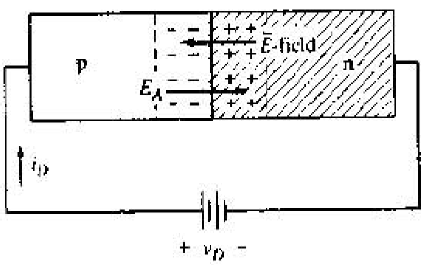


Figure 1.13 A pn junction with an applied forward-bias voltage, showing the direction of the electric field E_A induced by v_D and of the net space-charge electric field E

This applied voltage polarity (i.e., bias) is known as **forward bias**. The forward-bias voltage v_D must always be less than the built-in potential barrier V_{bi}.

As the majority carriers cross into the opposite regions, they become minority carriers in those regions, causing the minority carrier concentrations to increase. Figure 1.14 shows the resulting excess minority carrier concentrations

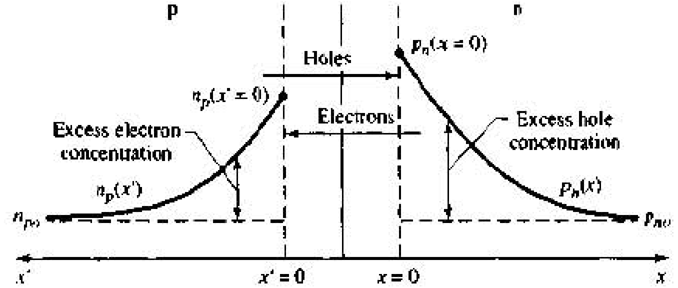


Figure 1.14 Steady-state minority carrier concentration in a pn junction under forward bias

at the space-charge region edges. These excess minority carriers diffuse into the neutral n- and p-regions, where they recombine with majority carriers, thus establishing a steady-state condition, as shown in Figure 1.14.

1.2.4 Ideal Current–Voltage Relationship

As shown in Figure 1.14, an applied voltage results in a gradient in the minority carrier concentrations, which in turn causes diffusion currents. The theoretical relationship between the voltage and the current in the pn junction is given by

$$i_D = I_S\left[e^{\left(\frac{v_D}{nV_T}\right)} - 1\right] \tag{1.15}$$

The parameter I_S is the **reverse-bias saturation current**. For silicon pn junctions, typical values of I_S are in the range of 10^{-15} to 10^{-13} A. The actual value depends on the doping concentrations and the cross-sectional area of the junction. The parameter V_T is the thermal voltage, as defined in Equation (1.13), and is approximately $V_T = 0.026$ V at room temperature. The parameter n is usually called the emission coefficient or ideality factor, and its value is in the range $1 \le n \le 2$.

The emission coefficient n takes into account any recombination of electrons and holes in the space-charge region. At very low current levels, recombination may be a significant factor and the value of n may be close to 2. At higher current levels, recombination is less a factor, and the value of n will be 1. Unless otherwise stated, we will assume the emission coefficient is $n = 1$.

Example 1.5 **Objective:** Determine the current in a pn junction.

Consider a pn junction at $T = 300\,°K$ in which $I_S = 10^{-14}$ A and $n = 1$. Find the diode current for $v_D = +0.70$ V and $v_D = -0.70$ V.

Solution: For $v_D = +0.70$ V, the pn junction is forward-biased and we find

$$i_D = I_S\left[e^{\left(\frac{v_D}{V_T}\right)} - 1\right] = (10^{-14})\left[e^{\left(\frac{+0.70}{0.026}\right)} - 1\right] \Rightarrow 4.93 \text{ mA}$$

For $v_D = -0.70$ V, the pn junction is reverse-biased and we find

$$i_D = I_S\left[e^{\left(\frac{v_D}{V_T}\right)} - 1\right] = (10^{-14})\left[e^{\left(\frac{-0.70}{0.026}\right)} - 1\right] \cong -10^{-14}\,\text{A}$$

Comment: Although I_S is quite small, even a relatively small value of forward-bias voltage can induce a moderate junction current. With a reverse-bias voltage applied, the junction current is virtually zero.

Test Your Understanding

1.10 A silicon pn junction diode at $T = 300°\text{K}$ has a reverse-saturation current of $I_S = 10^{-14}$ A. (a) Determine the forward-bias diode current for (i) $v_D = 0.5$ V, (ii) $v_D = 0.6$ V, and (iii) $v_D = 0.7$ V. (b) Find the reverse-bias diode current for (i) $v_D = -0.5$ V, and (ii) $v_D = -2$ V. (Ans. (a) (i) 2.25 μA, (ii) 105 μA, (iii) 4.93 mA; (b) (i) 10^{-14} A, (ii) 10^{-14} A)

1.11 A silicon pn junction diode at $T = 300°\text{K}$ has a reverse-saturation current of $I_S = 10^{-13}$ A. The diode is forward-biased with a resulting current of 1 mA. Determine v_D. (Ans. $v_D = 0.599$ V)

1.2.5 pn Junction Diode

Figure 1.15 is a plot of the derived current–voltage characteristics of a pn junction. For a forward-bias voltage, the current is an exponential function

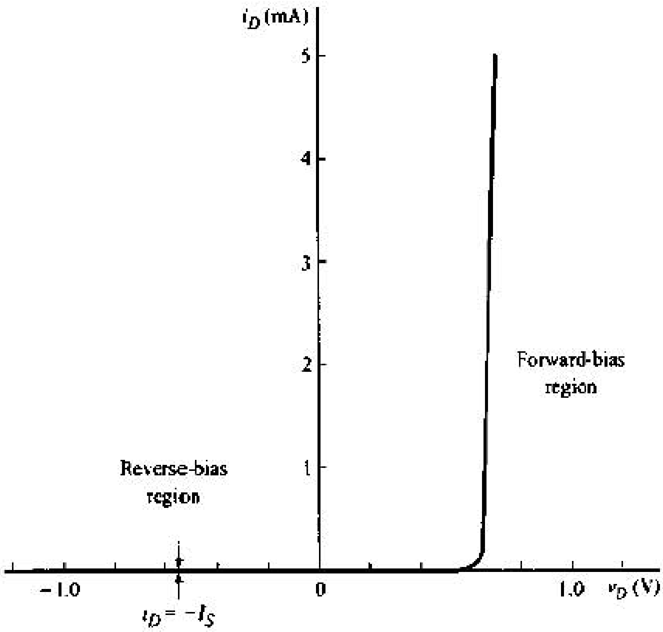


Figure 1.15 Ideal I–V characteristics of a pn junction diode for $I_S = 10^{-14}$ A

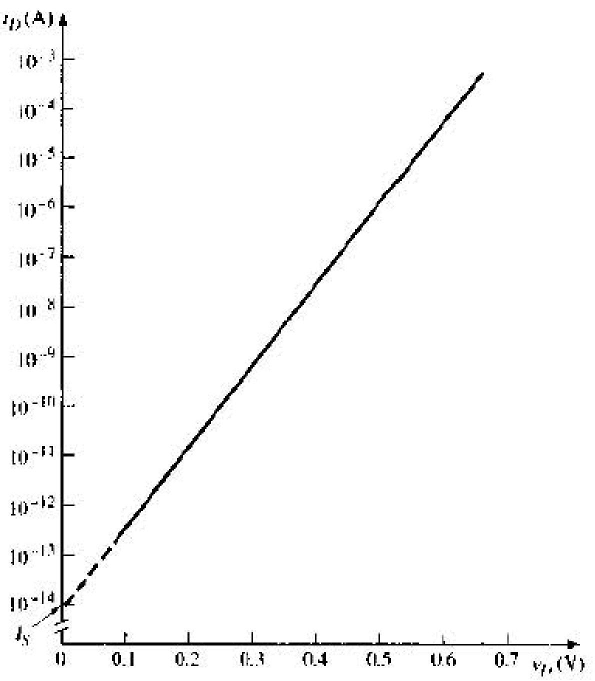


Figure 1.16 Ideal forward-biased I–V characteristics of a pn junction diode, with the current plotted on a log scale for $I_S = 10^{-14}$ A and $n = 1$

of voltage. Figure 1.16 depicts the forward-bias current plotted on a log scale. With only a small change in the forward-bias voltage, the corresponding forward-bias current increases by orders of magnitude. For a forward-bias voltage $v_D > +0.1$ V, the (-1) term in Equation (1.15) can be neglected. In the reverse-bias direction, the current is almost zero.

The semiconductor device that displays these I–V characteristics is called a **pn junction diode**. Figure 1.17 shows the diode circuit symbol and the conventional current direction and voltage polarity. The diode can be thought of and used as a voltage controlled switch that is "off" for a reverse-bias voltage and "on" for a forward-bias voltage. In the forward-bias or "on" state, a relatively large current is produced by a fairly small applied voltage; in the reverse-bias, or "off" state, only a very small current is created.

When a diode is reverse-biased by at least 0.1 V, the diode current is $i_D = -I_S$. The current is in the reverse direction and is a constant, hence the name reverse-bias saturation current. Real diodes, however, exhibit reverse-bias currents that are considerably larger than I_S. This additional current is called a generation current and is due to electrons and holes being generated within the space-charge region. Whereas a typical value of I_S may be 10^{-14} A, a typical value of reverse-bias current may be 10^{-9} A or 1 nA. Even though this current is much larger than I_S, it is still small and negligible in most cases.

Figure 1.17 The basic pn junction diode: (a) simplified geometry and (b) circuit symbol, and conventional current direction and voltage polarity

Temperature Effects

Since both I_S and V_T are functions of temperature, the diode characteristics also vary with temperature. The temperature-related variations in forward-bias characteristics are illustrated in Figure 1.18. For a given current, the required

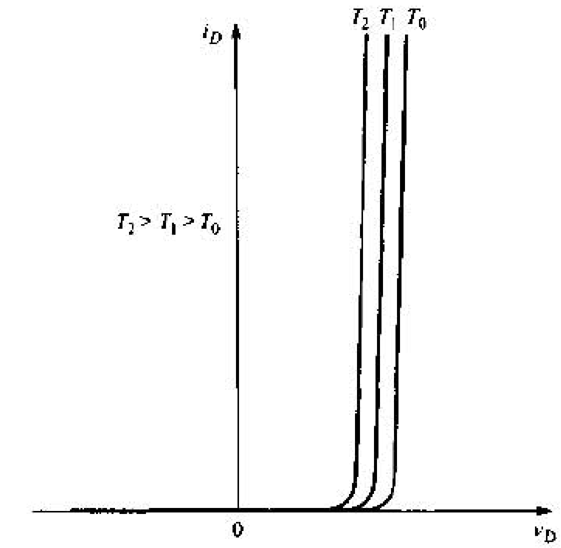


Figure 1.18 Forward-bias characteristics versus temperature

forward-bias voltage decreases as temperature increases. For silicon diodes, the change is approximately 2 mV/°C.

The parameter I_S is a function of the intrinsic carrier concentration n_i, which in turn is strongly dependent on temperature. Consequently, the value of I_S approximately doubles for every 5 °C increase in temperature. The actual reverse-bias diode current, as a general rule, doubles for every 10 °C rise in temperature. As an example of the importance of this effect, in germanium, the relative value of n_i is large, resulting in a large reverse-saturation current in germanium-based diodes. Increases in this reverse current with increases in the temperature make the germanium diode highly impractical for most circuit applications.

Breakdown Voltage

When a reverse-bias voltage is applied to a pn junction, the electric field in the space-charge region increases. The electric field may become large enough that covalent bonds are broken and electron-hole pairs are created. Electrons are swept to the n-region and holes to the p-region by the electric field generating a reverse-bias current. This breakdown mechanism is called the **Zener effect**. Another breakdown mechanism is called **avalanche breakdown**, which occurs when minority carriers crossing the space-charge region gain sufficient kinetic energy to be able to break covalent bonds during a collision process. The generated electron-hole pairs can themselves be involved in a collision process generating additional electron-hole pairs, thus, the avalanche process. The reverse-bias current for each breakdown mechanism will be limited by the external circuit.

The voltage at which breakdown occurs depends on fabrication parameters of the pn junction, but is usually in the range of 50 to 200 V for discrete devices, although breakdown voltages outside this range are possible—in excess of 1000 V, for example. A pn junction is usually rated in terms of its

peak inverse voltage or **PIV**. The PIV of a diode must never be exceeded in circuit operation if reverse breakdown is to be avoided.

Zener diodes are fabricated with a specifically designed breakdown voltage and are designed to operate in the breakdown region. These diodes are discussed later in this chapter.

Switching Transient

Since the pn junction diode can be used as an electrical switch, an important parameter is its transient response, that is, its speed and characteristics, as it is switched from one state to the other. Assume, for example, that the diode is switched from the forward-bias "on" state to the reverse-bias "off" state. Figure 1.19 shows a simple circuit that will switch the applied voltage at time $t = 0$. For $t < 0$, the forward-bias current i_D is

$$i_D = I_F = \frac{V_F - v_D}{R_F} \tag{1.16}$$

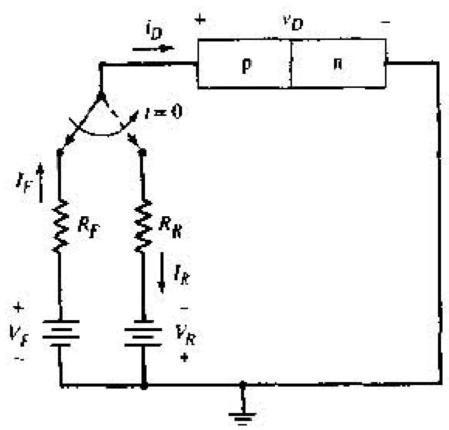


Figure 1.19 Simple circuit for switching a diode from forward to reverse bias

The minority carrier concentrations for an applied forward-bias voltage and an applied reverse-bias voltage are shown in Figure 1.20. Here, we neglect the change in the space charge region width. When a forward-bias voltage is applied, excess minority carrier charge is stored in both the p- and n-regions. The excess charge is the difference between the minority carrier concentrations for a forward-bias voltage and those for a reverse-bias voltage as indicated in the figure. This charge must be removed when the diode is switched from the forward to the reverse bias.

As the forward-bias voltage is removed, relatively large diffusion currents are created in the reverse-bias direction. This happens because the excess minority carrier electrons flow back across the junction into the n-region, and the excess minority carrier holes flow back across the junction into the p-region.

The large reverse-bias current is initially limited by resistor R_R to approximately

$$i_D = -I_R \cong \frac{-V_R}{R_R} \tag{1.17}$$

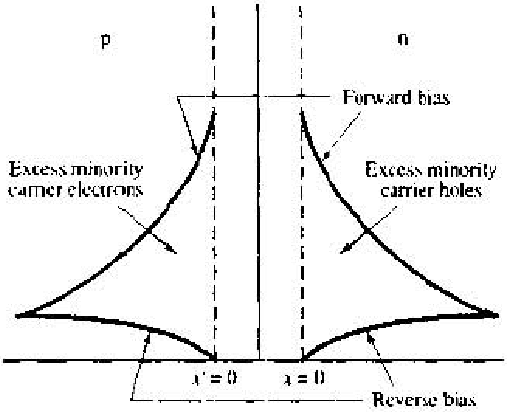


Figure 1.20 Stored excess minority carrier charge under forward bias compared to reverse bias

The junction capacitances do not allow the junction voltage to change instantaneously. The reverse current I_R is approximately constant for $0^+ < t < t_s$, where t_s is the **storage time**, which is the length of time required for the minority carrier concentrations at the space-charge region edges to reach the thermal equilibrium values. After this time, the voltage across the junction begins to change. The fall time t_f is typically defined as the time required for the current to fall to 10 percent of its initial value. The total **turn-off time** is the sum of the storage time and the fall time. Figure 1.21 shows the current characteristics as this entire process takes place.

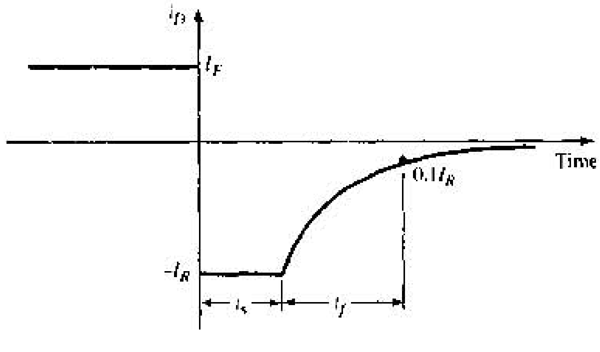


Figure 1.21 Current characteristics versus time during diode switching

In order to switch a diode quickly, the diode must have a small excess minority carrier lifetime, and we must be able to produce a large reverse current pulse. Therefore, in the design of diode circuits, we must provide a path for the transient reverse-bias current pulse. These same transient effects impact the switching of transistors. For example, the switching speed of transistors in digital circuits will affect the speed of computers.

The turn-on transient occurs when the diode is switched from the "off" state to the forward-bias "on" state, which can be initiated by applying a forward-bias current pulse. The transient **turn-on time** is the time required to establish the forward-bias minority carrier distributions. During this time, the voltage across the junction gradually increases toward its steady-state value. Although the turn-on time for the pn junction diode is not zero, it is usually less than the transient turn-off time.

Test Your Understanding

1.12 Recall that the forward-bias diode voltage decreases approximately by 2 mV/°C for silicon diodes with a given current. If $V_D = 0.650$ V at $I_D = 1$ mA for a temperature of 25 °C, determine the diode voltage at $I_D = 1$ mA for $T = 125$ °C. (Ans. $V_D = 0.450$ V)

1.3 DIODE CIRCUITS: DC ANALYSIS AND MODELS

In this section, we begin to study the diode in various circuit configurations. As we have seen, the diode is a two-terminal device with nonlinear i–v characteristics, as opposed to a two-terminal resistor, which has a linear relationship between current and voltage. The analysis of nonlinear electronic circuits is not as straightforward as the analysis of linear electric circuits. However, there are electronic functions that can be implemented only by nonlinear circuits. Examples include the generation of dc voltages from sinusoidal voltages and the implementation of logic functions.

Mathematical relationships, or **models**, that describe the current-voltage characteristics of electrical elements allow us to analyze and design circuits without having to fabricate and test them in the laboratory. An example is Ohm's law, which describes the properties of a resistor. In this section, we will develop the dc analysis and modeling techniques of diode circuits.

To begin to understand diode circuits, consider a simple diode application. The current-voltage characteristics of the pn junction diode were given in Figure 1.15. An **ideal diode** (as opposed to a diode with ideal I–V characteristics) has the characteristics shown in Figure 1.22(a). When a reverse-bias voltage is applied, the current through the diode is zero (Figure 1.22(b)); when current through the diode is greater than zero, the voltage across the diode is zero (Figure 1.22(c)). An external circuit connected to the diode must be designed to control the forward current through the diode.

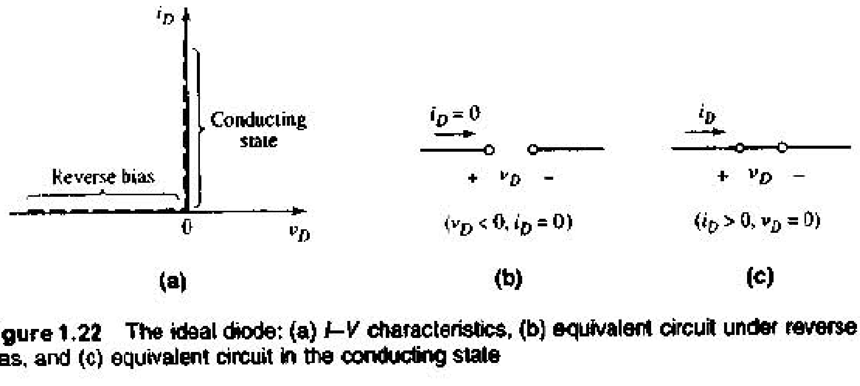


Figure 1.22 The ideal diode: (a) I–V characteristics, (b) equivalent circuit under reverse bias, and (c) equivalent circuit in the conducting state

One diode circuit is the **rectifier** circuit shown in Figure 1.23(a). Assume that the input voltage v_I is a sinusoidal signal, as shown in Figure 1.23(b), and the diode is an ideal diode (see Figure 1.22(a). During the positive half-cycle of the sinusoidal input, a forward-bias current exists in the diode and the voltage across the diode is zero. The equivalent circuit for this condition is shown in Figure 1.23(c). The output voltage v_O is then equal to the input voltage. During the negative half-cycle of the sinusoidal input, the diode is reverse biased. The equivalent circuit for this condition is shown in Figure 1.23(d). In this part of the cycle, the diode acts as an open circuit, the current is zero, and the output voltage is zero. The output voltage of the circuit is shown in Figure 1.23(e).

Over the entire cycle, the input signal is sinusoidal and has a zero average value; however, the output signal contains only positive values and therefore has a positive average value. Conseqently, this circuit is said to **rectify** the input signal, which is the first step in generating a dc voltage from a sinusoidal (ac) voltage. A dc voltage is required in virtually all electronic circuits.

As mentioned, the analysis of nonlinear circuits is not as straightforward as that of linear circuits. In this section, we will look at four approaches to the dc analysis of diode circuits: (a) iteration; (b) graphical techniques; (c) a piecewise linear modeling method; and (d) a computer analysis. Methods (a) and (b) are closely related and are therefore presented together.

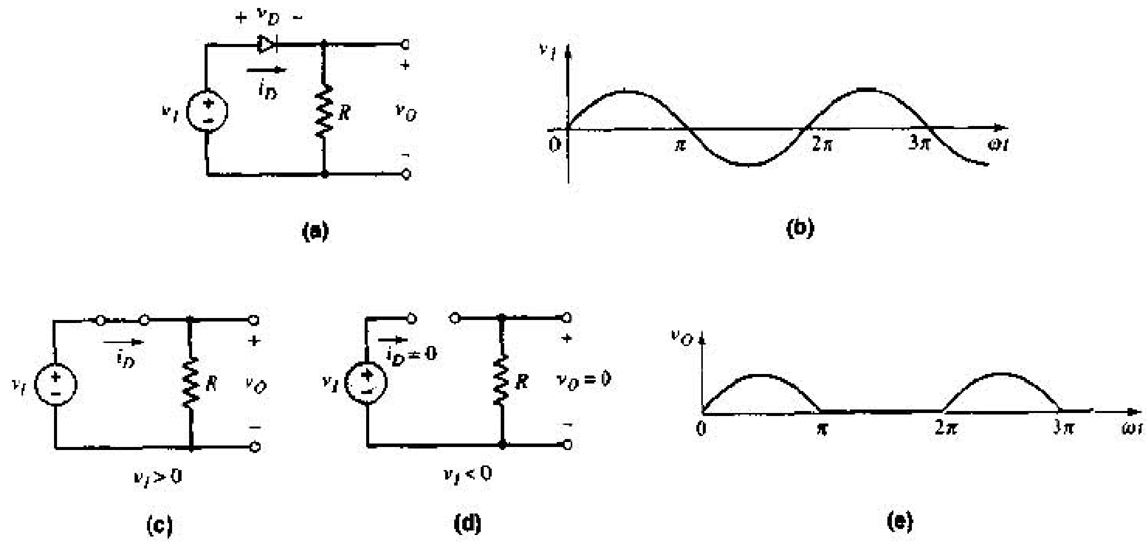


Figure 1.23 The diode rectifier: (a) circuit, (b) sinusoidal input signal, (c) equivalent circuit for $v_I > 0$, (d) equivalent circuit for $v_I < 0$, and (e) rectified output signal

1.3.1 Iteration and Graphical Analysis Techniques

Iteration means using trial and error to find a solution to a problem. The graphical analysis technique involves plotting two simultaneous equations and locating their point of intersection, which is the solution to the two equations. We will use both techniques to solve the circuit equations, which include the diode equation. These equations are difficult to solve by hand because they contain both linear and exponential terms.

Consider, for example, the circuit shown in Figure 1.24, with a dc voltage V_{PS} applied across a resistor and a diode. **Kirchhoff's voltage law** applies both to nonlinear and linear circuits, so we can write

$$V_{PS} = I_D R + V_D \tag{1.18(a)}$$

which can be rewritten as

$$I_D = \frac{V_{PS}}{R} - \frac{V_D}{R} \tag{1.18(b)}$$

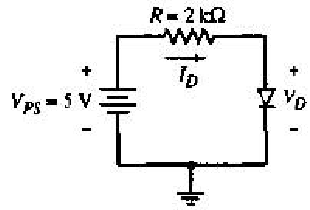


Figure 1.24 A simple diode circuit

[Note: In the remainder of this section in which dc analysis is emphasized, the dc variables are denoted by uppercase letters and uppercase subscripts.]

The diode voltage V_D and current I_D are related by the ideal diode equation as

$$I_D = I_S\left[e^{\left(\frac{V_D}{V_T}\right)} - 1\right] \tag{1.19}$$

where I_S is assumed to be known for a particular diode.

Combining Equations (1.18(a)) and (1.19), we obtain

$$V_{PS} = I_S R\left[e^{\left(\frac{V_D}{V_T}\right)} - 1\right] + V_D \tag{1.20}$$

which contains only one unknown, V_D. However, Equation (1.20) is a transcendental equation and cannot be solved directly. The use of iteration to find a solution to this equation is demonstrated in the following example.

Example 1.6 **Objective:** Determine the diode voltage and current for the circuit shown in Figure 1.24.

Consider a diode with a given reverse-saturation current of $I_S = 10^{-13}$ A.

Solution: We can write Equation (1.20) as

$$5 = (10^{-13})(2 \times 10^3)\left[e^{\left(\frac{V_D}{0.026}\right)} - 1\right] + V_D \tag{1.21}$$

If we first try $V_D = 0.60$ V, the right side of Equation (1.21) is 2.7 V, so the equation is not balanced and we must try again. If we next try $V_D = 0.65$ V, the right side of Equation (1.21) is 15.1 V. Again, the equation is not balanced, but we can see that the solution for V_D is between 0.6 and 0.65 V. If we continue refining our guesses, we will be able to show that, when $V_D = 0.619$ V, the right side of Equation (1.21) is 4.99 V, which is essentially equal to the value of the left side of the equation.

The current in the circuit can then be determined by dividing the voltage difference across the resistor by the resistance, or

$$I_D = \frac{V_{PS} - V_D}{R} = \frac{5 - 0.619}{2} = 2.19 \text{ mA}$$

Comment: Once the diode voltage is known, the current can also be determined from the ideal diode equation. However, dividing the voltage difference across a resistor by the resistance is usually easier, and this approach is used extensively in the analysis of diode and transistor circuits.

To use a graphical approach to analyze the circuit, we go back to Kirchhoff's voltage law, as expressed by Equation (1.18(b)), which produces a straight-line relationship between current I_D and voltage V_D for a given V_{PS} and R. This equation is referred to as the circuit **load line**, which can be plotted on a graph with I_D and V_D as the axes. From Equation (1.18(b)), we see that if $I_D = 0$, then $V_D = V_{PS}$. Also from this equation, if $V_D = 0$, then $I_D = V_{PS}/R$. The load line can be drawn between these two points. Using the values given in Example (1.6), we can plot the straight line shown in Figure 1.25. The second plot in the figure is that of Equation (1.19), which is the ideal diode equation relating the diode current and voltage. The intersection of the load line and the device characteristics curve provides the dc current $I_D \approx 2.2\,\text{mA}$ through the diode and the dc voltage $V_D \approx 0.62$ V across the diode. This point is referred to as the **quiescent point**, or the **Q-point**.

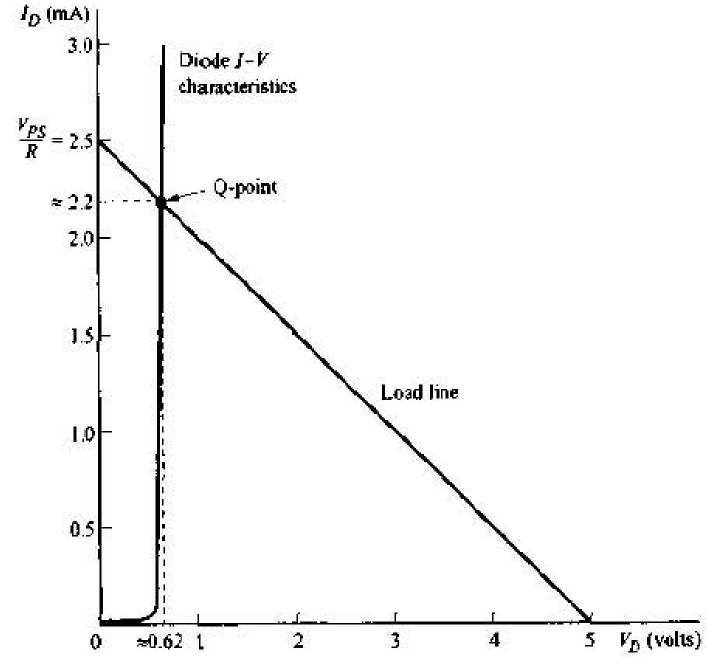


Figure 1.25 The diode and load line characteristics for the circuit shown in Figure 1.24

The graphical analysis method can yield accurate results, but it is somewhat cumbersome. However, the concept of the load line and the graphical approach are useful for "visualizing" the response of a circuit, and the load line is used extensively in the evaluation of electronic circuits.

Test Your Understanding

***1.13** Consider the circuit in Figure 1.24. Let $V_{PS} = 4$ V, $R = 40\,\text{k}\Omega$, and $I_S = 10^{-12}$ A. Determine V_D and I_D, using the ideal diode equation and the iteration method. (Ans. $V_D = 0.535$ V, $I_D = 0.864$ mA)

1.14 Consider the diode and circuit in Exercise 1.13. Determine V_D and I_D, using the graphical technique. (Ans. $V_D \approx 0.54$ V, $I_D \approx 0.87$ mA)

1.3.2 Piecewise Linear Model

Another, simpler way to analyze diode circuits is to *approximate* the diode's current–voltage characteristics, using linear relationships or straight lines. Figure 1.26, for example, shows the ideal current–voltage characteristics and two linear approximations.

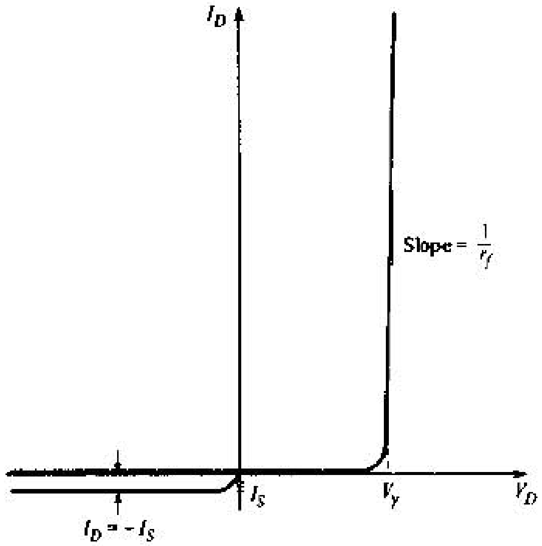


Figure 1.26 The ideal diode I–V characteristics and two linear approximations

For $V_D \geq V_\gamma$, we assume a straight-line approximation whose slope is $1/r_f$, where V_γ is the **turn-on, or cut-in, voltage** of the diode, and r_f is the **forward diode resistance**. The equivalent circuit for this linear approximation is a constant-voltage source in series with a resistor (Figure 1.27(a)).[2] For $V_D < V_\gamma$, we assume a straight-line approximation parallel to the V_D axis at the zero current level. In this case, the equivalent circuit is an open circuit (Figure 1.27(b)).

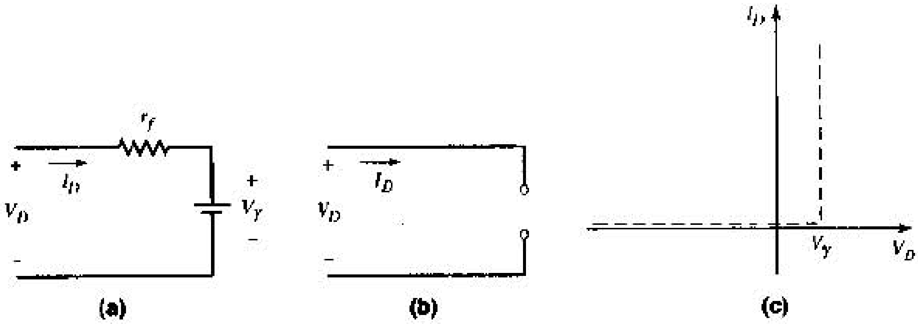


Figure 1.27 The diode equivalent circuit (a) in the "on" condition when $V_D \geq V_\gamma$, (b) in the "off" condition when $V_D < V_\gamma$, and (c) piecewise linear approximation when $r_f = 0$

[2] It is important to keep in mind that the voltage source in Figure 1.27(a) only represents a voltage drop for $V_D \geq V_\gamma$. When $V_D < V_\gamma$, the V_γ source does *not* produce a negative diode current. For $V_D < V_\gamma$, the equivalent circuit in Figure 1.27(b) must be used.

This method models the diode with segments of straight lines; thus the name **piecewise linear model**. If we assume $r_f = 0$, the piecewise linear diode characteristics are shown in Figure 1.27(c).

Example 1.7 **Objective:** Determine the diode voltage and current in the circuit shown in Figure 1.24, using a piecewise linear model.

Assume piecewise linear diode parameters of $V_\gamma = 0.6\,\text{V}$ and $r_f = 10\,\Omega$.

Solution: With the given input voltage polarity, the diode is forward biased or "turned on," so $I_D > 0$. The equivalent circuit is shown in Figure 1.27(a). The diode current is determined by

$$I_D = \frac{V_{PS} - V_\gamma}{R + r_f} = \frac{5 - 0.6}{2 \times 10^3 + 10} \Rightarrow 2.19\,\text{mA}$$

and the diode voltage is

$$V_D = V_\gamma + I_D r_f = 0.6 + (2.19 \times 10^{-3})(10) = 0.622\,\text{V}$$

Comment: This solution, obtained using the piecewise linear model, is nearly equal to the solution obtained in Example 1.6, in which the ideal diode equation was used. However, the analysis using the piecewise-linear model in this example is by far easier than using the actual diode I–V characteristics as was done in Example 1.6. In general, we are willing to accept some slight analysis inaccuracy for ease of analysis.

Because the forward diode resistance r_f in Example 1.7 is much smaller than the circuit resistance R, the diode current I_D is essentially independent of the value of r_f. In addition, if the cut-in voltage is 0.7 V instead of 0.6 V, the calculated diode current will be 2.15 mA, which is not significantly different from the previous results. Therefore, the calculated diode current is not a strong function of the cut-in voltage. Consequently, we will often assume a cut-in voltage of 0.7 V for silicon pn junction diodes.

The concept of the load line and the piecewise linear model can be combined in diode circuit analyses. Using Kirchhoff's voltage law, expressed as Equation 1.14(b), and the circuit in Figure 1.24, assume $V_\gamma = 0.7\,\text{V}$, $r_f = 0$, $V_{PS} = +5\,\text{V}$, and $R = 2\,\text{k}\Omega$. Figure 1.28(a) shows the resulting load line and the piecewise linear characteristic curves of the diode. The two curves intersect

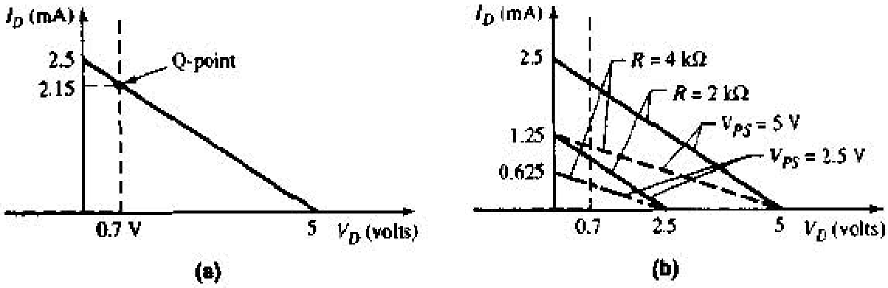


Figure 1.28 Piecewise linear approximation (a) load line for $V_{PS} = 5\,\text{V}$ and $R = 2\,\text{k}\Omega$ and (b) several load lines

at the Q-point, or diode current, $I_{DQ} \cong 2.15$ mA, which is essentially a function of only V_{PS} and R. Figure 1.28(b) shows the same piecewise linear characteristics of the diode but with four different load lines, corresponding to: $V_{PS} = 5$ V, $R = 4$ kΩ; $V_{PS} = 5$ V, $R = 2$ kΩ; $V_{PS} = 2.5$ V, $R = 4$ kΩ; and $V_{PS} = 2.5$ V, $R = 2$ kΩ. The Q-point changes for each load line.

The load line concept is also useful when the diode is reverse biased. Figure 1.29(a) shows the same diode circuit as before, but with the direction of the diode reversed. The diode current I_D and voltage V_D shown are the usual forward-biased parameters. Applying Kirchhoff's voltage law, we can write

$$V_{PS} = I_{PS}R - V_D = -I_D R - V_D \tag{1.22(a)}$$

or

$$I_D = -\frac{V_{PS}}{R} - \frac{V_D}{R} \tag{1.22(b)}$$

where $I_D = -I_{PS}$. Equation (1.22(b)) is the load line equation. The two end points are found by setting $I_D = 0$, which yields $V_D = -V_{PS} = -5$ V, and by setting $V_D = 0$, which yields $I_D = -V_{PS}/R = -5/2 = -2.5$ mA. The diode characteristics and the load line are plotted in Figure 1.29(b). We see that the load is now in the third quadrant, where it intersects the diode characteristics curve at $V_D = -5$ V and $I_D = 0$, demonstrating that the diode is reverse biased.

Although the piecewise linear model may yield solutions that are less accurate than those obtained with the ideal diode equation, the analysis is much easier.

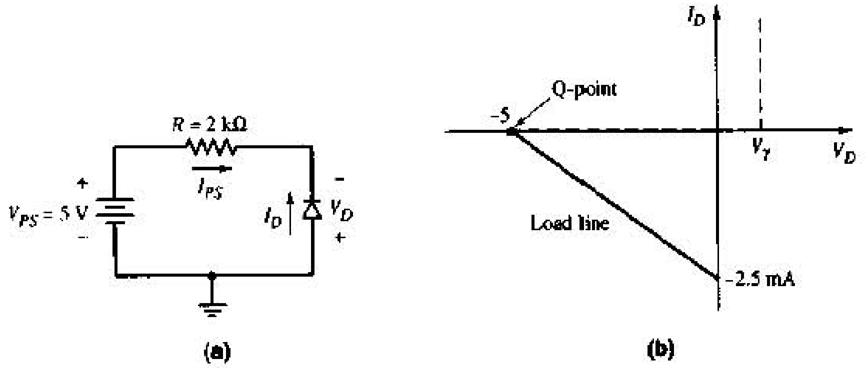


Figure 1.29 Reverse-biased diode (a) circuit and (b) piecewise linear approximation and load line

Test Your Understanding

1.16 (a) Consider the circuit in Figure 1.24. Let $V_{PS} = 5$ V, $R = 4$ kΩ, and $V_\gamma = 0.7$ V. Assume $r_f = 0$. Determine I_D. (b) If V_{PS} is increased to 8 V, what must be the new value of R such that I_D is the same value as in part (a)? (c) Draw the diode characteristics and load lines for parts (a) and (b). (Ans. (a) $I_D = 1.08$ mA, (b) $R = 6.79$ kΩ)

1.16 The power supply (input) voltage in the circuit of Figure 1.24 is $V_{PS} = 10$ V and the diode cut-in voltage is $V_\gamma = 0.7$ V (assume $r_f = 0$). The power dissipated in the diode is to be no more than 1.05 mW. Determine the maximum diode current and the minimum value of R to meet the power specification. (Ans. $I_D = 1.5$ mA, $R = 6.2$ kΩ)

1.3.3 Computer Simulation and Analysis

Today's computers are capable of using detailed simulation models of various components and performing complex circuit analyses quickly and relatively easily. Such models can factor in many diverse conditions, such as the temperature dependence of various parameters. One of the earliest, and now the most widely used, circuit analysis programs is the *s*imulation *p*rogram with *i*ntegrated *c*ircuit *e*mphasis (SPICE). This program, developed at the University of California at Berkeley, was first released about 1973, and has been continuously refined since that time. One outgrowth of SPICE is PSpice, which is designed for use on personal computers.

Example 1.8 **Objective:** Determine the diode current and voltage characteristics of the circuit shown in Figure 1.24 using a PSpice analysis.

Solution: Figure 1.30(a) is the PSpice circuit schematic. A standard 1N4002 diode from the PSpice library was used in the analysis. The input voltage $V1$ was varied (dc sweep) from 0 to 5 V. Figure 1.30(b) and (c) shows the diode voltage and diode current characteristics versus the input voltage.

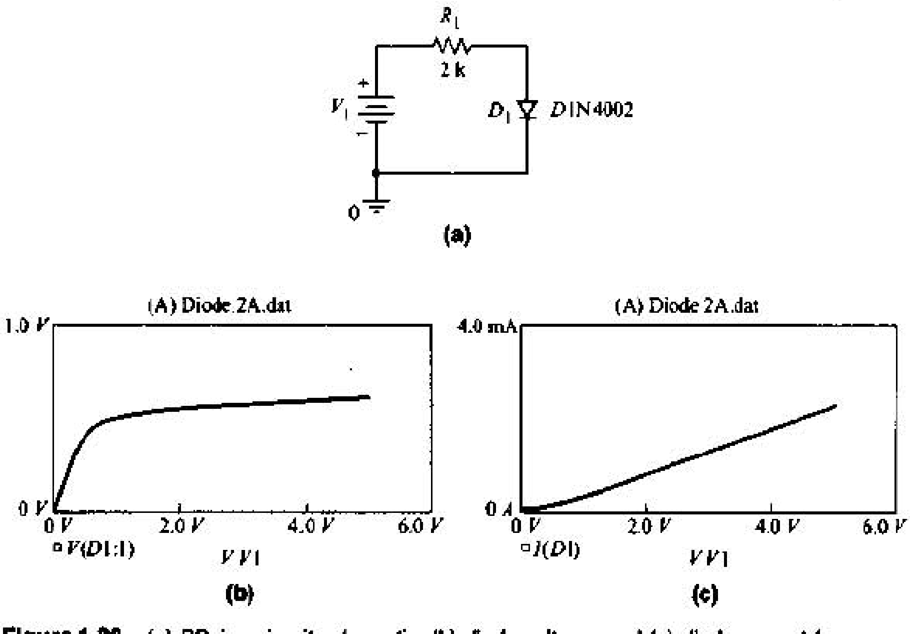


Figure 1.30 (a) PSpice circuit schematic, (b) diode voltage, and (c) diode current for Example 1.8

Discussion: Several observations may be made from the results. The diode voltage increases at almost a linear rate up to approximately 400 mV without any discernible (mA) current being measured. For an input voltage greater than approximately 500 mV, the diode voltage increases gradually to a value of about 610 mV at the maximum input voltage. The current also increases to a maximum value of approximately 2.2 mA at the maximum input voltage. The piecewise linear model predicts quite accurate results at the maximum input voltage. However, these results show that there is definitely a nonlinear relation between the diode current and diode voltage. We must keep in mind that the piecewise linear model is an approximation technique that works very well in many applications.

1.3.4 Summary of Diode Models

The two dc diode models used in the hand analysis of diode circuits are: the ideal diode equation and the piecewise linear approximation. For the ideal diode equation, the reverse-saturation current I_S must be specified. For the piecewise linear model, the cut-in voltage V_γ and forward diode resistance r_f must be specified. In most cases, however, r_f is assumed to be zero unless otherwise given.

1.4 DIODE CIRCUITS: AC EQUIVALENT CIRCUIT

Up to this point, we have only looked at the dc characteristics of the pn junction diode. When semiconductor devices with pn junctions are used in linear amplifier circuits, the time-varying, or ac, characteristics of the pn junction become important, because sinusoidal signals may be superimposed on the dc currents and voltages. The following sections examine these ac characteristics.

1.4.1 Sinusoidal Analysis

In the circuit shown in Figure 1.31(a), the voltage source v_i is assumed to be a sinusoidal, or time-varying, signal. The total input voltage v_I is composed of a dc component V_{PS} and an ac component v_i superimposed on the dc value. To investigate this circuit, we will look at two types of analyses: a dc analysis involving only the dc voltages and currents, and an ac analysis involving only the ac voltages and currents. (We should point out that the circuit in the figure is not a practical circuit, since it is not desirable to have a dc current flowing through an ac signal source. However, the circuit is useful for a discussion of dc and ac analyses.)

Current–Voltage Relationships

Since the input voltage contains a dc component with an ac signal superimposed, the diode current will also contain a dc component with an ac signal superimposed, as shown in Figure 1.31(b). Here, I_{DQ} is the dc quiescent diode current. In addition, the diode voltage will contain a dc value with an ac signal superimposed, as shown in Figure 1.31(c). For this analysis, assume that the ac

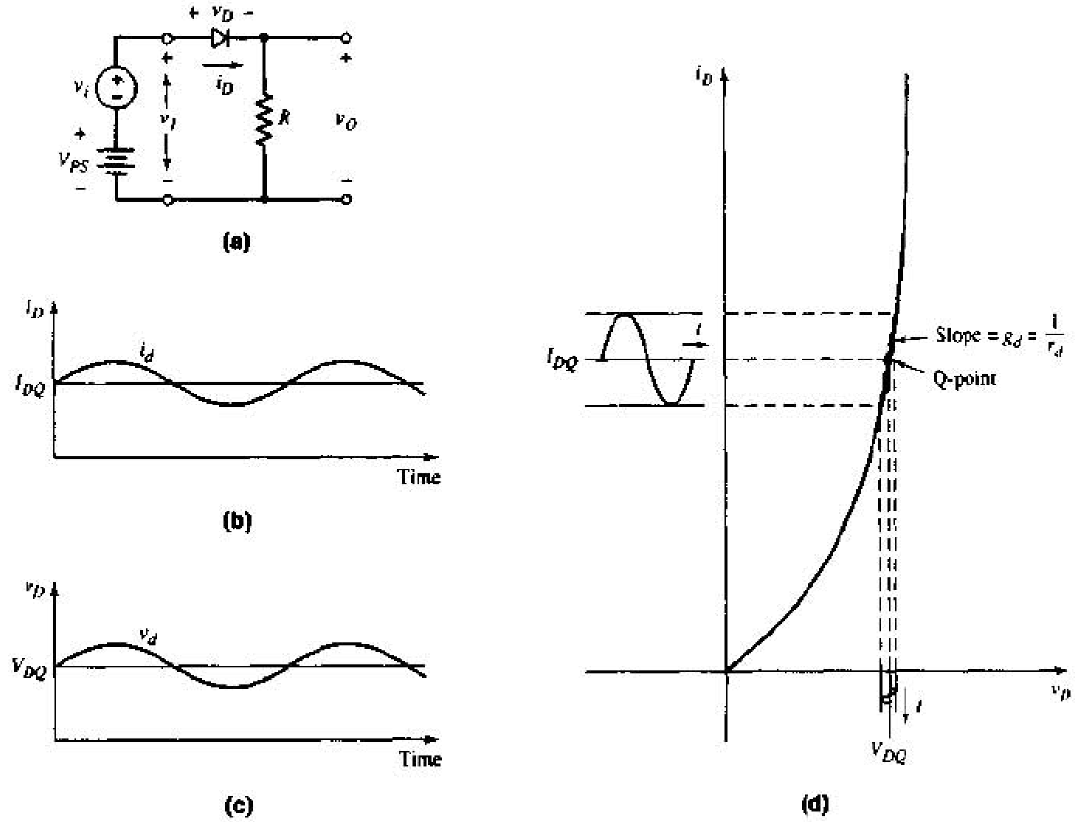


Figure 1.31 AC circuit analysis: (a) circuit with combined dc and sinusoidal input voltages, (b) sinusoidal diode current superimposed on the quiescent current, (c) sinusoidal diode voltage superimposed on the quiescent value, and (d) forward-biased diode I–V characteristics with a sinusoidal current and voltage superimposed on the quiescent values

signal is small compared to the dc component, so that a linear ac model can be developed from the nonlinear diode.

The relationship between the diode current and voltage can be written as

$$i_D \cong I_S e^{\left(\frac{v_D}{V_T}\right)} = I_S e^{\left(\frac{V_{DQ}+v_d}{V_T}\right)} \tag{1.23}$$

where V_{DQ} is the dc quiescent voltage and v_d is the ac component. We are neglecting the -1 term in the diode equation. Equation (1.23) can be rewritten as

$$i_D = I_S\left[e^{\left(\frac{V_{DQ}}{V_T}\right)}\right]\cdot\left[e^{\left(\frac{v_d}{V_T}\right)}\right] \tag{1.24}$$

If the ac signal is "small," then $v_d \ll V_T$, and we can expand the exponential function into a linear series, as follows:

$$e^{\left(\frac{v_d}{V_T}\right)} \cong 1 + \frac{v_d}{V_T} \tag{1.25}$$

We may also write the quiescent diode current as

$$I_{DQ} = I_S e^{\left(\frac{V_{DQ}}{V_T}\right)} \tag{1.26}$$

The diode current–voltage relationship from Equation (1.24) can then be written as

$$i_D = I_{DQ}\left(1 + \frac{v_d}{V_T}\right) = I_{DQ} + \frac{I_{DQ}}{V_T} \cdot v_d = I_{DQ} + i_d \tag{1.27}$$

where i_d is the ac component of the diode current. The relationship between the ac components of the diode voltage and current is then

$$i_d = \left(\frac{I_{DQ}}{V_T}\right) \cdot v_d = g_d \cdot v_d \tag{1.28(a)}$$

or

$$v_d = \left(\frac{V_T}{I_{DQ}}\right) \cdot i_d = r_d \cdot i_d \tag{1.28(b)}$$

The parameters g_d and r_d, respectively, are the diode **small-signal incremental conductance** and **resistance**, also called the **diffusion conductance** and **diffusion resistance**. We see from these two equations that

$$r_d = \frac{1}{g_d} = \frac{V_T}{I_{DQ}} \tag{1.29}$$

This equation tells us that the incremental resistance is a function of the dc bias current I_{DQ} and is inversely proportional to the slope of the I–V characteristics curve, as shown in Figure 1.31(d).

Circuit Analysis

To analyze the circuit shown in Figure 1.31(a), we can use the piecewise linear model for the dc calculations and Equation (1.29) for the ac calculation.

Example 1.9 **Objective:** Analyze the circuit shown in Figure 1.31(a).

Assume circuit and diode parameters of $V_{PS} = 5\text{ V}$, $R = 5\text{ k}\Omega$, $V_\gamma = 0.6\text{ V}$, and $v_i = 0.1 \sin \omega t(\text{V})$.

Solution: Divide the analysis into two parts: the dc analysis and the ac analysis.

For *the dc analysis*, we set $v_i = 0$ and then determine the dc quiescent current as

$$I_{DQ} = \frac{V_{PS} - V_\gamma}{R} = \frac{5 - 0.6}{5} = 0.88\text{ mA}$$

The dc value of the output voltage is

$$V_O = I_{DQ}R = (0.88)(5) = 4.4\text{ V}$$

For *the ac analysis*, we consider only the ac signals and parameters in the circuit. In other words, we effectively set $V_{PS} = 0$. The ac Kirchhoff voltage law (KVL) equation becomes

$$v_i = i_d r_d + i_d R = i_d(r_d + R)$$

where r_d is again the small-signal diode diffusion resistance. From Equation (1.29), we have

$$r_d = \frac{V_T}{I_{DQ}} = \frac{0.026}{0.88} = 0.0295\,\text{k}\Omega$$

The ac diode current is

$$i_d = \frac{v_i}{r_d + R} = \frac{0.1 \sin \omega t}{0.0295 + 5} \Rightarrow 19.9 \sin \omega t (\mu\text{A})$$

The ac component of the output voltage is

$$v_o = i_d R = 0.0995 \sin \omega t (\text{V})$$

Comment: Throughout the text, we will divide the circuit analysis into a dc analysis and an ac analysis. To do so, we will use separate equivalent circuit models for each analysis.

Frequency Response

In the previous analysis, we implicitly assumed that the frequency of the ac signal was small enough that capacitance effects in the circuit would be negligible. If the frequency of the ac input signal increases, the **diffusion capacitance** associated with a forward-biased pn junction becomes important. The source of the diffusion capacitance is shown in Figure 1.32, which displays the dc values of the minority carrier concentrations and the changes caused by an ac component being superimposed. The ΔQ charge is alternately being charged and discharged through the junction as the voltage across the junction changes. The diffusion capacitance is the change in the stored minority carrier charge that is caused by a change in the voltage, or

$$C_d = \frac{dQ}{dV_D} \tag{1.30}$$

The diffusion capacitance C_d is normally much larger than the junction capacitance C_j, because of the magnitude of the charges involved.

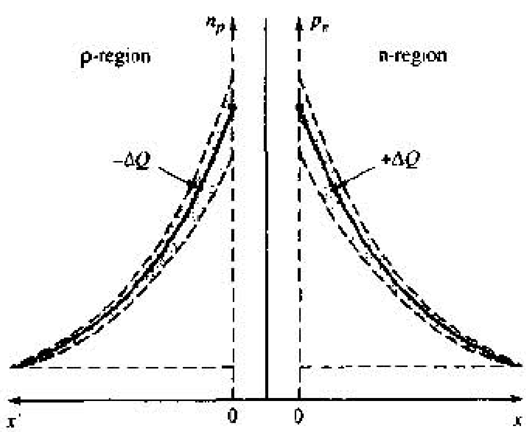


Figure 1.32 Change in minority carrier stored charge, leading to diffusion capacitance

1.4.2 Small-Signal Equivalent Circuit

The small-signal equivalent circuit of the forward-biased pn junction is shown in Figure 1.33 and is developed partially from the equation for the **admittance**, which is given by

$$Y = g_d + j\omega C_d \tag{1.31}$$

where g_d and C_d are the diffusion conductance and capacitance, respectively. We must also add the junction capacitance, which is in parallel with the diffusion resistance and capacitance, and a series resistance, which is required because of the finite resistances in the neutral n- and p-regions.

The small-signal equivalent circuit of the pn junction is used to obtain the ac response of a diode circuit subjected to ac signals superimposed on the Q-point values. Small-signal equivalent circuits of pn junctions are also used to develop small-signal models of transistors, and these models are used in the analysis and design of transistor amplifiers.

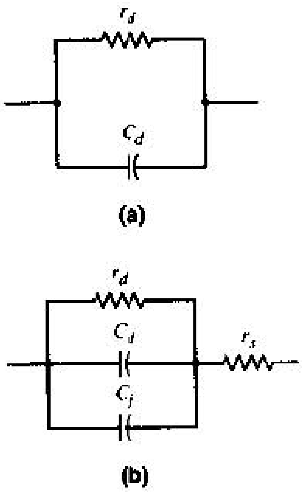


Figure 1.33 Small-signal equivalent circuit of the diode: (a) simplified version and (b) complete circuit

Test Your Understanding

1.17 Determine the diffusion conductance of a pn junction diode at $T = 300°K$ and biased at a current of 0.8 mA. (Ans. $g_d = 30.8$ mS)

1.18 The diffusion resistance of a pn junction diode at $T = 300°K$ is determined to be $r_d = 50\ \Omega$. What is the quiescent diode current? (Ans. $I_{DQ} = 0.52$ mA)

1.5 OTHER DIODE TYPES

Other types of diodes with specialized characteristics include the solar cell, photodiode, light-emitting diode, Schottky diode, and Zener diode. The solar cell, photodiode, light-emitting diode, and Zener diode are types of pn junction diodes with specific characteristics that make them useful in particular circuit applications.

1.5.1 Solar Cell

A **solar cell** is a pn junction device with no voltage directly applied across the junction. The pn junction, which converts solar energy into electrical energy, is connected to a load as indicated in Figure 1.34. When light hits the space-charge region, electrons and holes are generated. They are quickly separated and swept out of the space-charge region by the electric field, thus creating a **photocurrent**. The generated photocurrent will produce a voltage across the load, which means that the solar cell has supplied power. Solar cells are usually fabricated from silicon, but may be made from GaAs or other III–V compound semiconductors.

Solar cells have long been used to power the electronics in satellites and space vehicles, and also as the power supply to some calculators. Solar cells are also used to power race cars in a Sunrayce event. Collegiate teams in the United States design, build and drive the race cars. Typically, a Sunrayce car

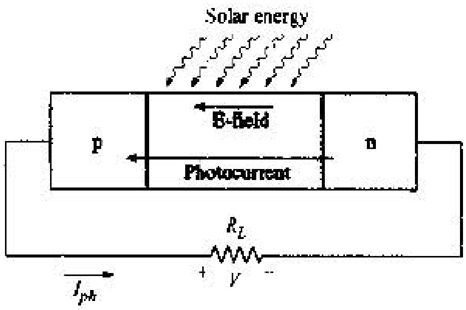


Figure 1.34 A pn junction solar cell connected to load

has 8 m^2 of solar cell arrays that can produce 800 W of power on a sunny day at noon. The power from the solar array can be used either to power an electric motor or to charge a battery pack.

1.5.2 Photodiode

Photodetectors are devices that convert optical signals into electrical signals. An example is the **photodiode**, which is similar to a solar cell except that the pn junction is operated with a reverse-bias voltage. Incident photons or light waves create excess electrons and holes in the space-charge region. These excess carriers are quickly separated and swept out of the space-charge region by the electric field, thus creating a "photocurrent." This generated photocurrent is directly proportional to the incident photon flux.

1.5.3 Light-Emitting Diode

The **light-emitting diode** (LED) converts current to light. As previously explained, when a forward-bias voltage is applied across a pn junction, electrons and holes flow across the space-charge region and become excess minority carriers. These excess minority carriers diffuse into the neutral semiconductor regions, where they recombine with majority carriers. If the semiconductor is a **direct bandgap material**, such as GaAs, the electron and hole can recombine with no change in momentum, and a photon or light wave can be emitted. Conversely, in an **indirect bandgap material**, such as silicon, when an electron and hole recombine, both energy and momentum must be conserved, so the emission of a photon is very unlikely. Therefore, LEDs are fabricated from GaAs or other compound semiconductor materials. In an LED, the diode current is directly proportional to the recombination rate, which means that the output light intensity is also proportional to the diode current.

Monolithic arrays of LEDs are fabricated for numeric and alphanumeric displays, such as the readout of a digital voltmeter.

An LED may be integrated into an optical cavity to produce a coherent photon output with a very narrow bandwidth. Such a device is a laser diode, which is used in optical communications applications.

The LED can be used in conjunction with a photodiode to create an optical system such as that shown in Figure 1.35. The light signal created

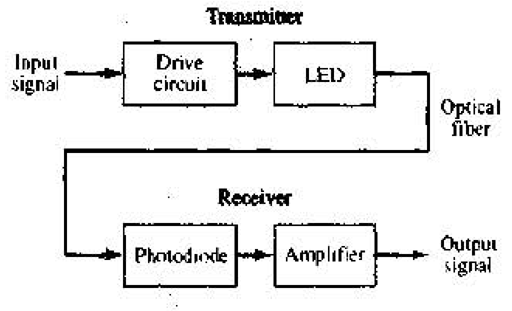


Figure 1.35 Basic elements in an optical transmission system

may travel over relatively long distances through the optical fiber, because of the low optical absorption in high-quality optical fibers.

1.5.4 Schottky Barrier Diode

A **Schottky barrier diode**, or simply a Schottky diode, is formed when a metal, such as aluminium, is brought into contact with a *moderately* doped n-type semiconductor. Figure 1.36(a) shows the metal-semiconductor contact, and Figure 1.36(b) shows the circuit symbol with the current direction and voltage polarity.

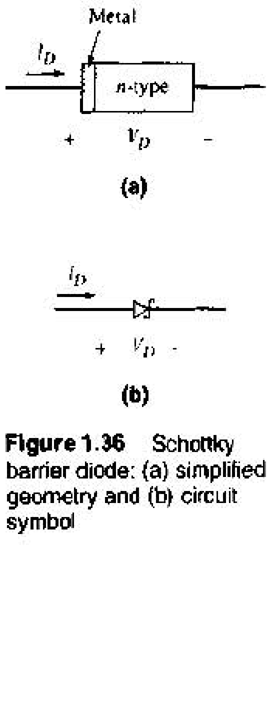


Figure 1.36 Schottky barrier diode: (a) simplified geometry and (b) circuit symbol

The current–voltage characteristics of a Schottky diode are very similar to those of a pn junction diode. The same ideal diode equation can be used for both devices. However, there are two important differences between the two diodes that directly affect the response of the Schottky diode.

First, the current mechanism in the two devices is different. The current in a pn junction diode is controlled by the diffusion of minority carriers. The current in a Schottky diode results from the flow of majority carriers over the potential barrier at the metallurgical junction. This means that there is no minority carrier storage in the Schottky diode, so the switching time from a forward bias to a reverse bias is very short compared to that of a pn junction diode. The storage time, t_s, for a Schottky diode is essentially zero.

Second, the reverse-saturation current I_S for a Schottky diode is larger than that of a pn junction diode for comparable device areas. This property means that the current in a Schottky diode is larger than that in a pn junction diode for the same forward-bias voltage.

Figure 1.37 compares the characteristics of the two diodes. Applying the piecewise linear model, we can determine that the Schottky diode has a smaller turn-on voltage than the pn junction diode. In later chapters, we will see how this lower turn-on voltage and the shorter switching time make the Schottky diode useful in integrated-circuit applications.

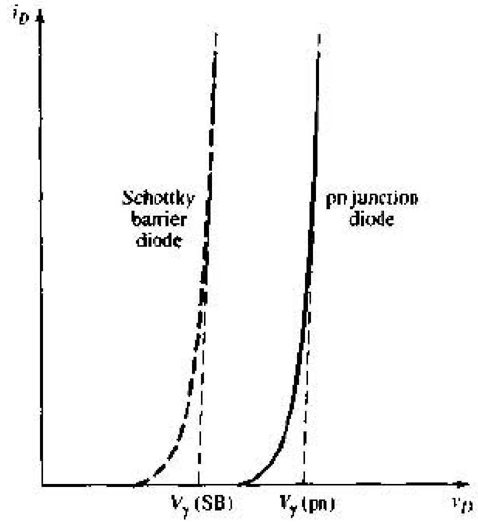


Figure 1.37 Comparison of the forward-bias *I–V* characteristics of a pn junction diode and a Schottky barrier diode

Example 1.10 **Objective:** Calculate the currents in a circuit containing both a pn junction diode and a Schottky diode.

Consider the circuit shown in Figure 1.38. Assume the cut-in voltages for the pn junction diode and the Schottky diode are $V_\gamma = 0.7$ V and $V_\gamma = 0.3$ V, respectively. Let $r_f = 0$ for both diodes.

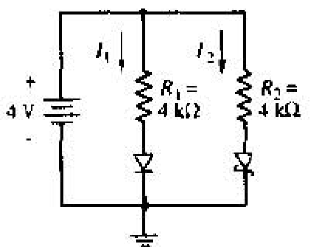


Figure 1.38 Simple circuit with both a pn junction diode and a Schottky barrier diode

Solution: The current I_1 is the voltage difference across R_1 divided by the resistance R_1, or

$$I_1 = \frac{4 - 0.7}{4} = 0.825 \text{ mA}$$

Similarly, the current I_2 is the voltage difference across R_2 divided by the resistance R_2, or

$$I_2 = \frac{4 - 0.3}{4} = 0.925 \text{ mA}$$

Comment: The dc calculations for a circuit containing a Schottky diode are the same as those for a circuit containing a pn junction diode.

Another type of metal–semiconductor junction is also possible. A metal applied to a heavily doped semiconductor forms, in most cases, an *ohmic contact*: that is, a contact that conducts current equally in both directions, with very little voltage drop across the junction. Ohmic contacts are used to connect one semiconductor device to another on an IC, or to connect an IC to its external terminals.

Test Your Understanding

1.19 The reverse-saturation currents of a pn junction diode and a Schottky diode are $I_S = 10^{-12}$ A and 10^{-8} A, respectively. Determine the forward-bias voltages required to produce 1 mA in each diode. (Ans. pn diode, $V_D = 0.539$ V; Schottky diode, $V_D = 0.299$ V)

1.20 A pn junction diode and a Schottky diode both have forward-bias currents of 1.2 mA. The reverse-saturation current of the pn junction diode is $I_S = 4 \times 10^{-15}$ A. The difference in forward-bias voltages is 0.265 V. Determine the reverse-saturation current of the Schottky diode. (Ans. $I_S = 1.07 \times 10^{-10}$ A)

1.5.5 Zener Diode

As mentioned earlier in this chapter, the applied reverse-bias voltage cannot increase without limit. At some point, breakdown occurs and the current in the reverse-bias direction increases rapidly. The voltage at this point is called the breakdown voltage. The diode I–V characteristics, including breakdown, are shown in Figure 1.39.

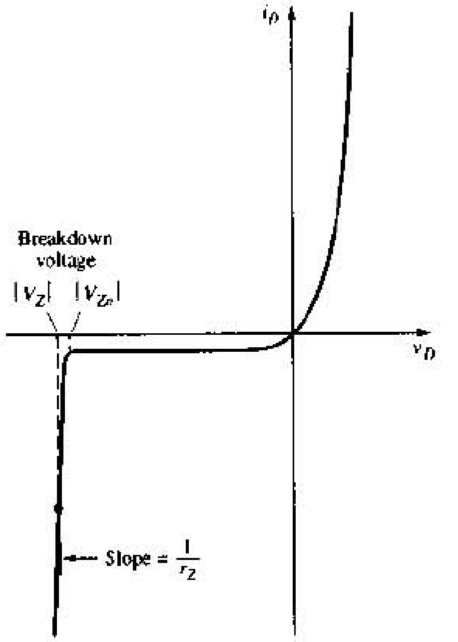


Figure 1.39 Diode I–V characteristics showing breakdown effects

Diodes, called **Zener diodes**, can be designed and fabricated to provide a specified breakdown voltage V_{Zo}. (Although the breakdown voltage is on the negative voltage axis (reverse-bias), its value is given as a positive quantity.) The large current that may exist at breakdown can cause heating effects and catastrophic failure of the diode due to the large power dissipation in the device. However, diodes can be operated in the breakdown region by limiting the current to a value within the capabilities of the device. Such a diode can be used as a constant-voltage reference in a circuit. The diode breakdown voltage is essentially constant over a wide range of currents and temperatures. The slope of the I–V characteristics curve in breakdown is quite large, so the incremental resistance r_z is small. Typically, r_z is in the range of a few ohms or tens of ohms.

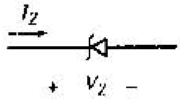


Figure 1.40 Circuit symbol of the Zener diode

The circuit symbol of the Zener diode is shown in Figure 1.40. (Note the difference between this symbol and the Schottky diode symbol.) The voltage V_Z is the Zener breakdown voltage, and the current I_Z is the reverse-bias current when the diode is operating in the breakdown region.

Design Example 1.11 **Objective:** Consider a simple constant-voltage reference circuit and design the value of resistance required to limit the current in this circuit.

Consider the circuit shown in Figure 1.41. Assume that the Zener diode breakdown voltage is $V_Z = 5.6$ V and the Zener resistance is $r_z = 0$. The current in the diode is to be limited to 3 mA.

Solution: As before, we can determine the current from the voltage difference across R divided by the resistance. That is,

$$I = \frac{V_{PS} - V_Z}{R}$$

The resistance is then

$$R = \frac{V_{PS} - V_Z}{I} = \frac{10 - 5.6}{3} = 1.47\,\text{k}\Omega$$

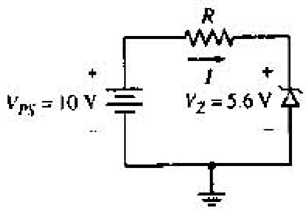


Figure 1.41 Simple circuit containing a Zener diode

Comment: The resistance external to the Zener diode limits the current when the diode is operating in the breakdown region. In the circuit shown in the figure, the output voltage will remain constant at 5.6 V, even though the power supply voltage and the resistance may change over a limited range. Hence, this circuit provides a constant output voltage. We will see further applications of the Zener diode in the next chapter.

Test Your Understanding

1.21 Consider the circuit shown in Figure 1.41. Determine the value of resistance R required to limit the power dissipated in the Zener diode to 10 mW. (Ans. $R = 2.46\,\text{k}\Omega$)

1.22 A Zener diode has an equivalent series resistance of 20 Ω. If the voltage across the Zener diode is 5.20 V at $I_Z = 1$ mA, determine the voltage across the diode at $I_Z = 10$ mA. (Ans. $V_Z = 5.38$ V)

1.6 SUMMARY

- We initially considered some of the characteristics and properties of semiconductor materials. We discussed the concept of electrons (negative charge) and holes (positive charge) as two distinct charge carriers in a semiconductor. The doping of pure semiconductor crystals with specific types of impurity atoms produces either n-type materials, which have a preponderance of electrons, or p-type materials, which have a preponderance of holes. The concepts of n-type and p-type materials are used throughout the text.
- A pn junction diode is formed when an n-doped region and a p-doped region are directly adjacent to each other. The current–voltage characteristics of the diode are nonlinear: The current is an exponential function of voltage in the forward-bias condition, and is essentially zero in the reverse-bias condition.
- Since the i–v relationship of the diode is nonlinear, the analysis of circuits containing diodes is not as straightfoward as that of linear circuits that contain only linear resistors. A piecewise-linear model of the diode was developed so that approximate hand calculation results can be easily obtained. The i–v characteristics of the diode are broken into linear segments, which are valid over particular regions of operation. The concept of a diode turn-on voltage was introduced as part of the piecewise linear model.
- Time-varying, or ac signals, may be superimposed on a dc diode current and voltage. A small-signal linear equivalent circuit was developed and is used to determine the relationship between the ac current and ac voltage. This same equivalent circuit will be applied extensively when the frequency response of transistors is discussed.

CHECKPOINT

After studying this chapter, the reader should have the ability to:

✔ Understand the concept of intrinsic carrier concentration, the difference between n-type and p-type materials, and the concept of drift and diffusion currents. (Section 1.1)

✔ Analyze a simple diode circuit using the ideal diode current–voltage characteristics and using the iteration analysis technique. (Section 1.3)

✔ Analyze a diode circuit using the piecewise linear approximation model for the diode. (Section 1.3)

✔ Determine the small-signal characteristics of a diode using the small-signal equivalent circuit. (Section 1.4)

✔ Understand the general characteristics of a solar cell, light-emitting diode, Schottky barrier diode, and Zener diode. (Section 1.5)

REVIEW QUESTIONS

1. Describe an intrinsic semiconductor material. What is meant by the intrinsic carrier concentration?
2. Describe the concept of an electron and a hole as charge carriers in the semiconductor material.
3. Describe an extrinsic semiconductor material. What is the value of the electron concentration in an n-type material, and what is the value of the hole concentration in a p-type material?
4. Describe the concepts of drift current and diffusion current in a semiconductor material.

5. How is a pn junction formed? What is meant by a built-in potential barrier, and how is it formed?
6. How is a junction capacitance created in a reverse-biased pn junction diode?
7. Write the ideal diode current–voltage relationship. Describe the meaning of I_S and V_T.
8. Describe the iteration method of analysis and when it must be used to analyze a diode circuit.
9. Describe the piecewise linear model of a diode and why it is useful. What is the diode turn-on voltage?
10. Define a load line in a simple diode circuit.
11. Under what conditions is the small-signal model of a diode used in the analysis of a diode circuit?
12. Describe the operation of a simple solar cell circuit.
13. How do the i–v characteristics of a Schottky barrier diode differ from those of a pn junction diode?
14. What characteristic of a Zener diode is used in the design of a Zener diode circuit?

PROBLEMS

[Note: Unless otherwise specified, assume that $T = 300\,^\circ\text{K}$ in the following problems. Also, assume the emission coefficient is $n = 1$ unless otherwise stated.]

Section 1.1 Semiconductor Materials and Properties

1.1 (a) Calculate the intrinsic carrier concentration in silicon at (i) $T = 275\,^\circ\text{K}$ and (ii) $T = 325\,^\circ\text{K}$. (b) Repeat part (a) for gallium arsenide.

1.2 (a) The intrinsic carrier concentration in silicon is to be no larger than $n_i = 10^{12}\,\text{cm}^{-3}$. Determine the maximum allowable temperature. (b) Repeat part (a) for $n_i = 10^9\,\text{cm}^{-3}$.

1.3 (a) Find the concentrations of electrons and holes in a sample of silicon that has a concentration of donor atoms equal to $5 \times 10^{15}\,\text{cm}^{-3}$. Is the semiconductor n-type or p-type? (b) Repeat part (a) for gallium arsenide.

1.4 (a) Calculate the concentration of electrons and holes in a silicon semiconductor sample that has a concentration of acceptor atoms equal to $10^{16}\,\text{cm}^{-3}$. Is the semiconductor n- or p-type? (b) Repeat part (a) for germanium.

1.5 The electron concentration in silicon at $T = 300\,^\circ\text{K}$ is $n_o = 5 \times 10^{15}\,\text{cm}^{-3}$. (a) Determine the hole concentration. (b) Is the material n-type or p-type? (c) What is the impurity doping concentration?

D1.6 (a) A silicon semiconductor material is to be designed such that the majority carrier electron concentration is $n_o = 7 \times 10^{15}\,\text{cm}^{-3}$. Should donor or acceptor impurity atoms be added to intrinsic silicon to achieve this electron concentration? What concentration of dopant impurity atoms is required? (b) In this silicon material, the minority carrier hole concentration is to be no larger than $p_o = 10^6\,\text{cm}^{-3}$. Determine the maximum allowable temperature.

1.7 The applied electric field in p-type silicon is $E = 15\,\text{V/cm}$. The semiconductor conductivity is $\sigma = 2.2\,(\Omega\text{–cm})^{-1}$ and the cross-sectional area is $A = 10^{-4}\,\text{cm}^2$. Determine the drift current in the semiconductor.

1.8 A drift current density of $85\ A/cm^2$ is established in n-type silicon with an applied electric field of $E = 12\ V/cm$. Determine the conductivity of the semiconductor.

1.9 In GaAs, the mobilities are $\mu_n = 8500\ cm^2/V\text{-}s$ and $\mu_p = 400\ cm^2/V\text{-}s$. (a) Determine the range in conductivity for a range in donor concentration of $10^{15} \leq N_d \leq 10^{19}\ cm^{-3}$. (b) Using the results of part (a), determine the range in drift current density if the applied electric field is $E = 0.10\ V/cm$.

1.10 GaAs is doped to $N_a = 10^{17}\ cm^{-3}$. (a) Calculate n_o and p_o. (b) Excess electrons and holes are generated such that $\delta n = \delta p = 10^{15}\ cm^{-3}$. Determine the total concentration of electrons and holes.

Section 1.2 The pn Junction

1.11 Calculate V_{bi} in a silicon pn junction for: (a) $N_d = N_a = 10^{15}\ cm^{-3}$; (b) $N_d = 10^{15}\ cm^{-3}$, $N_a = 10^{18}\ cm^{-3}$; and (c) $N_d = N_a = 10^{18}\ cm^{-3}$.

1.12 Repeat Problem 1.11 for gallium arsenide.

1.13 The donor concentration in the n-region of a silicon pn junction is $N_d = 10^{16}\ cm^{-3}$. Plot V_{bi} versus N_a over the range $10^{15} \leq N_a \leq 10^{18}\ cm^{-3}$ where N_a is the acceptor concentration in the p-region.

1.14 Consider a uniformly doped GaAs pn junction with doping concentrations of $N_a = 5 \times 10^{18}\ cm^{-3}$ and $N_d = 5 \times 10^{16}\ cm^{-3}$. Plot the built-in potential barrier V_{bi} versus temperature for $200\ °K \leq T \leq 500\ °K$.

1.15 A silicon pn junction has zero-bias junction capacitance of $C_{jo} = 1\ pF$ and doping concentrations of $N_a = 2 \times 10^{16}\ cm^{-3}$ and $N_d = 2 \times 10^{15}\ cm^{-3}$. Calculate the junction capacitance at: (a) $V_R = 1\ V$ and (b) $V_R = 5\ V$.

***1.16** The zero-bias capacitance of a silicon pn junction diode is $C_{jo} = 0.02\ pF$ and the built-in potential is $V_{bi} = 0.80\ V$. The diode is reverse biased through a 47-kΩ resistor and a voltage source. (a) For $t < 0$, the applied voltage is 5 V and, at $t = 0$, the applied voltage drops to zero volts. Estimate the time it takes for the diode voltage to change from 5 V to 1.5 V. (As an approximation, use the average diode capacitance between the two voltage levels.) (b) Repeat part (a) for an input voltage change from 0 V to 5 V and a diode voltage change from 0 V to 3.5 V. (Use the average diode capacitance between these two voltage levels.)

1.17 A silicon pn junction is doped at $N_a = 10^{18}\ cm^{-3}$ and $N_d = 10^{15}\ cm^{-3}$. The zero-bias junction capacitance is $C_{jo} = 0.25\ pF$. An inductance of 2.2 mH is placed in parallel with the pn junction. Calculate the resonant frequency f_o of the circuit for reverse-bias voltages of: (a) $V_R = 1\ V$, and (b) $V_R = 10\ V$.

1.18 (a) At what reverse bias voltage does the reverse-bias current in a silicon pn junction diode reach 90 percent of its saturation value? (b) What is the ratio of the current for a forward-bias voltage of 0.2 V to the current for a reverse-bias voltage of 0.2 V?

1.19 (a) Determine the current in a silicon pn junction diode for forward-bias voltages of 0.5, 0.6, and 0.7 V if the reverse-saturation current is $I_S = 10^{-11}\ A$. (b) Repeat part (a) for $I_S = 10^{-13}\ A$.

1.20 For a pn junction diode, what must be the forward-bias voltage to produce a current of 150 μA if (a) $I_S = 10^{-11}\ A$ and (b) $I_S = 10^{-13}\ A$.

1.21 A silicon pn junction diode has an emission coefficient of $n = 2$. The diode current is 1 mA when $V_D = 0.7\ V$. (a) Find the reverse-saturation current. (b) Determine the diode current when the voltage is increased to 0.8 V. (c) Repeat parts (a) and (b) when the emission coefficient is $n = 1$.

1.22 The reverse-saturation current of a silicon pn junction diode at $T = 300$ °K is $I_S = 10^{-12}$ A. Determine the temperature range over which I_S varies from 0.5×10^{-12} A to 50×10^{-12} A.

1.23 A silicon pn junction diode has an applied forward-bias voltage of 0.6 V. Determine the ratio of current at 100 °C to that at −55 °C.

1.24 (a) Consider a silicon pn junction diode operating in the forward-bias region. Determine the increase in forward-bias voltage that will cause a factor of 10 increase in current. (b) Repeat part (a) for a factor of 100 increase in current.

Section 1.3 DC Diode Analysis

1.25 A pn junction diode is in series with a 10 MΩ resistor and a 1.5 V power supply. The reverse-saturation current of the diode is $I_S = 30$ nA. (a) Determine the diode current and voltage if the diode is forward biased. (b) Repeat part (a) if the diode is reverse biased.

***1.26** (a) The diode in the circuit shown in Figure P1.26 has a reverse-saturation current of $I_S = 5 \times 10^{-13}$ A. Determine the diode voltage and current. (b) Repeat part (a) with a computer simulation analysis.

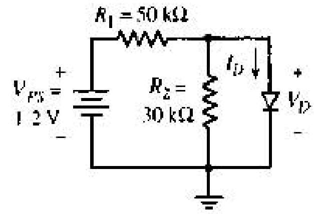


Figure P1.26

1.27 The reverse-saturation current of each diode in the circuit shown in Figure P1.27 is $I_S = 2 \times 10^{-13}$ A. Determine the input voltage V_I required to produce an output voltage of $V_O = 0.60$ V.

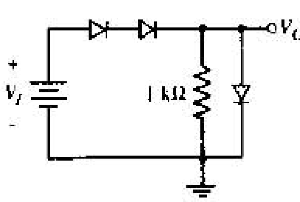


Figure P1.27

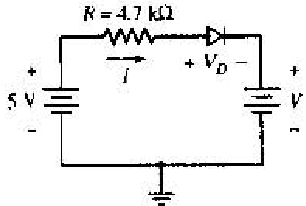


Figure P1.28

1.28 (a) In the circuit shown in Figure P1.28, find the diode voltage V_D and the supply voltage V such that the current is $I = 0.50$ mA. Assume the reverse-saturation current is $I_S = 5 \times 10^{-12}$ A. (b) From the results of part (a), determine the power dissipated in the diode.

1.29 (a) Consider the circuit shown in Figure P1.26. The value of R_1 is reduced to $R_1 = 10$ kΩ and the cut-in voltage of the diode is $V_\gamma = 0.7$ V. Determine I_D and V_D. (b) Repeat part (a) if $R_1 = 50$ kΩ. (c) Repeat parts (a) and (b) with a computer simulation analysis.

1.30 The cut-in voltage for each diode in the circuits shown in Figure P1.30 is $V_\gamma = 0.6\,\text{V}$. For each circuit, determine the diode current I_D and the voltage V_O (measured with respect to ground potential).

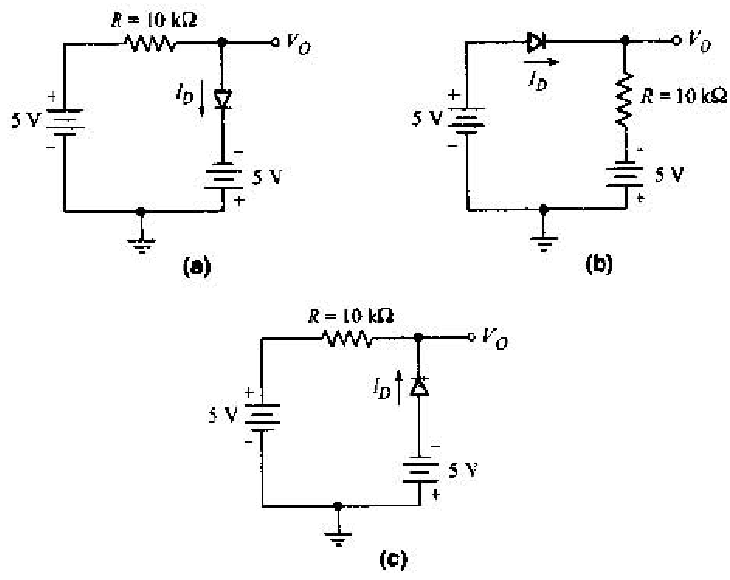


Figure P1.30

***D1.31** The cut-in voltage of the diode shown in the circuit in Figure P1.31 is $V_\gamma = 0.7\,\text{V}$. The diode is to remain biased "on" for a power supply voltage in the range $5 \le V_{PS} \le 10\,\text{V}$. The minimum diode current is to be $I_D(\text{min}) = 2\,\text{mA}$. The maximum power dissipated in the diode is to be no more than 10 mW. Determine appropriate values of R_1 and R_2.

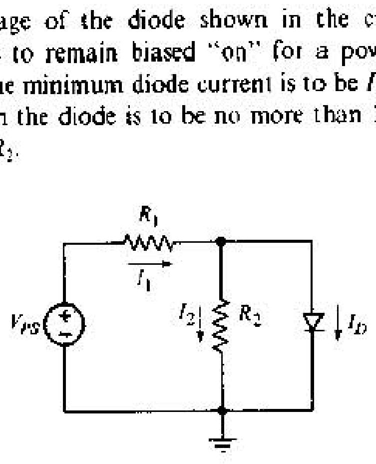


Figure P1.31

1.32 Assume each diode in the circuit shown in Figure P1.32 has a cut-in voltage of $V_\gamma = 0.65\,\text{V}$. The input voltage is $V_I = 5\,\text{V}$. Determine the value of R_1 required such that I_{D1} is one-half the value of I_{D2}. What are the values of I_{D1} and I_{D2}?

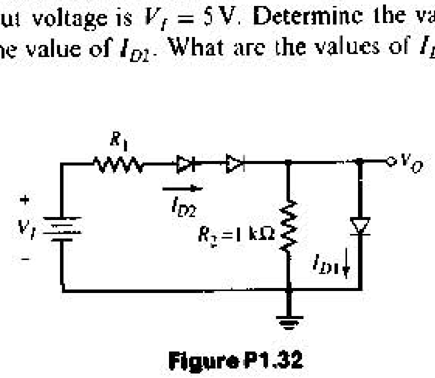


Figure P1.32

1.33 The voltage V in Figure P1.28 is $V = 1.7\,V$. If the cut-in voltage of the diode is $V_\gamma = 0.65\,V$, determine the new value of R required to limit the power dissipation in the diode to no more than 0.20 mW.

1.34 Repeat Problem 1.25 if the diode cut-in voltage is $V_\gamma = 0.7\,V$. Compare these results to those obtained in Problem 1.25. Discuss any discrepancies.

Section 1.4 Small-Signal Diode Analysis

1.35 (a) Consider a pn junction diode biased at $I_{DQ} = 1$ mA. A sinusoidal voltage is superimposed on V_{DQ} such that the peak-to-peak sinusoidal current is $0.05 I_{DQ}$. Find the value of the applied peak-to-peak sinusoidal voltage. (b) Repeat part (a) if $I_{DQ} = 0.1$ mA.

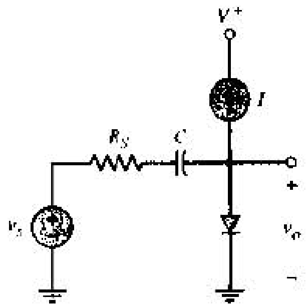


Figure P1.36

***1.36** The diode in the circuit shown in Figure P1.36 is biased with a constant current source I. A sinusoidal signal v_s is coupled through R_S and C. Assume that C is large so that it acts as a short circuit to the signal. (a) Show that the sinusoidal component of the diode voltage is given by

$$v_o = v_s\left(\frac{V_T}{V_T + IR_S}\right)$$

(b) If $R_S = 260\,\Omega$, find v_o/v_s for $I = 1$ mA, $I = 0.1$ mA, and $I = 0.01$ mA.

Section 1.5 Other Types of Diodes

1.37 The reverse-saturation currents of a pn junction diode and a Schottky diode are $I_S = 10^{-14}$ A and 10^{-9} A, respectively. Determine the forward-bias voltages required to produce a current of 100 μA in each diode.

1.38 A pn junction diode and a Schottky diode have equal cross-sectional areas and have forward-bias currents of 0.5 mA. The reverse-saturation current of the Schottky diode is $I_S = 5 \times 10^{-7}$ A. The difference in forward-bias voltages between the two diodes is 0.30 V. Determine the reverse-saturation current of the pn junction diode.

1.39 Consider the circuit shown in Figure P1.39. The reverse-saturation currents of the Schottky diode and pn junction diode are 10^{-8} A and 10^{-12} A, respectively. Determine the value of R such that the currents in the diodes are equal. What is the voltage across each diode?

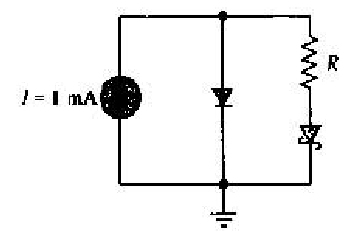


Figure P1.39

1.40 The reverse-saturation currents of a Schottky diode and a pn junction diode are $I_S = 5 \times 10^{-8}$ A and 10^{-12} A, respectively. (a) The diodes are connected in parallel and the parallel combination is driven by a constant current of 0.5 mA. (i) Determine the current in each diode. (ii) Determine the voltage across each diode. (b) Repeat part (a) for the diodes connected in series, with a voltage of 0.90 V connected across the series combination.

***1.41** Consider the Zener diode circuit shown in Figure P1.41. The Zener breakdown voltage is $V_Z = 5.6$ V at $I_Z = 0.1$ mA, and the incremental Zener resistance is $r_z = 10\,\Omega$. (a) Determine V_O with no load ($R_L = \infty$). (b) Find the change in the output voltage if V_{PS} changes by ± 1 V. (c) Find V_O if $V_{PS} = 10$ V and $R_L = 2\,\text{k}\Omega$.

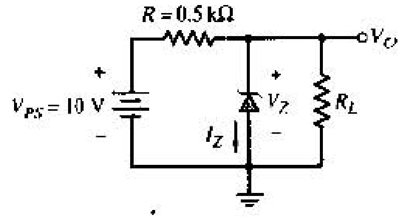


Figure P1.41

1.42 A voltage regulator consists of a 6.8 V Zener diode in series with a 200 Ω resistor and a 9 V power supply. (a) Neglecting r_z, calculate the diode current and power dissipation. (b) If the power supply is increased to 12 V, calculate the percentage increase in diode current and power dissipation.

***1.43** Consider the Zener diode circuit shown in Figure P1.41. The Zener diode voltage is $V_Z = 6.8$ V at $I_Z = 0.1$ mA and the incremental Zener resistance is $r_z = 20\,\Omega$. (a) Calculate V_O with no load ($R_L = \infty$). (b) Find the change in the output voltage when a load resistance of $R_L = 1\,\text{k}\Omega$ is connected.

COMPUTER SIMULATION PROBLEMS

1.44 Use a computer simulation to generate the ideal current–voltage characteristics of a diode from a reverse-bias voltage of 5 V to a forward-bias current of 10 mA, for an I_S parameter value of: (a) 10^{-14} A and (b) 10^{-10} A. Use the default values for all other parameters.

1.45 Use a computer simulation to generate the I–V characteristics of a diode with $I_S = 10^{-12}$ A at temperatures of: (a) $T = 0\,^\circ$C, (b) $T = 25\,^\circ$C, (c) $T = 75\,^\circ$C, and (d) $T = 125\,^\circ$C. Plot the characteristics from a reverse-bias voltage of 5 V to a forward-bias current of 10 mA.

1.46 Consider the circuit shown in Figure 1.31(a) with $V_{PS} = 5$ V. Let $I_S = 10^{-14}$ A and assume that v_i is a sinusoidal source with a peak value of 0.25 V. Choose values of R to generate quiescent diode currents of approximately 0.1, 1.0, and 10 mA. From a computer simulation analysis, determine the peak values of the sinusoidal diode current and sinusoidal diode voltage for each dc diode current. Compare the relationship between the ac diode current and voltage to Equation (1.28(b)), where r_d is given by Equation (1.29). Do the computer simulation results compare favorably with the theoretical predictions?

1.47 Repeat Problem 1.16 using the actual C versus V_R characteristics.

DESIGN PROBLEMS

[Note: Each design should be verified by a computer simulation.]

***D1.48** Design a circuit to produce the characteristics shown in Figure P1.48, where i_D is the diode current and v_I is the input voltage. Assume the diode has piecewise linear parameters of $V_\gamma = 0.7$ V and $r_f = 0$.

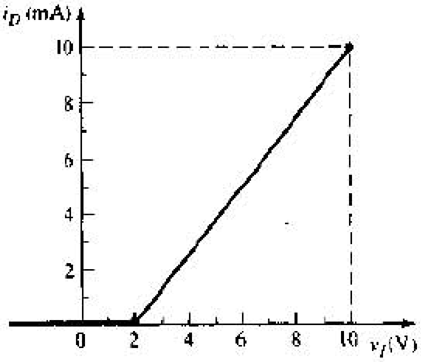


Figure P1.48

***D1.49** Design a circuit to produce the characteristics shown in Figure P1.49 where v_I is the input voltage and i_I is the current supplied by v_I. Assume any diodes in the circuit have piecewise linear parameters of $V_\gamma = 0.7$ V and $r_f = 0$.

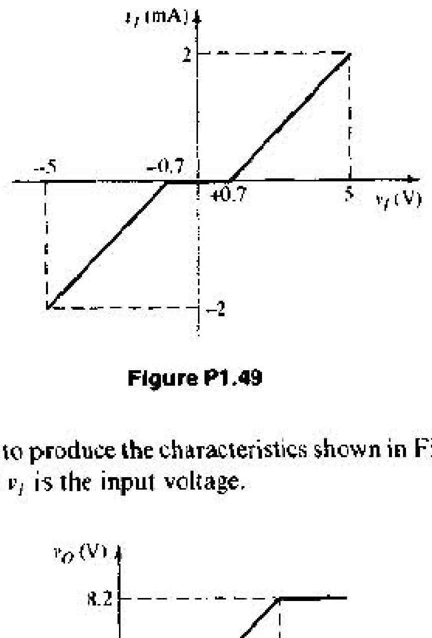


Figure P1.49

***D1.50** Design a circuit to produce the characteristics shown in Figure P1.50, where v_O is an output voltage and v_I is the input voltage.

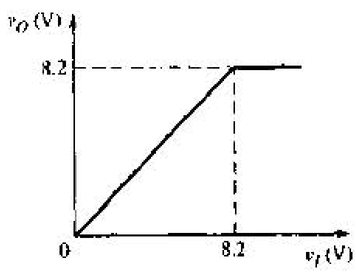


Figure P1.50

CHAPTER

2

Diode Circuits

2.0 PREVIEW

In the last chapter, we discussed some of the properties of semiconductor materials, and introduced the diode. We presented the ideal current–voltage relationship, and considered the piecewise linear model, which simplifies the dc analysis of diode circuits. In this chapter, the techniques and concepts developed in Chapter 1 are used to analyze and design electronic circuits containing diodes. A general goal of this chapter is to develop the ability to use the piecewise linear model and approximation techniques in the hand analysis and design of various diode circuits.

Each circuit to be considered accepts an input signal at a set of input terminals and produces an output signal at a set of output terminals. This process is called **signal processing**. The circuit "processes" the input signal and produces an output signal that is a different shape or a different function compared to the input signal. We will see in this chapter how diodes are used to perform these various signal processing functions.

Circuits to be considered perform functions such as rectification, clipping, and clamping. These functions are possible only because of the nonlinear properties of the pn junction diode. The conversion of an ac voltage to a dc voltage, such as for a dc power supply, is called rectification. Clipper diode circuits clip portions of a signal that are above or below some reference level. Clamper circuits shift the entire signal by some dc value.

Zener diodes, which operate in the reverse-bias breakdown region, have the advantage that the voltage across the diode in this region is nearly constant over a wide range of currents. Such diodes are used in voltage reference or voltage regulator circuits. Finally, we look at the circuits of two special diodes: the light-emitting diode (LED) and the photodiode. An LED circuit is used in visual displays, such as the seven-segment numerical display. The photodiode circuit is used to detect the presence or absence of light and convert this information into an electrical signal.

Although diodes are useful electronic devices, we will begin to see the limitations of these devices and the desirability of having some type of "amplifying" device.

2.1 RECTIFIER CIRCUITS

One important application of diodes is in the design of rectifier circuits. A diode rectifier forms the first stage of a dc power supply as shown in Figure 2.1. As we will see throughout the text, a dc power supply is required to bias all electronic circuits. The dc output voltage v_O will usually be in the range of 3 to 24 V depending on the particular electronics application. Throughout the first part of this chapter, we will analyze and design the various stages in the power supply circuit.

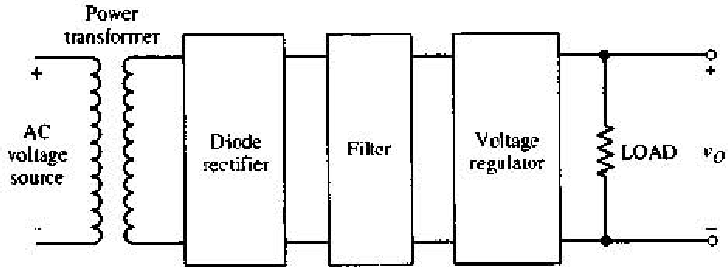


Figure 2.1 Block diagram of an electronic power supply

Rectification is the process of converting an alternating (ac) voltage into one that is limited to one polarity. The diode is useful for this function because of its nonlinear characteristics, that is, current exists for one voltage polarity, but is essentially zero for the opposite polarity. Rectification is classified as **half-wave** or **full-wave**, with half-wave being the simplest.

2.1.1 Half-Wave Rectification

Figure 2.2(a) shows a power transformer with a diode and resistor connected to the secondary of the transformer. We will use the piecewise linear approach in analyzing this circuit, assuming the diode forward resistance is $r_f = 0$ when the diode is "on."

The input signal, v_I, is, in general, a 120 V(rms), 60 Hz ac signal. Recall that the secondary voltage, v_S, and primary voltage, v_I, of an ideal transformer are related by

$$\frac{v_I}{v_S} = \frac{N_1}{N_2} \tag{2.1}$$

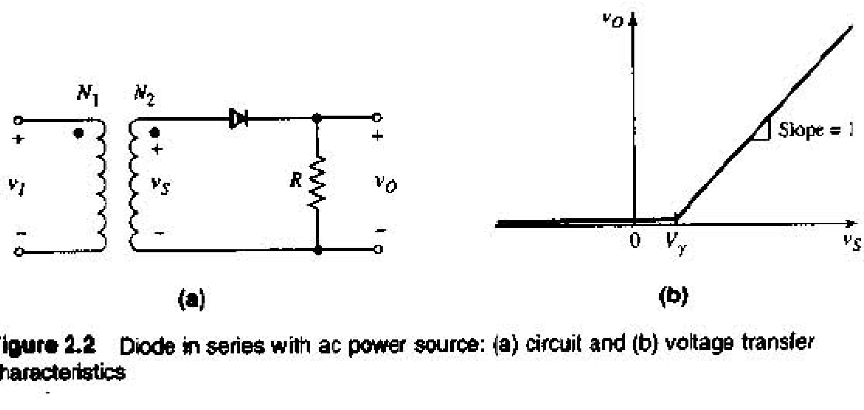


Figure 2.2 Diode in series with ac power source: (a) circuit and (b) voltage transfer characteristics

where N_1 and N_2 are the number of primary and secondary turns, respectively. The ratio N_1/N_2 is called the **transformer turns ratio**. The transformer turns ratio will be designed to provide a particular secondary voltage, v_S, which in turn will produce a particular output voltage v_O.

Problem-Solving Technique: Diode Circuits

In using the piecewise linear model of the diode, the first objective is to determine the linear region (conducting or not conducting) in which the diode is operating. To do this, we can:

1. Determine the input voltage condition such that a diode is conducting (on). Then find the output signal for this condition.
2. Determine the input voltage condition such that a diode is not conducting (off). Then find the output signal for this condition.

[Note: Item 2 can be performed before item 1 if desired.]

Figure 2.2(b) shows the voltage transfer characteristics, v_O versus v_S, for the circuit. For $v_S < 0$, the diode is reverse biased, which means that the current is zero and the output voltage is zero. As long as $v_S < V_\gamma$, the diode will be nonconducting, so the output voltage will remain zero. When $v_S > V_\gamma$, the diode becomes forward biased and a current is induced in the circuit. In this case, we can write

$$i_D = \frac{v_S - V_\gamma}{R} \tag{2.2(a)}$$

and

$$v_O = i_D R = v_S - V_\gamma \tag{2.2(b)}$$

For $v_S > V_\gamma$, the slope of the transfer curve is 1.

If v_S is a sinusoidal signal, as shown in Figure 2.3(a), the output voltage can be found using the voltage transfer curve in Figure 2.2(b). For $v_S \le V_\gamma$ the output voltage is zero; for $v_S > V_\gamma$, the output is given by Equation (2.2(b)), or

$$v_O = v_S - V_\gamma$$

and is shown in Figure 2.3(b). We can see that while the input signal v_S alternates polarity and has a time-average value of zero, the output voltage v_O is unidirectional and has an average value that is not zero. The input signal is therefore rectified. Also, since the output voltage appears only during the positive cycle of the input signal, the circuit is called a **half-wave rectifier**.

When the diode is cut off and nonconducting, no voltage drop occurs across the resistor R; therefore, the entire input signal voltage appears across the diode (Figure 2.3(c)). Consequently, the diode must be capable of handling the peak current in the forward direction and sustaining the largest peak inverse voltage (PIV) without breakdown. For the circuit shown in Figure 2.2(a), the value of PIV is equal to the peak value of v_S.

The load line concept can help in visualizing the operation of the half-wave rectifier circuit. Figure 2.4(a) shows the sine wave input. Figure 2.4(b) shows the piecewise linear characteristics of the diode, along with the load lines at

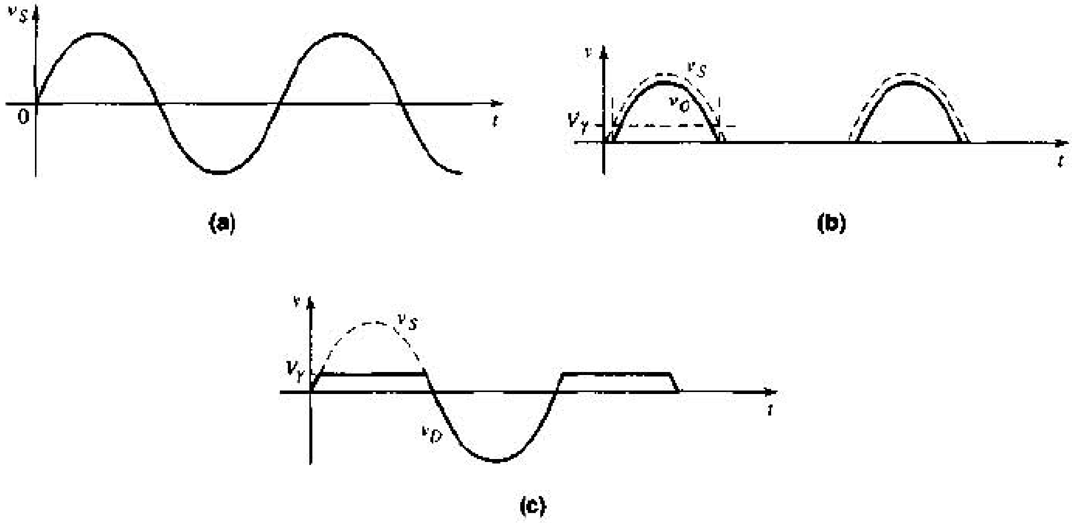


Figure 2.3 Half-wave rectifier circuit: (a) sinusoidal input voltage, (b) output voltage, and (c) diode voltage

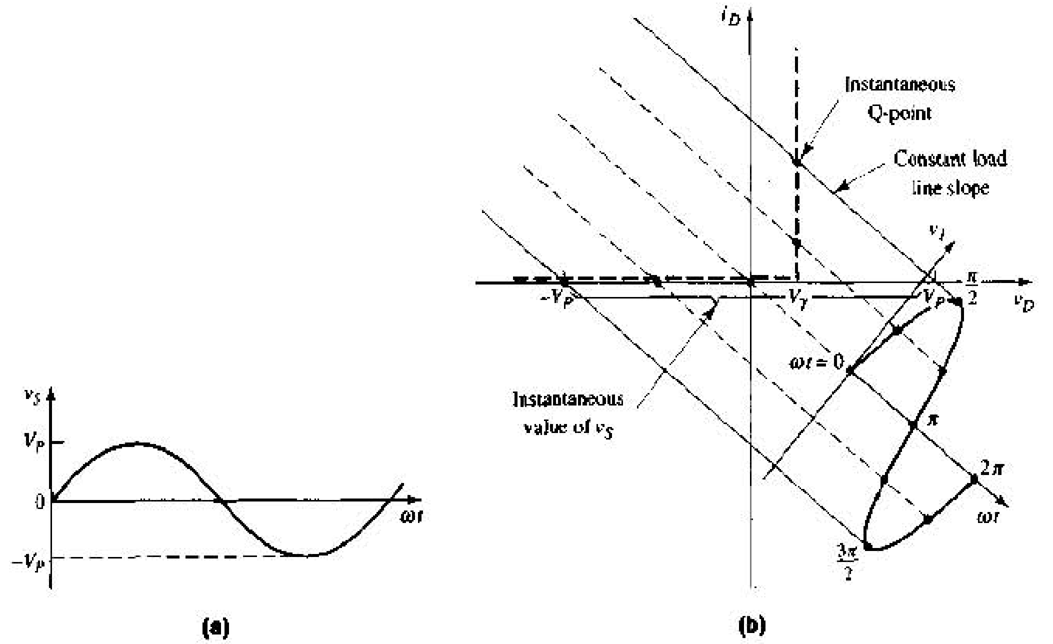


Figure 2.4 Operation of half-wave rectifier circuit: (a) sinusoidal input voltage and (b) diode piecewise linear characteristics and circuit load line at various times

various times. Because the resistance R is a constant, the slope of the load lines remains constant. However, since the magnitude of the power supply voltage varies with time, the magnitude of the load line also changes with time. As the load line sweeps across the diode I–V characteristics, the output voltage, diode voltage, and diode current can be determined as a function of time.

We can use a half-wave rectifier circuit to charge a battery as shown in Figure 2.5(a). Charging current exists whenever the instantaneous ac source voltage is greater than the battery voltage plus the diode cut-in voltage as

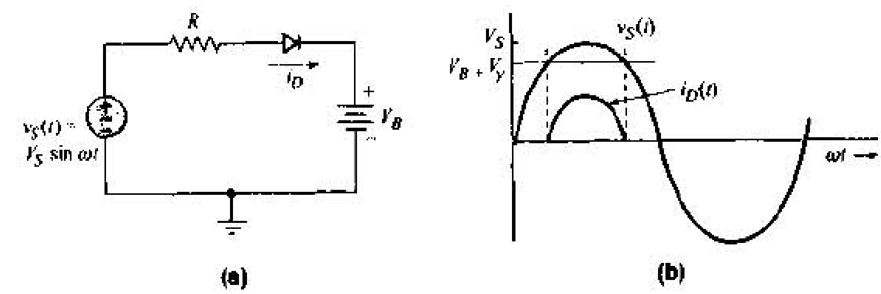


Figure 2.5 (a) Half-wave rectifier used as a battery charger; (b) input voltage and diode current waveforms

shown in Figure 2.5(b). The resistance R in the circuit is to limit the current. When the ac source voltage is less than V_B, the current is zero. Thus current flows only in the direction to charge the battery. One disadvantage of the half-wave rectifier is that we "waste" the negative half-cycles. The current is zero during the negative half-cycles, so there is no energy dissipated, but at the same time, we are not making use of any possible available energy.

Test Your Understanding

2.1 Figure 2.5(a) shows a simple circuit for charging a battery. Assume $V_B = 12$ V and $R = 100\ \Omega$. Also assume that v_S is a sinusoidal signal with a peak amplitude of 24 V and that the diode has piecewise linear parameters of $V_\gamma = 0.6$ V and $r_f = 0$. Determine: (a) the peak diode current; (b) the maximum reverse-bias diode voltage; and (c) the fraction (percent) of the cycle over which the diode conducts. (Ans. (a) 114 mA, (b) 36 V, (c) 32.4%)

2.1.2 Full-Wave Rectification

The full-wave rectifier inverts the negative portions of the sine wave so that a unipolar output signal is generated during both halves of the input sinusoid. One example of a full-wave rectifier circuit appears in Figure 2.6(a). The input to the rectifier consists of a power transformer, in which the input is normally a 120 V (rms), 60 Hz ac signal, and the two outputs are from a center-tapped secondary winding that provides equal voltages v_S, with the polarities shown. When the input line voltage is positive, both output signal voltages v_S are also positive.

The primary winding connected to the 120 V ac source has N_1 windings, and each half of the secondary winding has N_2 windings. The value of the v_S output voltage is 120 (N_2/N_1) volts (rms). The **turns ratio** of the transformer, usually designated (N_1/N_2) can be designed to "step down" the input line voltage to a value that will produce a particular dc output voltage from the rectifier.

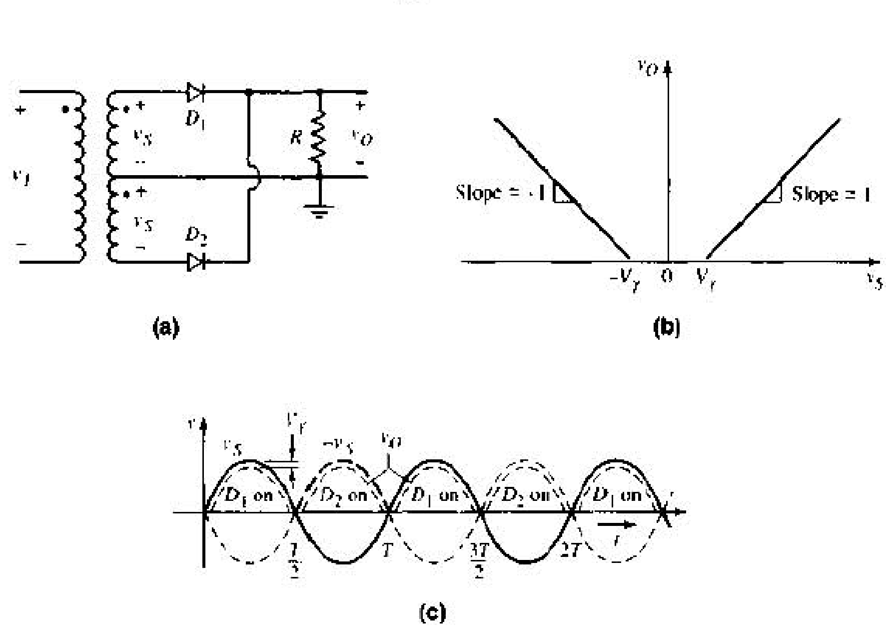


Figure 2.6 Full-wave rectifier: (a) circuit with center-tapped transformer, (b) voltage transfer characteristics, and (c) input and output waveforms

The input power transformer also provides electrical isolation between the powerline circuit and the electronic circuits to be biased by the rectifier circuit. This isolation reduces the risk of electrical shock.

During the positive half of the input voltage cycle, both output voltages v_S are positive; therefore, diode D_1 is forward biased and conducting and D_2 is reverse biased and cut off. The current through D_1 and the output resistance produce a positive output voltage. During the negative half cycle, D_1 is cut off and D_2 is forward biased, or "on," and the current through the output resistance again produces a positive output voltage. If we assume that the forward diode resistance r_f of each diode is small and negligible, we obtain the voltage transfer characteristics, v_O versus v_S, shown in Figure 2.6(b).

For a sinusoidal input voltage, we can determine the output voltage versus time by using the voltage transfer curve shown in Figure 2.6(b). When $v_S > V_\gamma$, D_1 is on and the output voltage is $v_O = v_S - V_\gamma$. When v_S is negative, then for $v_S < -V_\gamma$ or $-v_S > V_\gamma$, D_2 is on and the output voltage is $v_O = -v_S - V_\gamma$. The corresponding input and output voltage signals are shown in Figure 2.6(c). Since a rectified output voltage occurs during both the positive and negative cycles of the input signal, this circuit is called a **full-wave rectifier.**

Another example of a full-wave rectifier circuit appears in Figure 2.7(a). This circuit is a **bridge rectifier**, which still provides electrical isolation between the input ac powerline and the rectifier output, but does not require a center-tapped secondary winding. However, it does use four diodes, compared to only two in the previous circuit.

During the positive half of the input voltage cycle, v_S is positive, D_1 and D_2 are forward biased, D_3 and D_4 are reverse biased, and the direction of the current is as shown in Figure 2.7(a). During the negative half-cycle of the input voltage, v_S is negative, and D_3 and D_4 are forward biased. The direction of the current, shown in Figure 2.7(b), produces the same output voltage polarity as before.

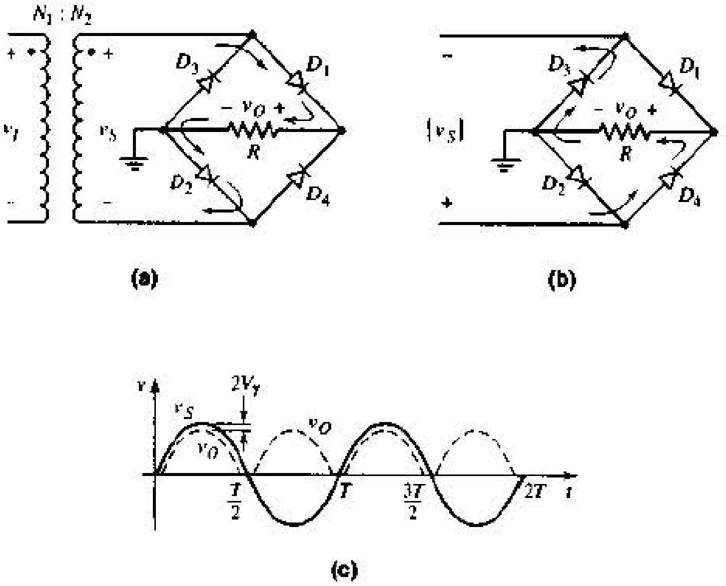


Figure 2.7 A full-wave bridge rectifier: (a) circuit showing the current direction for a positive input cycle, (b) current direction for a negative input cycle, and (c) input and output voltage waveforms

Figure 2.7(c) shows the sinusoidal voltage v_S and the rectified output voltage v_O. Because two diodes are in series in the conduction path, the magnitude of v_O is two diode drops less than the magnitude of v_S.

One difference to be noted in the bridge rectifier circuit in Figure 2.7(a) and the rectifier in Figure 2.6(a) is the ground connection. Whereas the center tap of the secondary winding of the circuit in Figure 2.6(a) is at ground potential, the secondary winding of the bridge circuit (Figure 2.7(a)) is not directly grounded. One side of the load R is grounded, but the secondary of the transformer is not.

Example 2.1 **Objective:** Compare voltages and the transformer turns ratio in two full-wave rectifier circuits.

Consider the rectifier circuits shown in Figures 2.6(a) and 2.7(a). Assume the input voltage is from a 120 V (rms), 60 Hz ac source. The desired peak output voltage v_O is 9 V, and the diode cut-in voltage is assumed to be $V_\gamma = 0.7$ V.

Solution: For the center-tapped transformer circuit shown in Figure 2.6(a), a peak voltage of $v_O(\max) = 9$ V means that the peak value of v_S is

$$v_S(\max) = v_O(\max) + V_\gamma = 9 + 0.7 = 9.7\text{ V}$$

For a sinusoidal signal, this produces an rms value of

$$v_{S,\text{rms}} = \frac{9.7}{\sqrt{2}} = 6.86\text{ V}$$

The turns ratio of the primary to each secondary winding must then be

$$\frac{N_1}{N_2} = \frac{120}{6.86} \cong 17.5$$

For the bridge circuit shown in Figure 2.7(a), a peak voltage of $v_O(\max) = 9$ V means that the peak value of v_S is

$$v_S(\max) = v_O(\max) + 2V_\gamma = 9 + 2(0.7) = 10.4\text{ V}$$

For a sinusoidal signal, this produces an rms value of

$$v_{S,rms} = \frac{10.4}{\sqrt{2}} = 7.35\text{ V}$$

The turns ratio should then be

$$\frac{N_1}{N_2} = \frac{120}{7.35} \cong 16.3$$

For the center-tapped rectifier, the peak inverse voltage (PIV) of a diode is

$$\text{PIV} = v_R(\max) = 2v_S(\max) - V_\gamma = 2(9.7) - 0.7 = 18.7\text{ V}$$

For the bridge rectifier, the peak inverse voltage of a diode is

$$\text{PIV} = v_R(\max) = v_S(\max) - V_\gamma = 10.4 - 0.7 = 9.7\text{ V}$$

Comment: These calculations demonstrate the advantages of the bridge rectifier over the center-tapped transformer circuit. First, only half as many turns are required for the secondary winding in the bridge rectifier. This is true because only half of the secondary winding of the center-tapped transformer is utilized at any one time. Second, for the bridge circuit, the peak inverse voltage that any diode must sustain without breakdown is only half that of the center-tapped transformer circuit.

Because of the advantages demonstrated in Example 2.1, the bridge rectifier circuit is used more often than the center-tapped transformer circuit.

2.1.3 Filters, Ripple Voltage, and Diode Current

If a capacitor is added in parallel with the load resistor of a half-wave rectifier to form a simple filter circuit (Figure 2.8(a)), we can begin to transform the half-wave sinusoidal output into a dc voltage. Figure 2.8(b) shows the positive half of the output sine wave, and the beginning portion of the voltage across the capacitor, assuming the capacitor is initially uncharged. If we assume that the diode forward resistance is $r_f = 0$, which means that the r_fC time constant is zero, the voltage across the capacitor follows this initial portion of the signal voltage. When the signal voltage reaches its peak and begins to decrease, the voltage across the capacitor also starts to decrease, which means the capacitor starts to discharge. The only discharge current path is through the resistor. If the RC time constant is large, the voltage across the capacitor discharges exponentially with time (Figure 2.8(c)). During this time period, the diode is cut off.

A more detailed analysis of the circuit response when the input voltage is near its peak value indicates a subtle difference between actual circuit operation and the qualitative description. If we assume that the diode turns off immediately when the input voltage starts to decrease from its peak value, then the output voltage will decrease exponentially with time, as previously indicated. An exaggerated sketch of these two voltages is shown in Figure 2.8(d). The output voltage decreases at a faster rate than the input voltage,

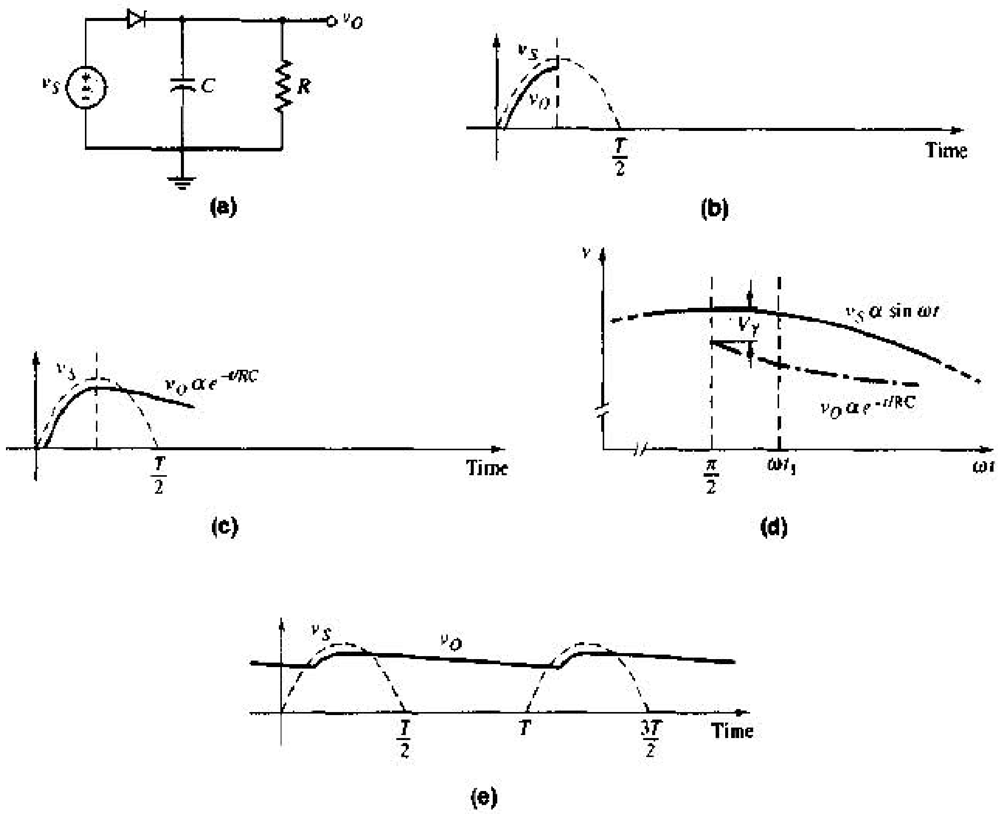


Figure 2.8 Simple filter circuit: (a) half-wave rectifier with an RC filter, (b) positive input voltage and initial portion of output voltage, (c) output voltage resulting from capacitor discharge, (d) expanded view of input and output voltages assuming capacitor discharge begins at $\omega t = \pi/2$, and (e) steady-state input and output voltages

which means that at time t_1, the difference between v_I and v_O, that is, the voltage across the diode, is greater than V_γ. However, this condition cannot exist. Therefore, the diode does not turn off immediately. If the RC time constant is large, there is only a small difference between the time of the peak input voltage and the time the diode turns off. In this situation, a computer analysis may provide more accurate results than an approximate hand analysis.

During the next positive cycle of the input voltage, there is a point at which the input voltage is greater than the capacitor voltage, and the diode turns back on. The diode remains on until the input reaches its peak value and the capacitor voltage is completely recharged.

Since the capacitor filters out a large portion of the sinusoidal signal, it is called a **filter capacitor**. The steady-state output voltage of the RC filter is shown in Figure 2.8(e).

The ripple effect in the output from a full-wave filtered rectifier circuit can be seen in the output waveform in Figure 2.9. The capacitor charges to its peak voltage value when the input signal is at its peak value. As the input decreases, the diode becomes reverse biased and the capacitor discharges through the output resistance R. Determining the ripple voltage is necessary for the design of a circuit with an acceptable amount of ripple.

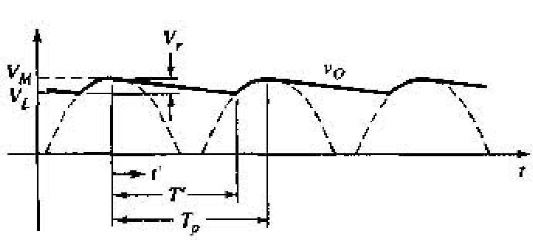


Figure 2.9 Output voltage of a full-wave rectifier with an *RC* filter

To a good approximation, the output voltage, that is, the voltage across the capacitor or the *RC* circuit, can be written as

$$v_O(t) = V_M e^{-t'/\tau} = V_M e^{-t'/RC} \tag{2.3}$$

where t' is the time after the output has reached its peak value, and RC is the time constant of the circuit.

The smallest output voltage is

$$V_L = V_M e^{-T'/RC} \tag{2.4}$$

where T' is the discharge time, as indicated in the figure.

The **ripple voltage** V_r is defined as the difference between V_M and V_L, and is determined by

$$V_r = V_M - V_L = V_M\left(1 - e^{-T'/RC}\right) \tag{2.5}$$

Normally, we will want the discharge time T' to be small compared to the time constant, or $T' \ll RC$. Expanding the exponential in a series and keeping only the linear terms of that expansion, we have the approximation

$$e^{-T'/RC} \cong 1 - \frac{T'}{RC} \tag{2.6}$$

The ripple voltage can now be written as

$$V_r \cong V_M\left(\frac{T'}{RC}\right) \tag{2.7}$$

Since the discharge time T' depends on the RC time constant, Equation (2.7) is difficult to solve. However, if the ripple effect is small, then as an approximation, we can let $T' = T_p$, so that

$$V_r \cong V_M\left(\frac{T_p}{RC}\right) \tag{2.8}$$

where T_p is the time between peak values of the output voltage. For a full-wave rectifier, T_p is one-half the signal period. Therefore, we can relate T_p to the signal frequency,

$$f = \frac{1}{2T_p}$$

The ripple voltage is then

$$V_r = \frac{V_M}{2fRC} \tag{2.9}$$

For a half-wave rectifier, the time T_p corresponds to a full period (not a half period) of the signal, so the factor 2 does not appear in Equation (2.9).

Equation (2.9) can be used to determine the capacitor value required for a particular ripple voltage.

Example 2.2 **Objective:** Determine the capacitance required to yield a particular ripple voltage.

Consider a full-wave rectifier circuit with a 60 Hz input signal and a peak output voltage of $V_M = 10\text{ V}$. Assume the output load resistance is $R = 10\text{ k}\Omega$ and the ripple voltage is to be limited to $V_r = 0.2\text{ V}$.

Solution: From Equation (2.9), we can write

$$C = \frac{V_M}{2fRV_r} = \frac{10}{2(60)(10 \times 10^3)(0.2)} \Rightarrow 41.7\,\mu\text{F}$$

Comment: If the ripple voltage is to be limited to a smaller value, a larger filter capacitor must be used.

The diode in a filtered rectifier circuit conducts for a brief interval Δt near the peak of the sinusoidal input signal (Figure 2.10(a)). The diode current supplies the charge lost by the capacitor during the discharge time. Figure 2.11 shows the equivalent circuit of the full-wave rectifier during the charging time. We see that

$$i_D = i_C + i_R = C\frac{dv_O}{dt} + \frac{v_O}{R} \tag{2.10}$$

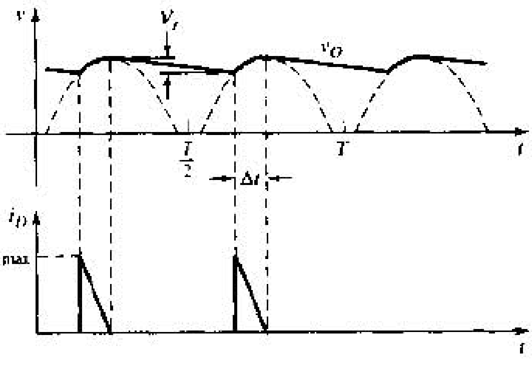


Figure 2.10 Output of a full-wave rectifier with an *RC* filter: (a) diode conduction time and (b) diode current

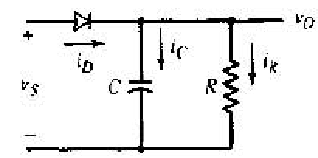


Figure 2.11 Equivalent circuit of a full-wave rectifier during capacitor charging cycle

If the ripple voltage is small, then the resistor or load current is $i_R \cong V_M/R$. If we neglect the diode cut-in voltage, then

$$V_M \cos(\omega\Delta t) = V_M - V_r \tag{2.11}$$

If the ripple voltage is small, then $\omega\Delta t$ is small. Therefore, $\cos(\omega\Delta t) \cong 1 - \frac{1}{2}(\omega\Delta t)^2$. Using Equation (2.11), we find that

$$\omega\Delta t = \sqrt{\frac{2V_r}{V_M}} \tag{2.12}$$

The charge supplied to the capacitor through the diode is

$$Q_{\text{sup}} = i_{C,\text{avg}}\Delta t \tag{2.13(a)}$$

The charge lost by the capacitor during the discharge time is

$$Q_{\text{lost}} = CV_r \tag{2.13(b)}$$

To find the average diode current $i_{D,\text{avg}}$ during conduction, we equate these two equations, which yields

$$i_{C,\text{avg}} = \frac{CV}{\Delta t} \tag{2.14}$$

where $i_{C,\text{avg}}$ is the average current through the capacitor during the charging cycle.

The average currents in the diode and capacitor during the charging cycle are related by

$$i_{D,\text{avg}} = i_{C,\text{avg}} + \frac{V_M}{R} \tag{2.15}$$

The average diode current during the diode conduction time in a full-wave rectifier circuit is then

$$i_{D,\text{avg}} = \frac{V_M}{R}\left(1 + \pi\sqrt{\frac{V_M}{2V_r}}\right) \tag{2.16}$$

where we have used the frequency from Equation (2.9). The average capacitor current $i_{C,\text{avg}}$ is zero if the ripple voltage is zero. From Equation (2.15), the average diode current in this ideal case is then V_M/R, and does not become infinite as Equation (2.16) might suggest.

The peak diode current is found to be

$$i_{D,\text{max}} = \frac{V_M}{R}\left(1 + 2\pi\sqrt{\frac{V_M}{2V_r}}\right) \tag{2.17}$$

During the diode conduction time, for small ripple voltages, the current through the capacitor is much larger than the load current. Comparing Equations (2.16) and (2.17), we see that

$$i_{D,\text{max}} \cong 2i_{D,\text{avg}}$$

The resulting diode current approximates a triangular wave, as shown in Figure 2.10(b). The average diode current over the entire input signal period is then

$$i_D(\text{avg}) = \frac{1}{2\pi}\sqrt{\frac{2V_r}{V_M}}\left[\left(\frac{V_M}{R}\right)\left(1 + \pi\sqrt{\frac{V_M}{2V_r}}\right)\right] \tag{2.18}$$

Design Example 2.3 **Objective:** Design a full-wave rectifier to meet particular specifications.

A full-wave rectifier is to be designed to produce a peak output voltage of 12 V, deliver 120 mA to the load, and produce an output with a ripple of not more than 5 percent. An input line voltage of 120 V (rms), 60 Hz is available.

Solution: A full-wave bridge rectifier will be used, because of the advantages previously discussed. The effective load resistance is

$$R = \frac{V_O}{I_L} = \frac{12}{0.12} = 100\,\Omega$$

Assuming a diode cut-in voltage of 0.7 V, the peak value of v_S is

$$v_S(\max) = v_O(\max) + 2V_\gamma = 12 + 2(0.7) = 13.4\text{ V}$$

For a sinusoidal signal, this produces an rms voltage value of

$$v_{S,\text{rms}} = \frac{13.4}{\sqrt{2}} = 9.48\text{ V}$$

The transformer turns ratio is then

$$\frac{N_1}{N_2} = \frac{120}{9.48} = 12.7$$

For a 5 percent ripple, the ripple voltage is

$$V_r = (0.05)V_M = (0.05)(12) = 0.6\text{ V}$$

The required filter capacitor is found to be

$$C = \frac{V_M}{2fRV_r} = \frac{12}{2(60)(100)(0.6)} \Rightarrow 1667\,\mu\text{F}$$

The peak diode current is

$$i_{D,\max} = \frac{12}{100}\left[1 + 2\pi\sqrt{\frac{12}{2(0.6)}}\right] = 2.50\text{ A}$$

and the average diode current over the entire signal period is

$$i_D(\text{avg}) = \frac{1}{2\pi}\sqrt{\frac{2(0.6)}{12}}\left[\left(\frac{12}{100}\right)\left(1 + \pi\sqrt{\frac{12}{2(0.6)}}\right)\right] \Rightarrow 66\text{ mA}$$

Finally, the peak inverse voltage that each diode must sustain is

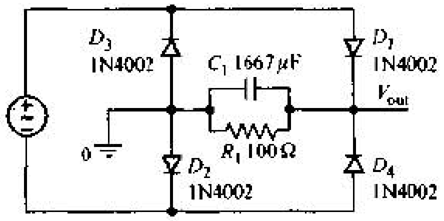
$$\text{PIV} = v_R(\max) = v_S(\max) - V_\gamma = 13.4 - 0.7 = 12.7\text{ V}$$

Comment: The minimum specifications for the diodes in this full-wave rectifier circuit are: a peak current of 2.50 A, an average current of 66 mA, and a peak inverse voltage of 12.7 V. In order to meet the desired ripple specification, the required filter capacitance must be large, since the effective load resistance is small.

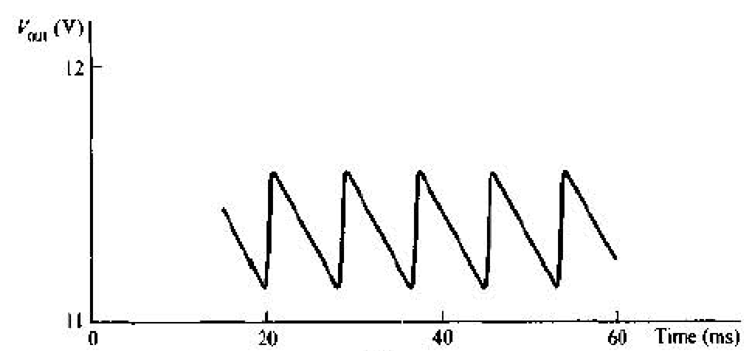
Design Pointer: (1) A particular turns ratio was determined for the transformer. However, this particular transformer design is probably not commercially available. This means an expensive custom transformer design would be required, or if a standard transformer is used, then additional circuit design is required to meet the output voltage specification. (2) A constant 120 V (rms) input voltage is assumed to be available. However, this voltage can fluctuate, so the output voltage will also fluctuate.

We will see later how more sophisticated designs will solve these two problems.

Computer Verification: Since we simply used an assumed cut-in voltage for the diode and used approximations in the development of the ripple voltage equations, we can use PSpice to give us a more accurate evaluation of the circuit. The PSpice circuit schematic and the steady-state output voltage are shown in Figure 2.12. We see that the peak output voltage is 11.6 V, which is close to the desired 12 V. One reason for the slight discrepancy is that the diode voltage drop for the maximum input voltage is slightly greater than 0.8 V rather than the assumed 0.7 V. The ripple voltage is approximately 0.5 V, which is within the 0.6 V specification.



(a)



(b)

Figure 2.12 (a) PSpice circuit schematic of diode bridge circuit; (b) Steady-state output voltage of PSpice analysis of diode bridge circuit for a 60 Hz input sine wave with a peak value of 13.4 V

Discussion: In the PSpice simulation, a standard diode, 1N4002, was used. In order for the computer simulation to be valid, the diode used in the simulation and in the actual circuit must match. In this example, to reduce the diode voltage and increase the peak output voltage, a diode with a larger cross-sectional area should be used.

Test Your Understanding

2.2 The input voltage to the half-wave rectifier in Figure 2.8(a) is $v_S = 75 \sin [2\pi(60)t]$ V. Assume a diode cut-in voltage of $V_\gamma = 0$. The ripple voltage is to be no more than $V_r = 4$ V. If the filter capacitor is 50 μF, determine the minimum load resistance that can be connected to the output. (Ans. $R = 6.25$ kΩ)

2.3 The circuit in Figure 2.6(a) is used to rectify a sinusoidal input signal with a peak voltage of 120 V and a frequency of 60 Hz. A filter capacitor is connected in parallel with

R. If the output voltage cannot drop below 100 V, determine the required value of the capacitance C. The transformer has a turns ratio of $N_1 : N_2 = 1 : 1$, where N_2 is the number of turns on each of the secondary windings. Assume the diode cut-in voltage is 0.7 V and the output resistance is 2.5 kΩ. (Ans. $C = 20.6\,\mu F$)

2.4 The secondary transformer voltage of the rectifier circuit shown in Figure 2.7(a) is $v_S = 50\sin[2\pi(60)t]$ V. Each diode has a cut-in voltage of $V_\gamma = 0.7$ V, and the load resistance is $R = 10$ kΩ. Determine the value of the filter capacitor that must be connected in parallel with R such that the ripple voltage is no greater than $V_r = 2$ V. (Ans. $C = 20.3\,\mu F$)

2.5 Determine the fraction (percent) of the cycle that each diode is conducting in (a) Exercise 2.2, (b) Exercise 2.3, and (c) Exercise 2.4. (Ans. (a) 5.2%, (b) 18.1%, (c) 9.14%)

2.1.4 Voltage Doubler Circuit

A **voltage doubler circuit** is very similar to the full-wave rectifier, except that two diodes are replaced by capacitors, and it can produce a voltage equal to approximately twice the peak output of a transformer (Figure 2.13).

Figure 2.14(a) shows the equivalent circuit when the voltage polarity at the "top" of the transformer is negative; Figure 2.14(b) shows the equivalent circuit for the opposite polarity. In the circuit in Figure 2.14(a), the forward diode resistance of D_2 is small; therefore, the capacitor C_1 will charge to almost the peak value of v_S. Terminal 2 on C_1 is positive with respect to terminal 1. As the magnitude of v_S decreases from its peak value, C_1 discharges through R_L

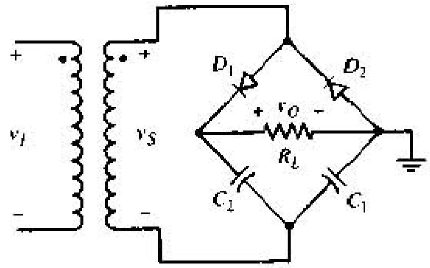


Figure 2.13 A voltage doubler circuit

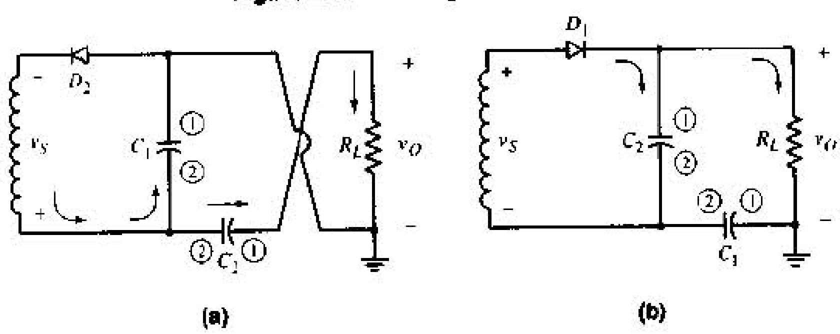


Figure 2.14 Equivalent circuit of the voltage doubler circuit: (a) negative input cycle and (b) positive input cycle

and C_2. We assume that the time constant $R_L C_2$ is very long compared to the period of the input signal.

As the polarity of v_S changes to that shown in Figure 2.14(b), the voltage across C_1 is essentially constant at V_M, with terminal 2 remaining positive. As v_S reaches its maximum value, the voltage across C_2 essentially becomes V_M. By Kirchhoff's voltage law, the peak voltage across R_L is now essentially equal to $2V_M$, or twice the peak output of the transformer. The same ripple effect occurs as in the output voltage of the rectifier circuits, but if C_1 and C_2 are relatively large, then the ripple voltage V_r, is quite small.

There are also voltage tripler and voltage quadrupler circuits. These circuits provide a means by which multiple dc voltages can be generated from a single ac source and power transformer.

2.2 ZENER DIODE CIRCUITS

In Chapter 1, we saw that the breakdown voltage of a Zener diode was nearly constant over a wide range of reverse-bias currents. This makes the Zener diode useful in a **voltage regulator**, or a constant-voltage reference circuit. In this chapter, we will look at an ideal voltage reference circuit, and the effects of including a nonideal **Zener resistance**.

The results of this section will then complete the design of the electronic power supply in Figure 2.1. We should note that in actual power supply designs, the voltage regulator will be a more sophisticated integrated circuit rather than the simpler Zener diode design that will be developed here. One reason is that a standard Zener diode with a particular desired breakdown voltage may not be available. However, this section will provide the basic concept of a voltage regulator.

2.2.1 Ideal Voltage Reference Circuit

Figure 2.15 shows a Zener voltage regulator circuit. For this circuit, the output voltage should remain constant, even when the output load resistance varies over a fairly wide range, and when the input voltage varies over a specific range.

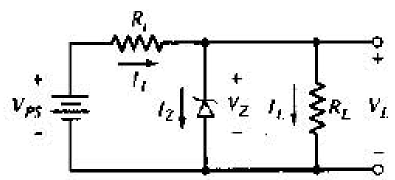


Figure 2.15 A Zener diode voltage regulator circuit

We determine, initially, the proper input resistance R_i. The resistance R_i limits the current through the Zener diode and drops the "excess" voltage between V_{PS} and V_Z. We can write

$$R_i = \frac{V_{PS} - V_Z}{I_I} = \frac{V_{PS} - V_Z}{I_Z + I_L} \tag{2.19}$$

which assumes that the Zener resistance is zero for the ideal diode. Solving this equation for the diode current, I_Z, we get

$$I_Z = \frac{V_{PS} - V_Z}{R_i} - I_L \tag{2.20}$$

where $I_L = V_Z/R_L$, and the variables are the input voltage source V_{PS} and the load current I_L.

For proper operation of this circuit, the diode must remain in the breakdown region and the power dissipation in the diode must not exceed its rated value. In other words:

1. The current in the diode is a minimum, $I_Z(\text{min})$, when the load current is a maximum, $I_L(\text{max})$, and the source voltage is a minimum, $V_{PS}(\text{min})$.
2. The current in the diode is a maximum, $I_Z(\text{max})$, when the load current is a minimum, $I_L(\text{min})$, and the source voltage is a maximum, $V_{PS}(\text{max})$.

Inserting these two specifications into Equation (2.19), we obtain

$$R_i = \frac{V_{PS}(\text{min}) - V_Z}{I_Z(\text{min}) + I_L(\text{max})} \tag{2.21(a)}$$

and

$$R_i = \frac{V_{PS}(\text{max}) - V_Z}{I_Z(\text{max}) + I_L(\text{min})} \tag{2.21(b)}$$

Equating these two expressions, we then obtain

$$\begin{aligned} &[V_{PS}(\text{min}) - V_Z] \cdot [I_Z(\text{max}) + I_L(\text{min})] \\ &= [V_{PS}(\text{max}) - V_Z] \cdot [I_Z(\text{min}) + I_L(\text{max})] \end{aligned} \tag{2.22}$$

Reasonably, we can assume that we know the range of input voltage, the range of output load current, and the Zener voltage. Equation (2.22) then contains two unknowns, $I_Z(\text{min})$ and $I_Z(\text{max})$. Further, as a minimum requirement, we can set the minimum Zener current to be one-tenth the maximum Zener current, or $I_Z(\text{min}) = 0.1 I_Z(\text{max})$. (More stringent design requirements may require the minimum Zener current to be 20 to 30 percent of the maximum value.) We can then solve for $I_Z(\text{max})$, using Equation (2.22), as follows:

$$I_Z(\text{max}) = \frac{I_L(\text{max}) \cdot [V_{PS}(\text{max}) - V_Z] - I_L(\text{min}) \cdot [V_{PS}(\text{min}) - V_Z]}{V_{PS}(\text{min}) - 0.9 V_Z - 0.1 V_{PS}(\text{max})} \tag{2.23}$$

Using the maximum current thus obtained from Equation (2.23), we can determine the maximum required power rating of the Zener diode. Then, combining Equation (2.23) with either Equation (2.21(a)) or (2.21(b)), we can determine the required value of the input resistance R_i.

Design Example 2.4 **Objective:** Design a voltage regulator using the circuit in Figure 2.15.

The voltage regulator is to power a car radio at $V_L = 9$ V from an automobile battery whose voltage may vary between 11 and 13.6 V. The current in the radio will vary between 0 (off) to 100 mA (full volume).

The equivalent circuit is shown in Figure 2.16.

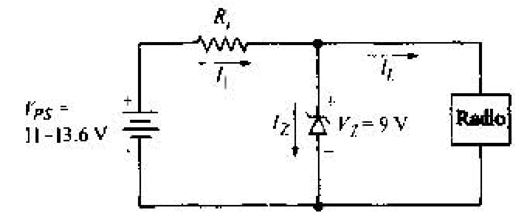


Figure 2.16 Circuit for Design Example 2.4

Solution: The maximum Zener diode current can be determined from Equation (2.23) as

$$I_Z(\text{max}) = \frac{(100)[13.6 - 9] - 0}{11 - (0.9)(9) - (0.1)(13.6)} \cong 300 \text{ mA}$$

The maximum power dissipated in the Zener diode is then

$$P_Z(\text{max}) = I_Z(\text{max}) \cdot V_Z = (300)(9) \Rightarrow 2.7 \text{ W}$$

The value of the current-limiting resistor R_i, from Equation (2.21(b)), is

$$R_i = \frac{13.6 - 9}{0.3} = 15.3 \, \Omega$$

The maximum power dissipated in this resistor is

$$P_{Ri}(\text{max}) = \frac{(V_{PS}(\text{max}) - V_Z)^2}{R_i} = \frac{(13.6 - 9)^2}{15.3} \cong 1.4 \text{ W}$$

Comment: From this design, we see that the minimum power ratings of the Zener diode and input resistor are 2.7 W and 1.4 W, respectively. The minimum Zener diode current occurs for $V_{PS}(\text{min})$ and $I_L(\text{max})$. We find $I_Z(\text{min}) = 30.7$ mA, which is approximately 10 percent of $I_Z(\text{max})$ as specified by the design equations.

Design Pointer: (1) The variable input in this example was due to a variable battery voltage. However, referring back to Example 2.3, the variable input could also be a function of using a standard transformer with a given turns ratio as opposed to a custom design with a particular turns ratio and/or having a 120 V (rms) input voltage that is not exactly constant.

(2) The 9 V output is a result of using a 9 V Zener diode. However, a Zener diode with exactly a 9 V breakdown voltage may also not be available. We will again see later how more sophisticated designs can solve this problem.

Test Your Understanding

2.6 The Zener diode regulator circuit shown in Figure 2.15 has an input voltage that varies between 10 and 14 V, and a load resistance that varies between $R_L = 20$ and $100\,\Omega$. Assume a 5.6 Zener diode is used, and assume $I_Z(\text{min}) = 0.1 I_Z(\text{max})$. Find the value of R_i required and the minimum power rating of the diode. (Ans. $P_Z = 3.31$ W, $R_i \cong 13\,\Omega$)

2.7 Suppose the current-limiting resistor in Example 2.4 is replaced by one whose value is $R_i = 20\,\Omega$. Determine the minimum and maximum Zener diode current. Does the circuit operate "properly"?

2.8 Suppose the power supply voltage in the circuit shown in Figure 2.16 drops to $V_{PS} = 10$ V. Let $R_i = 15.3\,\Omega$. What is the maximum load current in the radio if the minimum Zener diode current is to be maintained at $I_Z(\text{min}) = 30$ mA?

2.2.2 Zener Resistance and Percent Regulation

In the ideal Zener diode, the Zener resistance is zero. In actual Zener diodes, however, this is not the case. The result is that the output voltage is a function of the Zener diode current or the load current.

Figure 2.17 shows the equivalent circuit of the voltage regulator in Figure 2.15. Because of the Zener resistance, the output voltage will not remain constant. We can determine the minimum and maximum values of output voltage. A figure of merit for a voltage regulator is called the **percent regulation**, and is defined as

$$\%\ \text{Regulation} = \frac{V_L(\text{max}) - V_L(\text{min})}{V_L(\text{nom})} \times 100 \tag{2.24}$$

where $V_L(\text{nom})$ is the nominal value of the output voltage. As the percent regulation approaches zero percent, the circuit approaches that of an ideal voltage regulator.

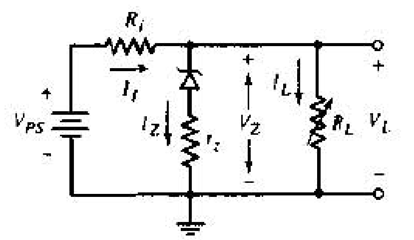


Figure 2.17 A Zener diode voltage regulator circuit with a nonzero Zener resistance

Example 2.5 **Objective:** Determine the percent regulation of a voltage regulator.

Consider the circuit described in Example 2.4 and assume a Zener resistance of $r_z = 4\,\Omega$. The nominal output voltage is $V_L(\text{nom}) = 9$ V.

Solution: As a first approximation, we can assume that the output voltage does not change significantly. Therefore, the minimum and maximum Zener diode currents will be the same, as in Example 2.4. Then,

$$V_L(\text{max}) = V_L(\text{nom}) + I_Z(\text{max})r_z = 9 + (0.30)(4) = 10.20\text{ V}$$

and

$$V_L(\text{min}) = V_L(\text{nom}) + I_Z(\text{min})r_z = 9 + (0.030)(4) = 9.12\text{ V}$$

The percent regulation is then

$$\%\text{ Regulation} = \frac{V_L(\text{max}) - V_L(\text{min})}{V_L(\text{nom})} \times 100 = \frac{10.2 - 9.12}{9} \times 100 = 12\%$$

Comment: Because of the relatively high current levels in this example, the percent regulation is fairly high. The percent regulation can be improved significantly by using amplifiers in conjunction with the voltage reference circuit. We will see examples of these circuits in later chapters.

Test Your Understanding

2.9 If the diode in Exercise 2.6 has a Zener resistance of $r_z = 1.5\,\Omega$, determine the percent regulation. (Ans. 14.3%)

2.3 CLIPPER AND CLAMPER CIRCUITS

In this section, we continue our discussion of nonlinear circuit applications of diodes. Diodes can be used in waveshaping circuits that either limit or "clip" portions of a signal, or shift the dc voltage level. The circuits are called **clippers** and **clampers**, respectively.

2.3.1 Clippers

Clipper circuits, also called **limiter circuits**, are used to eliminate portions of a signal that are above or below a specified level. For example, the half-wave rectifier is a clipper circuit, since all voltages below zero are eliminated. A simple application of a clipper is to limit the voltage at the input to an electronic circuit so as to prevent breakdown of the transistors in the circuit. The circuit may be used to measure the frequency of the signal, so the amplitude is not an important part of the signal.

Figure 2.18 shows the general voltage transfer characteristics of a limiter circuit. The limiter is a linear circuit if the input signal is in the range $V_O^-/A_v \le v_I \le V_O^+/A_v$, where A_v is the slope of the transfer curve. If $A_v \le 1$, as in diode circuits, the circuit is a **passive limiter**. If $v_I > V_O^+/A_v$, the output is limited to a maximum value of V_O^+. Similarly, if $v_I < V_O^-/A_v$, the output is limited to a minimum value of V_O^-. Figure 2.18 shows the general transfer curve of a double limiter, in which both the positive and negative peak values of the input signal are clipped.

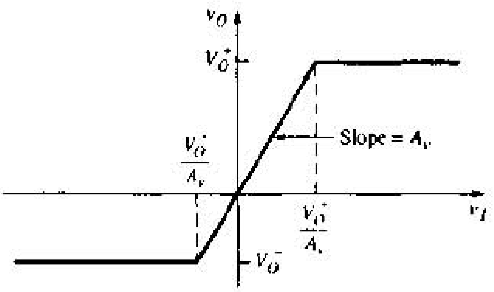


Figure 2.18 General voltage transfer characteristics of a limiter circuit

Various combinations of V_O^+ and V_O^- are possible. Both parameters may be positive, both negative, or one may be positive while the other negative, as indicated in the figure. If either V_O^- approaches minus infinity or V_O^+ approaches plus infinity, then the circuit reverts to a single limiter.

Figure 2.19(a) is a single-diode clipper circuit. The diode D_1 is off as long as $v_I < V_B + V_\gamma$. With D_1 off, the current is zero, the voltage drop across R is zero, and the output voltage follows the input voltage. When $v_I > V_B + V_\gamma$, the diode turns on, the output voltage is clipped, and v_O equals $V_B + V_\gamma$. The output signal is shown in Figure 2.19(b). In this circuit, the output is clipped above $V_B + V_\gamma$.

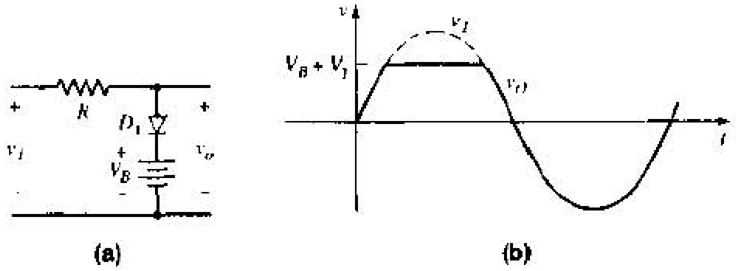


Figure 2.19 Single-diode clipper: (a) circuit and (b) output response

Other clipping circuits can be constructed by reversing the diode, the polarity of the voltage source, or both. Figures 2.20(a), (b), and (c) show these circuits, along with the corresponding input and output signals.

Positive and negative clipping can be performed simultaneously by using a double limiter or a **parallel-based clipper**, such as the circuit shown in Figure 2.21. The input and output signals are also shown in the figure. The parallel-based clipper is designed with two diodes and two voltage sources oriented in opposite directions.

Example 2.6 **Objective:** Find the output of the parallel-based clipper in Figure 2.22(a).

For simplicity, assume that $V_\gamma = 0$ and $r_f = 0$ for both diodes.

Solution: For $t = 0$, we see that $v_I = 0$ and both D_1 and D_2 are reverse biased. For $0 < v_I \leq 2$ V, D_1 and D_2 remain off; therefore, $v_O = v_I$. For $v_I > 2$V, D_1 turns on and

$$i_1 = \frac{v_I - 2}{10 + 10}$$

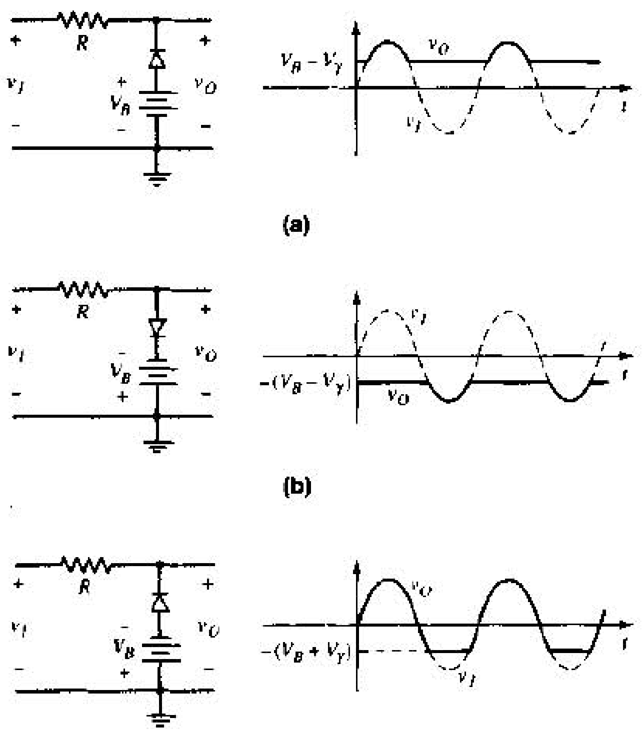


Figure 2.20 Additional diode clipper circuits and their corresponding output responses

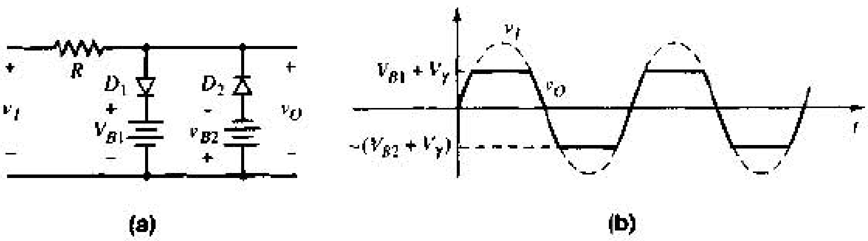


Figure 2.21 A parallel-based diode clipper circuit and its output response

Also,

$$v_O = i_1 R_2 + 2 = \tfrac{1}{2}(v_I - 2) + 2 = \tfrac{1}{2}v_I + 1$$

If $v_I = 6$ V, then $v_O = 4$ V.

For $-4 < v_I < 0$ V, both D_1 and D_2 are off and $v_O = v_I$. For $v_I \leq -4$ V, D_2 turns on and the output is constant at $v_O = -4$ V. The input and output waveforms are plotted in Figure 2.22(b).

Comment: If we assume that $V_\gamma \neq 0$, the output will be very similar to the results calculated here. The only difference will be the points at which the diodes turn on.

Diode clipper circuits can also be designed such that the dc power supply is in series with the input signals. Figure 2.23 shows various circuits based on this design. The battery in series with the input signal causes the input signal to be

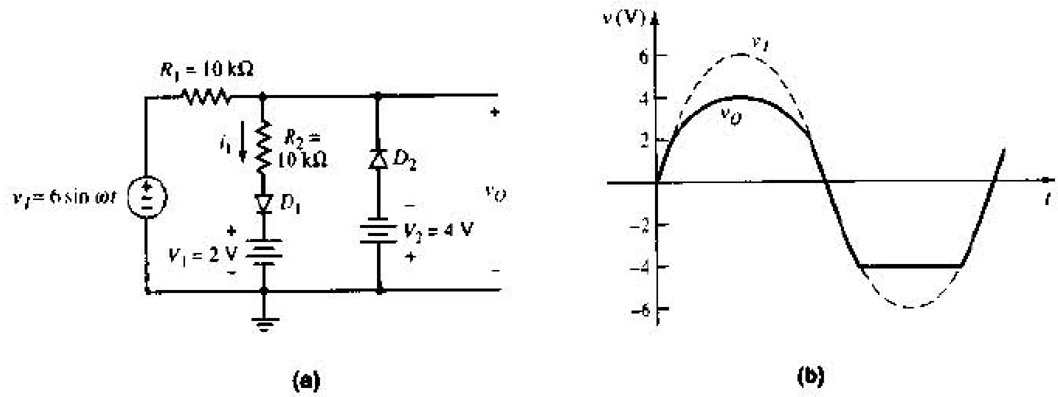


Figure 2.22 Figure for Example 2.6

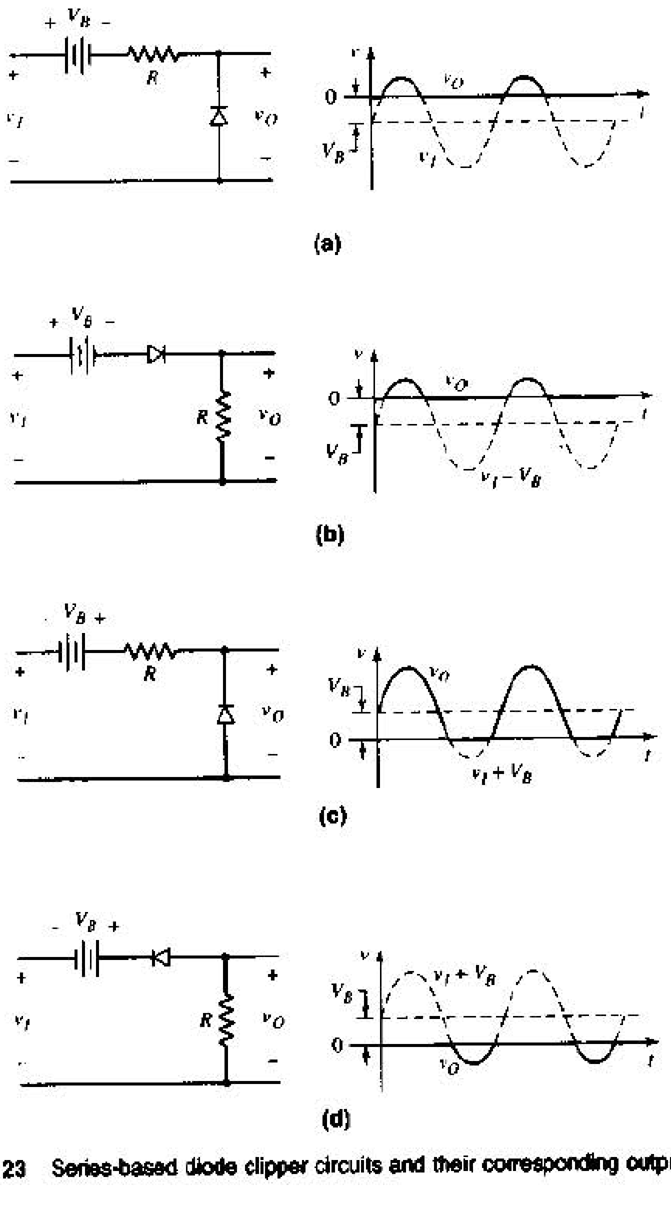


Figure 2.23 Series-based diode clipper circuits and their corresponding output responses

superimposed on the V_B dc voltage. The resulting conditioned input signals and corresponding output signals are also shown in Figure 2.23.

Test Your Understanding

2.10 Plot the voltage transfer characteristics (v_O versus v_I) for the circuit in Figure 2.22(a). Assume each diode cut-in voltage is $V_\gamma = 0.7\,\text{V}$. (Ans. For $-4.7 \le v_I \le 2.7\,\text{V}$, $v_O = v_I$; for $v_I > 2.7\,\text{V}$, $v_O = (\frac{1}{2})v_I + 1.35$; for $v_I < -4.7\,\text{V}$, $v_O = -4.7\,\text{V}$)

D2.11 Design a parallel-based clipper that will yield the voltage transfer function shown in Figure 2.24. Assume diode cut-in voltages of $V_\gamma = 0.7\,\text{V}$. (Ans. For Figure 2.22(a), $V_2 = 4.3$, $V_1 = 1.8\,\text{V}$, and $R_1 = 2R_2$)

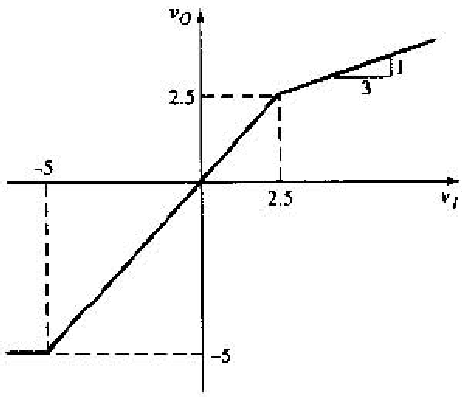


Figure 2.24 Figure for Exercise 2.11

2.3.2 Clampers

Clamping shifts the entire signal voltage by a dc level. In steady state, the output waveform is an exact replica of the input waveform, but the output signal is shifted by a dc value that depends on the circuit. The distinguishing feature of a clamper is that it adjusts the dc level without needing to know the exact waveform.

An example of clamping is shown in Figure 2.25(a). The sinusoidal input voltage signal is shown in Figure 2.25(b). Assume that the capacitor is initially uncharged. During the first 90 degrees of the input waveform, the voltage across the capacitor follows the input, and $v_C = v_I$ (assuming that $r_f = 0$ and $V_\gamma = 0$). After v_I and v_C reach their peak values, v_I begins to decrease and the diode becomes reverse biased. Ideally, the capacitor cannot discharge, so the voltage across the capacitor remains constant at $v_C = V_M$. By Kirchhoff's voltage law

$$v_O = -v_C + v_I = -V_M + V_M \sin \omega t \qquad \textbf{(2.25(a))}$$

or

$$v_O = V_M(\sin \omega t - 1) \qquad \textbf{(2.25(b))}$$

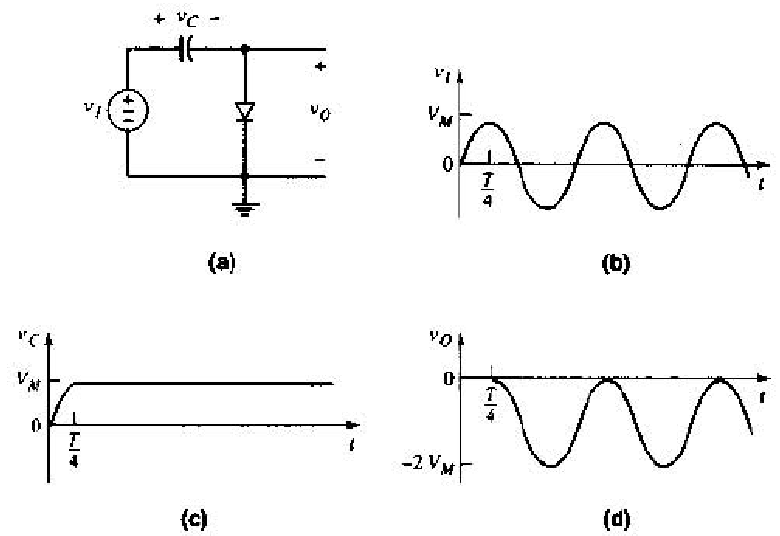


Figure 2.25 Action of a diode clamper circuit: (a) a typical diode clamper circuit, (b) the sinusoidal input signal, (c) the capacitor voltage, and (d) the output voltage

The capacitor and output voltages are shown in Figures 2.25(c) and (d). The output voltage is "clamped" at zero volts, that is, $v_O \le 0$. In steady state, the waveshapes of the input and output signals are the same, and the output signal is shifted by a certain dc level compared to the input signal.

A clamping circuit that includes an independent voltage source V_B is shown in Figure 2.26(a). In this circuit, the R_LC time constant is assumed to be large, where R_L is the load resistance connected to the output. If we assume, for simplicity, that $r_f = 0$ and $V_\gamma = 0$, then the output is clamped at V_B. Figure 2.26(b) shows an example of a sinusoidal input signal and the resulting output voltage signal. When the polarity of V_B is as shown, the output is shifted in a negative voltage direction. Similarly, Figure 2.26(c) shows a square-wave input signal and the resulting output voltage signal. For the square-wave signal, we neglect the diode capacitance effects and assume the voltage can change instantaneously.

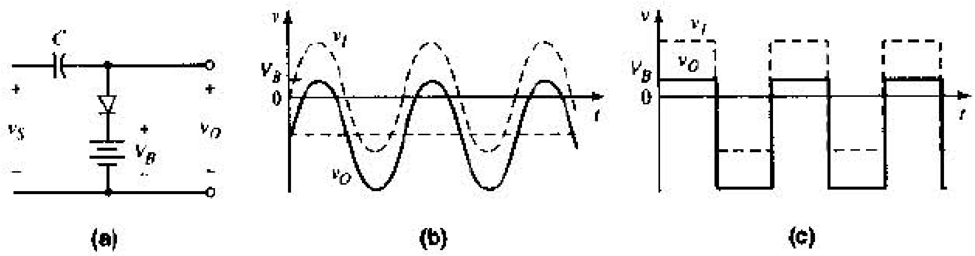


Figure 2.26 Action of a diode clamper circuit with a voltage source: (a) the circuit, (b) steady-state sinusoidal input and output signals, and (c) steady-state square-wave input and output signals

Electronic signals tend to lose their dc levels during signal transmission. For example, the dc level of a TV signal may be lost during transmission, so that the dc level must be restored at the TV receiver. The following example illustrates this effect.

Example 2.7 **Objective:** Find the steady-state output of the diode-clamper circuit shown in Figure 2.27(a).

The input v_I is assumed to be a sinusoidal signal whose dc level has been shifted with respect to a receiver ground by a value V_B during transmission. Assume $V_\gamma = 0$ and $r_f = 0$ for the diode.

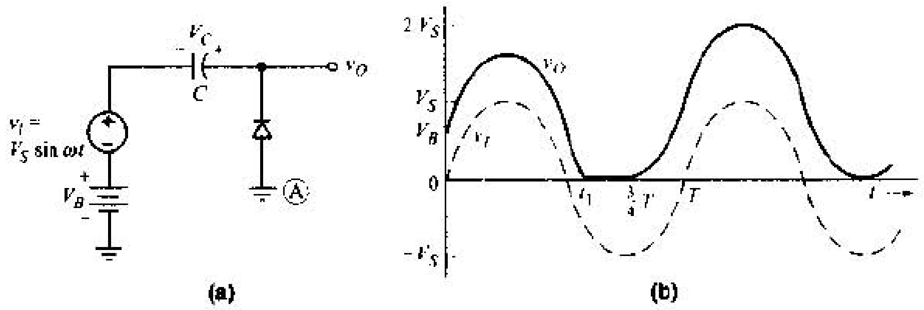


Figure 2.27 (a) Circuit for Example 2.7; (b) input and output waveforms

Solution: Figure 2.27(b) shows the sinusoidal input signal. If the capacitor is initially uncharged, then the output voltage is $v_O = V_B$ at $t = 0$ (diode reverse-biased). For $0 \le t \le t_1$, the effective RC time constant is infinite, the voltage across the capacitor does not change, and $v_O = v_I + V_B$.

At $t = t_1$, the diode becomes forward biased; the output cannot go negative, so the voltage across the capacitor changes (the $r_f C$ time constant is zero).

At $t = (\frac{3}{4})T$, the input signal begins increasing and the diode becomes reverse biased, so the voltage across the capacitor now remains constant at $V_S - V_B$ with the polarity shown. The output voltage is now given by

$$v_O = (V_S - V_B) + v_I + V_B = (V_S - V_B) + V_S \sin \omega t + V_B$$

or

$$v_O = V_S(1 + \sin \omega(t - (\tfrac{3}{4})T))$$

Comment: For $t > (\frac{3}{4})T$, steady state is reached. The output signal waveform is an exact replica of the input signal waveform and is now measured with respect to the reference ground at terminal **A**.

Test Your Understanding

2.12 Sketch the steady-state output voltage for the input signal given for the circuit in Figure 2.28. (Ans. Square wave between −2 and −10 V.)

2.13 Determine the steady-state output voltage v_O for the circuit in Figure 2.29(a), if the input is as shown in Figure 2.29(b). Assume the diode cut-in voltage is $V_\gamma = 0$. (Ans. Output is a square wave between +5 V and +35 V)

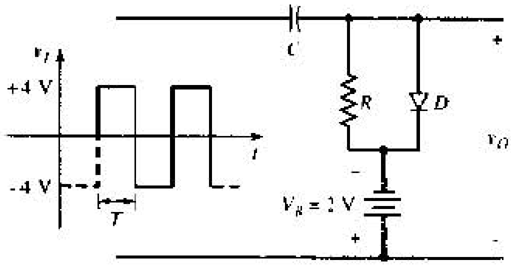


Figure 2.28 Figure for Exercise 2.12

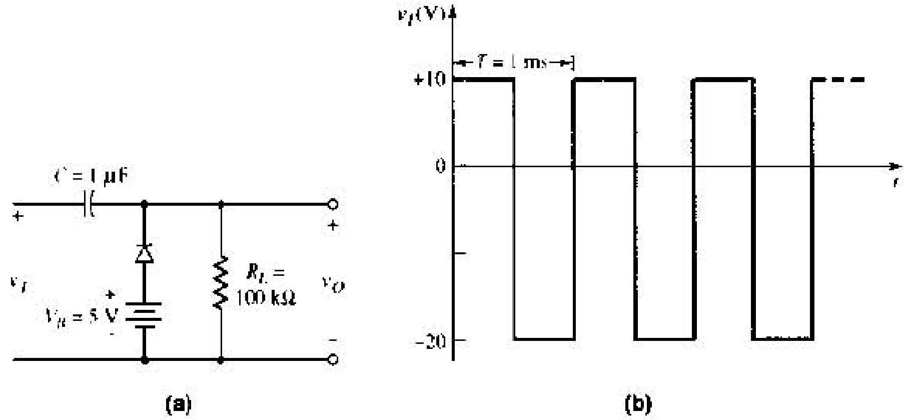


Figure 2.29 Figure for Exercise 2.13

2.4 MULTIPLE-DIODE CIRCUITS

Since a diode is a nonlinear device, part of the analysis of a diode circuit involves determining whether the diode is "on" or "off." If a circuit contains more than one diode, the analysis is complicated by the various possible combinations of "on" and "off."

In this section, we will look at several multiple-diode circuits. We will see, for example, how diode circuits can be used to perform logic functions. This section serves as an introduction to digital logic circuits that will be considered in detail in Chapters 16 and 17.

2.4.1 Example Diode Circuits

To review briefly, consider two single-diode circuits. Figure 2.30(a) shows a diode in series with a resistor. A plot of voltage transfer characteristics, v_O versus v_I, shows the piecewise linear nature of this circuit (Figure 2.30(b)). The diode does not begin to conduct until $v_I = V_\gamma$. Consequently, for $v_I \leq V_\gamma$, the output voltage is zero; for $v_I > V$, the output voltage is $v_O = v_I - V_\gamma$.

Figure 2.31(a) shows a similar diode circuit, but with the input voltage source explicitly included to show that there is a path for the diode current. The voltage transfer characteristic is shown in Figure 2.31(b). In this circuit, the diode remains conducting for $v_I < V_S - V_\gamma$, and the output voltage is $v_O =$

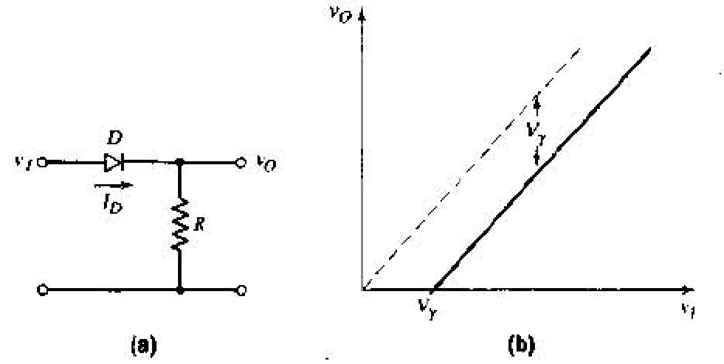


Figure 2.30 Diode and resistor in series: (a) circuit and (b) voltage transfer characteristics

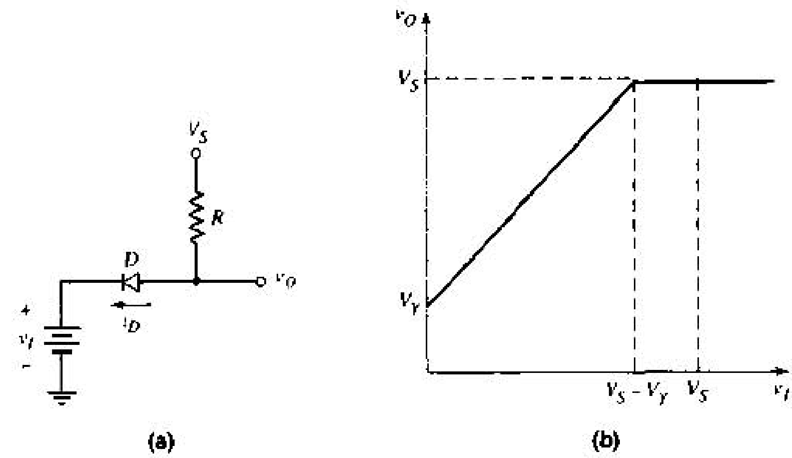


Figure 2.31 Diode with input voltage source: (a) circuit and (b) voltage transfer characteristics

$v_I + V_\gamma$. When $v_I > V_S - V_\gamma$, the diode turns off and the current through the resistor is zero; therefore, the output remains constant at V_S.

These two examples demonstrate the piecewise linear nature of the diode and the diode circuit. They also demonstrate that there are regions where the diode is "on," or conducting, and regions where the diode is "off," or nonconducting.

In multidiode circuits, each diode may be either "on" or "off." Consider the two-diode circuit in Figure 2.32. Since each diode may be either on or off, the circuit has four possible states. However, some of these states may not be feasible because of diode directions and voltage polarities.

If we assume that $V^+ > V^-$ and that $V^+ - V^- > V_\gamma$, there is at least a possibility that D_2 can be turned on. First, v' cannot be less than V^-. Then, for $v_I = V^-$, diode D_1 must be off. In this case, D_2 is on, $i_{R1} = i_{D2} = i_{R2}$, and

$$v_O = V^+ - i_{R1}R_1 \tag{2.26}$$

where

$$i_{R1} = \frac{V^+ - V_\gamma - V^-}{R_1 + R_2} \tag{2.27}$$

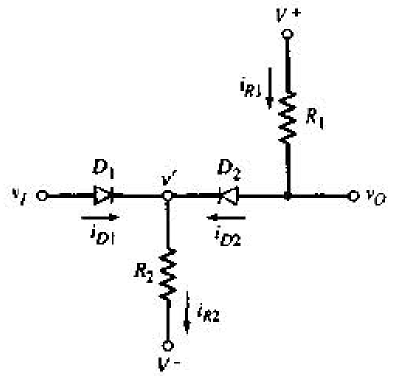


Figure 2.32 A two-diode circuit

Voltage v' is one diode drop below v_O, and D_1 remains off as long as v_I is less than the output voltage. As v_I increases and becomes equal to v_O, both D_1 and D_2 turn on. This condition or state is valid as long as $v_I < V^+$. When $v_I = V^+$, $i_{R1} = i_{D2} = 0$, at which point D_2 turns off and v_O cannot increase any further.

Figure 2.33 shows the resulting plot of v_O versus v_I. Three distinct regions, $v_O^{(1)}$, $v_O^{(2)}$, and $v_O^{(3)}$, correspond to the various conducting states of D_1 and D_2. The fourth possible state, corresponding to both D_1 and D_2 being off, is not feasible in this circuit.

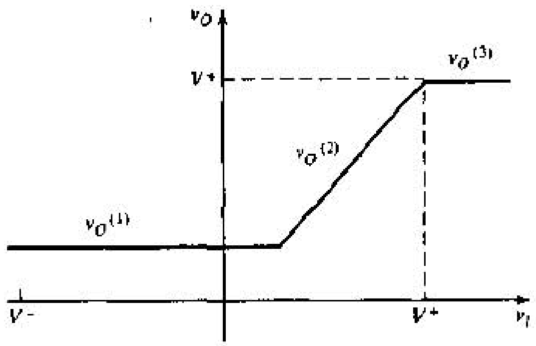


Figure 2.33 Voltage transfer characteristics for the two-diode circuit in Figure 2.32

Example 2.8 **Objective:** Determine the output voltage and diode currents for the circuit shown in Figure 2.32, for two values of input voltage.

Assume the circuit parameters are $R_1 = 5\,k\Omega$, $R_2 = 10\,k\Omega$, $V_\gamma = 0.7\,V$, $V^+ = +5\,V$, and $V^- = -5\,V$. Determine v_O, i_{D1}, and i_{D2} for $v_I = 0$ and $v_I = 4\,V$.

Solution: For $v_I = 0$, assume initially that D_1 is off. The currents are then

$$i_{R1} = i_{D2} = i_{R2} = \frac{V^+ - V_\gamma - V^-}{R_1 + R_2} = \frac{5 - 0.7 - (-5)}{5 + 10} = 0.62\,\text{mA}$$

The output voltage is

$$v_O = V^+ - i_{R1}R_1 = 5 - (0.62)(5) = 1.9\,\text{V}$$

and v' is

$$v' = v_O - V_\gamma = 1.9 - 0.7 = 1.2\text{ V}$$

From these results, we see that diode D_1 is indeed cut off, $i_{D1} = 0$, and our analysis is valid.

For $v_I = 4$ V, we see from Figure 2.33 that $v_O = v_I$; therefore, $v_O = v_I = 4$ V. In this region, both D_1 and D_2 are on, and

$$i_{R1} = i_{D2} = \frac{V^+ - v_O}{R_1} = \frac{5 - 4}{5} = 0.2\text{ mA}$$

Note that $v' = v_O - V_\gamma = 4 - 0.7 = 3.3$ V. Thus,

$$i_{R2} = \frac{v' - V^-}{R_2} = \frac{3.3 - (-5)}{10} = 0.83\text{ mA}$$

The current through D_1 is found from $i_{D1} + i_{D2} = i_{R2}$ or

$$i_{D1} = i_{R2} - i_{D2} = 0.83 - 0.2 = 0.63\text{ mA}$$

Comment: For $v_I = 0$, we see that $v_O = 1.9$ V and $v' = 1.2$ V. This means that D_1 is reverse biased, or off, as we initially assumed. For $v_I = 4$ V, we have $i_{D1} > 0$ and $i_{D2} > 0$, indicating that both D_1 and D_2 are forward biased, as we assumed.

Computer Analysis: For multidiode circuits, a PSpice analysis may be useful in determining the conditions under which the various diodes are conducting or not conducting. This avoids guessing the conducting state of each diode in a hand analysis. Figure 2.34 is the PSpice circuit schematic of the diode circuit in Figure 2.32. Figure 2.34 also shows the output voltage and the two diode currents as the input is varied between −1 V and +7 V. From these curves, we can determine when the diodes turn on and off.

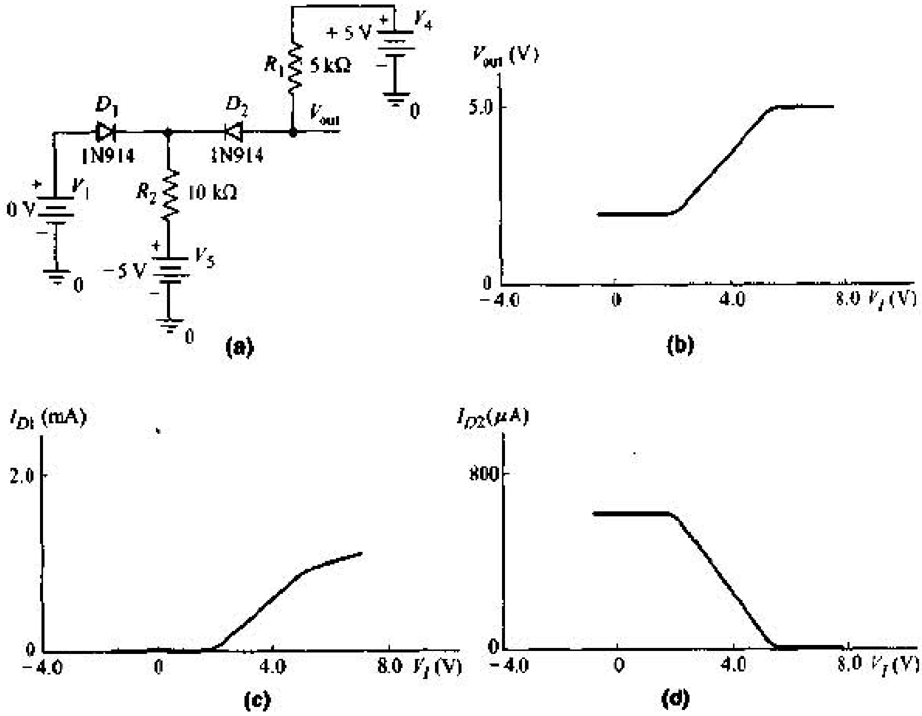


Figure 2.34 (a) PSpice circuit schematic; (b) output voltage; (c) current in diode 1, and (d) current in diode 2 for the diode circuit in Example 2.8

Comment: The hand analysis results, based on the piecewise linear model for the diode, agree very well with the computer simulation results. This gives us confidence in the piecewise linear model when quick hand calculations are made.

Problem-Solving Technique: Multiple Diode Circuits

Analyzing multidiode circuits requires determining if the individual devices are "on" or "off." In many cases, the choice is not obvious, so we must initially guess the state of each device, then analyze the circuit to determine if we have a solution consistent with our initial guess. To do this, we can:

1. Assume the state of a diode. If a diode is assumed "on," the voltage across the diode is assumed to be V_γ. If a diode is assumed to be "off," the current through the diode is assumed to be zero.
2. Analyze the "linear" circuit with *the assumed* diode states.
3. Evaluate the resulting state of each diode. If the initial assumption were that a diode is "off" and the analysis shows that $I_D = 0$ and $V_D \le V_\gamma$, then the assumption is correct. If, however, the analysis actually shows that $I_D > 0$ and/or $V_D > V_\gamma$, then the initial assumption is incorrect. Similarly, if the initial assumption were that a diode is "on" and the analysis shows that $I_D \ge 0$ and $V_D = V_\gamma$, then the initial assumption is correct. If, however, the analysis shows that $I_D < 0$ and/or $V_D < V_\gamma$, then the initial assumption is incorrect.
4. If any initial assumption is proven incorrect, then a new assumption must be made and the new "linear" circuit must be analyzed. Step 3 must then be repeated.

Example 2.9 **Objective:** Demonstrate how inconsistencies develop in a solution with incorrect assumptions.

For the circuit shown in Figure 2.32, assume that parameters are the same as those given in Example 2.8. Determine v_O, i_{D1}, i_{D2}, and i_{R2} for $v_I = 0$.

Solution: Assume initially that both D_1 and D_2 are conducting (i.e., on). Then, $v' = -0.7$ V and $v_O = 0$. The two currents are

$$i_{R1} = i_{D2} = \frac{V^+ - v_O}{R_1} = \frac{5 - 0}{5} = 1.0\,\text{mA}$$

and

$$i_{R2} = \frac{v' - V^-}{R2} = \frac{-0.7 - (-5)}{10} = 0.43\,\text{mA}$$

Summing the currents at the v' node, we find that

$$i_{D1} = i_{R2} - i_{D2} = 0.43 - 1.0 = -0.57\,\text{mA}$$

Since this analysis shows the D_1 current to be negative, which is an impossible or inconsistent solution, our initial assumption must be incorrect. If we go back to Example 2.8, we will see that the correct solution is D_1 off and D_2 on when $v_I = 0$.

Comment We can perform linear analyses on diode circuits, using the piecewise linear model. However, we must first determine if each diode in the circuit is operating in the "on" linear region or the "off" linear region.

Test Your Understanding

2.14 Consider the circuit shown in Figure 2.35, in which the diode cut-in voltages are $V_\gamma = 0.6\,V$. Plot v_O versus v_I for $0 \le v_I \le 10\,V$. (Ans. For $0 \le v_I \le 3.5\,V$, $v_O = 4.4\,V$; for $v_I > 3.5\,V$, D_2 turns off; and for $v_I \ge 9.4\,V$, $v_O = 10\,V$)

2.15 Determine V_O, I_{D1}, I_{D2}, and I in the circuit shown in Figure 2.36. Assume $V_\gamma = 0.6\,V$ for each diode. (Ans. $V_O = -0.6\,V$, $I_{D1} = 0$, $I_{D2} = I = 4.27\,mA$)

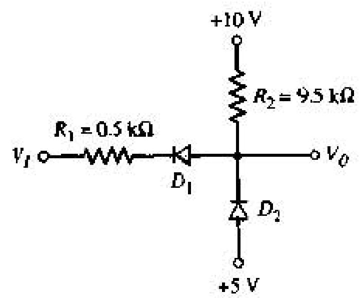


Figure 2.35 Figure for Exercise 2.14

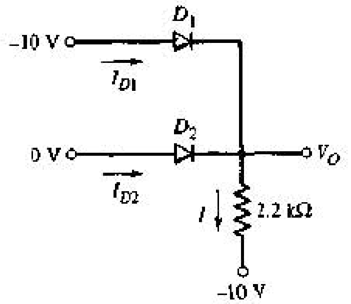


Figure 2.36 Figure for Exercise 2.15

2.4.2 Diode Logic Circuits

Diodes in conjunction with other circuit elements can perform certain **logic functions**, such as AND and OR. The circuit in Figure 2.37 is an example of a diode logic circuit. The four conditions of operation of this circuit depend on various combinations of input voltages, as follows:

$V_1 = V_2 = 0$: There is no excitation to the circuit; therefore, $V_O = 0$ and both diodes are off.

$V_1 = 5\,V$, $V_2 = 0$: Diode D_1 becomes forward biased and D_2 is reverse biased. Assuming a diode cut-in voltage of $V_\gamma = 0.7\,V$, the output voltage is $V_O = 4.3\,V$. The currents are $I_{D2} = 0$ and $I_{D1} = I = V_O/R$.

$V_1 = 0$, $V_2 = 5\,V$: Diode D_2 turns on, diode D_1 is cut off, and the output voltage is $V_O = 4.3\,V$. The currents are $I_{D1} = 0$ and $I_{D2} = I = V_O/R$.

$V_1 = V_2 = 5\,V$: Both diodes are forward biased, so the output is again $V_O = 4.3\,V$. The current in the resistor is $I = V_O/R$. Since both diodes are on, we assume that the current I splits evenly between the two diodes; therefore, $I_{D1} = I_{D2} = I/2$.

These results are shown in Table 2.1. By definition, in a positive logic system, a voltage near zero corresponds to a logic 0 and a voltage close to

Table 2.1 Two-diode OR logic circuit response

V_1(V)	V_2(V)	V_O(V)
0	0	0
5	0	4.3
0	5	4.3
5	5	4.3

the supply voltage of 5 V corresponds to a logic 1. The results shown in Table 2.1 indicate that this circuit performs the OR logic function. The circuit of Figure 2.37, then, is a two-input diode OR logic circuit.

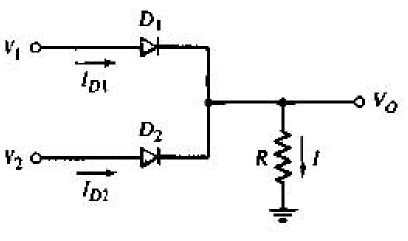


Figure 2.37 A two-input diode OR logic circuit

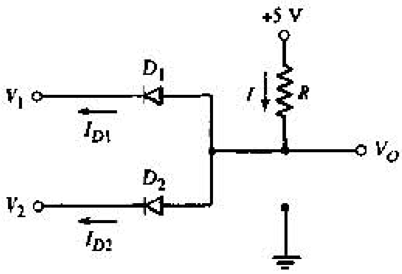


Figure 2.38 A two-input diode AND logic circuit

Next, consider the circuit in Figure 2.38. Assume a diode cut-in voltage of $V_\gamma = 0.7$ V. Again, there are four possible states, depending on the combination of input voltages, as follows:

$V_1 = V_2 = 0$: Both diodes are forward biased, and the output voltage is $V_O = 0.7$ V. The current in the resistor is $I = (5 - 0.7)/R$, which we assume splits evenly between the two diodes, so that $I_{D1} = I_{D2} = I/2$.

$V_1 = 5$ V, $V_2 = 0$: In this case, diode D_1 is off, D_2 is on, and the output voltage is $V_O = 0.7$ V. The currents are: $I_{D1} = 0$, and $I_{D2} = I = (5 - 0.7)/R$.

$V_1 = 0$, $V_2 = 5$ V: In this situation, D_1 is on, D_2 is off, and the output voltage is $V_O = 0.7$ V. The currents are: $I_{D1} = I = (5 - 0.7)/R$, and $I_{D2} = 0$.

$V_1 = V_2 = 5$ V: Since there is no potential difference between the supply voltage and the input voltages, all currents are zero and both diodes are off. Also, since there is no potential drop across the resistor R, the output voltage is $V_O = 5$ V.

These results are shown in Table 2.2. This circuit performs the AND logic function. The circuit of Figure 2.38 is a two-input diode AND logic circuit.

If we examine Tables 2.1 and 2.2, we see that the input "low" and "high" voltages may not be the same as the output "low" and "high" voltages. As an example, for the AND circuit (Table 2.2), the input "low" is 0 V, but the output "low" is 0.7 V. This can create a problem because the output of one logic gate is often the input to another logic gate. Another problem occurs when diode logic circuits are connected in cascade; that is, the output of one

Table 2.2 Two-diode AND logic circuit response

V_1(V)	V_2(V)	V_O(V)
0	0	0.7
5	0	0.7
0	5	0.7
5	5	5

OR gate is connected to the input of a second OR gate. The logic 1 levels of the two OR gates are not the same (see Problems 2.42 and 2.43). The logic 1 level degrades or decreases as additional logic gates are connected. However, these problems may be overcome with the use of amplifying devices (transistors) in digital logic systems.

Test Your Understanding

2.16 Consider the OR logic circuit shown in Figure 2.37. Assume a diode cut-in voltage of $V_\gamma = 0.6$ V. (a) Plot V_O versus V_1 for $0 \le V_1 \le 5$ V, if $V_2 = 0$. (b) Repeat part (a) if $V_2 = 3$ V. (Ans. (a) $V_O = 0$ for $V_1 \le 0.6$ V, $V_O = V_1 - 0.6$ for $0.6 \le V_1 \le 5$ V; (b) $V_O = 2.4$ V for $0 \le V_1 \le 3$ V, $V_O = V_1 - 0.6$ for $3 \le V_1 \le 5$ V)

2.17 Consider the AND logic circuit shown in Figure 2.38. Assume a diode cut-in voltage of $V_\gamma = 0.6$ V. (a) Plot V_O versus V_1 for $0 \le V_1 \le 5$ V, if $V_2 = 0$. (b) Repeat part (a) if $V_2 = 3$ V. (Ans. (a) $V_O = 0.6$ V for all V_1, (b) $V_O = V_1 + 0.6$ for $0 \le V_1 \le 3$ V, $V_O = 3.6$ V for $V_1 \ge 3$ V)

2.5 PHOTODIODE AND LED CIRCUITS

A photodiode converts an optical signal into an electrical current, and a light-emitting diode (LED) transforms an electrical current into an optical signal.

2.5.1 Photodiode Circuit

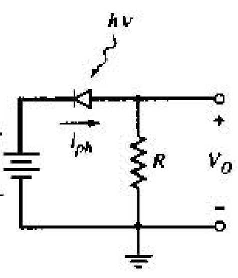


Figure 2.39 A photodiode circuit

Figure 2.39 shows a typical photodiode circuit in which a reverse-bias voltage is applied to the photodiode. If the photon intensity is zero, the only current through the diode is the reverse-saturation current, which is normally very small. Photons striking the diode create excess electrons and holes in the space-charge region. The electric field quickly separates these excess carriers and sweeps them out of the space-charge region, thus creating a **photocurrent** in the reverse-bias direction. The photocurrent is

$$I_{ph} = \eta e \Phi A \tag{2.28}$$

where η is the quantum efficiency, e is the electronic charge, Φ is the photon flux density (#/cm^2–s), and A is the junction area. This linear relationship between photocurrent and photon flux is based on the assumption that the reverse-bias voltage across the diode is constant. This in turn means that the voltage drop across R induced by the photocurrent must be small, or that the resistance R is small.

Example 2.10 **Objective:** Calculate the photocurrent generated in a photodiode.

For the photodiode shown in Figure 2.39 assume the quantum efficiency is 1, the junction area is $10^{-2}\,cm^2$, and the incident photon flux is $5 \times 10^{17}\,cm^{-2}-s^{-1}$.

Solution: From Equation (2.28), the photocurrent is

$$I_{ph} = \eta e \Phi A = (1)(1.6 \times 10^{-19})(5 \times 10^{17})(10^{-2}) \Rightarrow 0.8\,mA$$

Comment: The incident photon flux is normally given in terms of light intensity, in lumens, foot-candles, or W/cm^2. The light intensity includes the energy of the photons, as well as the photon flux.

Test Your Understanding

2.18 (a) Photons with an energy of $h\nu = 2\,eV$ are incident on the photodiode shown in Figure 2.39. The junction area is $A = 0.5\,cm^2$, the quantum efficiency is $\eta = 0.8$, and the light intensity is $6.4 \times 10^{-2}\,W/cm^2$. Determine the photocurrent I_{ph}. (b) If $R = 1\,k\Omega$, determine the minimum power supply voltage V_{PS} needed to ensure that the diode is reverse biased. (Ans. (a) $I_{ph} = 12.8\,mA$, (b) $V_{PS}(min) = 12.8\,V$)

2.5.2 LED Circuit

A light-emitting diode (LED) is the inverse of a photodiode; that is, a current is converted into an optical signal. If the diode is forward biased, electrons and holes are injected across the space-charge region, where they become excess minority carriers. These excess minority carriers diffuse into the neutral n- and p-regions, where they recombine with majority carriers, and the recombination can result in the emission of a photon.

LEDs are fabricated from compound semiconductor materials, such as gallium arsenide or gallium arsenide phosphide. Because these materials have higher bandgap energies than silicon, the forward-bias junction voltage is larger than that in silicon-based diodes.

It is common practice to use a seven-segment LED for the numeric readout of digital instruments, such as a digital voltmeter. The **seven-segment display** is sketched in Figure 2.40. Each segment is an LED normally controlled by IC logic gates.

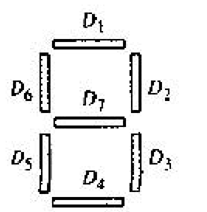


Figure 2.40 Seven-segment LED display

Figure 2.41 shows one possible circuit connection, known as a common-anode display. In this circuit, the anodes of all LEDs are connected to a 5 V source and the inputs are controlled by logic gates. If V_{I1} is "high," for example, D_1 is off and there is no light output. When V_{I1} goes "low," D_1 becomes forward biased and produces a light output.

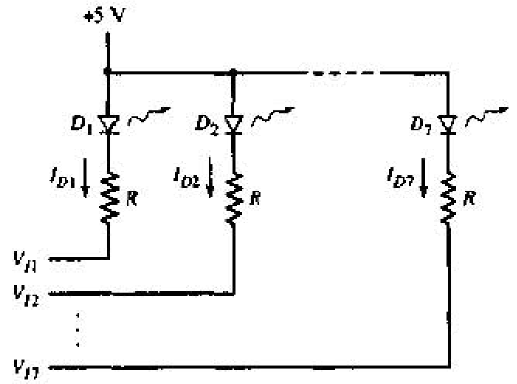


Figure 2.41 *Control circuit for the seven-segment LED display*

Example 2.11 **Objective:** Determine the value of R required to limit the current in the circuit in Figure 2.41 when the input is in the low state.

Assume that a diode current of 10 mA produces the desired light output, and that the corresponding forward-bias voltage drop is 1.7 V.

Solution: If $V_I = 0.2$ V in the "low" state, then the diode current is

$$I = \frac{5 - V_\gamma - V_I}{R}$$

The resistance R is then determined as

$$R = \frac{5 - V_\gamma - V_I}{I} = \frac{5 - 1.7 - 0.2}{10} \Rightarrow 310\ \Omega$$

Comment: Typical LED current-limiting resistor values are in the range of 300 to 350 Ω.

One application of LEDs and photodiodes is in **optoisolators**, in which the input signal is electrically decoupled from the output (Figure 2.42). An input signal applied to the LED generates light, which is subsequently detected by the photodiode. The photodiode then converts the light back to an electrical signal. There is no electrical feedback or interaction between the output and input portions of the circuit.

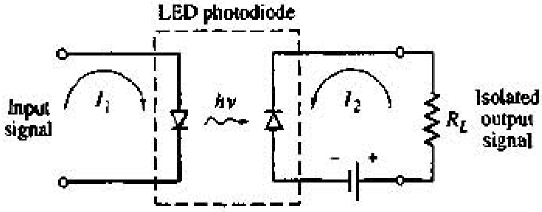


Figure 2.42 Optoisolator using an LED and a photodiode

Test Your Understanding

2.19 Determine the value of resistance R required to limit the current in the circuit shown in Figure 2.41 to $I = 15\,\text{mA}$. Assume $V_\gamma = 1.7\,\text{V}$, $r_f = 15\,\Omega$, and $V_I = 0.2\,\text{V}$ in the "low" state. (Ans. $R = 192\,\Omega$)

2.6 SUMMARY

- In this chapter, we analyzed several classes of diode circuits that can be used to produce various desired output signals. The resulting characteristics of each of the circuits considered rely on the nonlinear i–v relationship of the diode. We continued to use the piecewise linear model and approximation techniques in our hand analyses. Computer simulation can be used to obtain more accurate results when actual diode properties are known.
- Half-wave and full-wave rectifier circuits convert a sinusoidal (i.e., ac) signal to an approximate dc signal. A dc power supply, which is used to bias electronic circuits and systems, utilize these types of circuits. An RC filter can be connected to the output of the rectifier circuit to reduce the ripple effect. The ripple voltage in the output signal was determined as a function of the RC filter and other circuit parameters.
- Zener diodes operate in the reverse breakdown region. Since the breakdown voltage is nearly constant over a wide range of currents, these devices are useful in voltage reference or regulator circuits. The percent regulation, a figure of merit for the circuit, is a function of the range of input voltage and load resistance values, and of the individual device parameters.
- Techniques used to analyze multidiode circuits, which are used in various signal-processing applications, were discussed. The technique requires making assumptions as to whether a diode is conducting (on) or not conducting (off). After analyzing the circuit using these assumptions, we must go back and verify that the assumptions made were valid. This analysis technique is obviously not as straightfoward as the one for linear circuits.
- Diode circuits can be designed to perform basic digital logic functions. We considered the circuit that performs the OR logic function and the circuit that performs the AND logic function. However, we noted some inconsistencies between input and output logic values, which will limit the use of diode logic gates.
- The light-emitting diode (LED) converts an electrical current to light and is used extensively in such applications as the seven-segment alphanumeric display. Conversely, the photodiode detects an incident light signal and transforms it into an electrical current. Examples of these types of circuits were analyzed.

CHECKPOINT

After studying this chapter, the reader should have the ability to:

✔ In general, apply the diode piecewise linear model in the analysis of diode circuits.

✔ Analyze diode rectifier circuits, including the calculation of ripple voltage. (Section 2.1)

✔ Analyze Zener diode circuits, including the effect of a Zener resistance. (Section 2.2)

✔ Determine the output signal for a given input signal of diode clipper and clamper circuits. (Section 2.3)

✔ Analyze circuits with multiple diodes by making initial assumptions and then verifying these initial assumptions (Section 2.4)

REVIEW QUESTIONS

1. What characteristic of a diode is used in the design of diode signal processing circuits?
2. Describe a simple half-wave diode rectifier circuit and sketch the output voltage versus time.
3. Describe a simple full-wave diode rectifier circuit and sketch the output voltage versus time.
4. What is the advantage of connecting an RC filter to the output of a diode rectifier circuit?
5. Define ripple voltage. How can the magnitude of the ripple voltage be reduced?
6. Describe a simple Zener diode voltage reference circuit.
7. What effect does the Zener diode resistance have on the voltage reference circuit operation?
8. What are the general characteristics of diode clipper circuits?
9. Describe a simple diode clipper circuit to limit the negative portion of a sinusoidal input voltage to a specified value.
10. What are the general characteristics of diode clamper circuits?
11. What one circuit element, besides a diode, is present in all diode clamper circuits?
12. Describe the procedure used in the analysis of a circuit containing two diodes. How many initial assumptions concerning the state of the circuit are possible?
13. Describe a diode OR logic circuit. Compare a logic 1 value at the output compared to a logic 1 value at the input. Are they the same value?
14. Describe a diode AND logic circuit. Compare a logic 0 value at the output compared to a logic 0 value at the input. Are they the same value?
15. Describe a simple circuit that can be used to turn an LED on or off with a high or low input voltage.

PROBLEMS

[Note: In the following problems, assume $r_f = 0$ unless otherwise specified.]

Section 2.1 Rectifier Circuits

2.1 Assume the input to the circuit in Figure P2.1 is a triangular wave of 20 V peak-to-peak amplitude with a zero time-average value. Let $R = 1\ \text{k}\Omega$ and assume piecewise linear diode parameters of $V_\gamma = 0.6$ V and $r_f = 20\ \Omega$. Sketch the output voltage versus time over one cycle and label all appropriate voltages.

2.2 For the circuit shown in Figure P2.1, show that for $v_I \geq 0$, the output voltage is approximately given by

$$v_O = v_I - V_T \ln\left(\frac{v_O}{I_S R}\right)$$

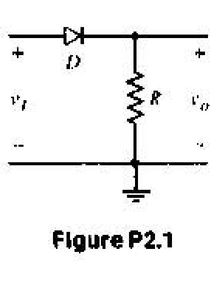


Figure P2.1

2.3 Consider the half-wave rectifier circuit in Figure 2.2(a). The input voltage is $v_I = 160\sin[2\pi(60)t]$ V and the transformer turns ratio is $N_1/N_2 = 4$. If $V_\gamma = 0$ and $r_f = 0$, determine (a) the peak diode current, (b) the value of PIV, (c) the average value of the output voltage, and (d) the fraction (percent) of a cycle that $v_O > 0$.

RD2.4 The input signal voltage to the full-wave rectifier circuit in Figure 2.7(a) in the text is $v_I = 160 \sin[2\pi(60)t]$ V. Assume $V_\gamma = 0.7$ V for each diode. Determine the required turns ratio of the transformer to produce a peak output voltage of (a) 25 V, and (b) 100 V. What must be the diode PIV rating for each case? Verify the results with a computer simulation analysis.

D2.5 The output resistance of the full-wave rectifier in Figure 2.7(a) in the text is $R = 150\,\Omega$. A filter capacitor is connected in parallel with R. Assume $V_\gamma = 0.7$ V. The peak output voltage is to be 24 V and the ripple voltage is to be no more than 0.5 V. The input frequency is 60 Hz. (a) Determine the required rms value of v_S. (b) Determine the required filter capacitance value. (c) Determine the peak current through each diode.

RD2.6 Repeat Problem 2.5 for the half-wave rectifier in Figure 2.2(a).

2.7 The circuit in Figure P2.7 is a complementary output rectifier. If $v_s = 26 \sin[2\pi(60)t]$ V, sketch the output waveforms v_o^+ and v_o^- versus time, assuming $V_\gamma = 0.6$ V for each diode.

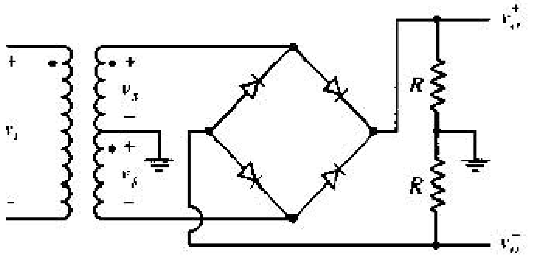


Figure P2.7

D2.8 Consider the battery charging circuit in Figure 2.5(a). Let $V_B = 12$ V, $V_\gamma = 0.7$ V, $V_S = 24$ V, and $\omega = 2\pi(60)$. The average battery charging current is to be $\bar{i}_D = 2$ A. Determine the required value of R and find the fraction of time the diode is conducting. What must be the power rating of the resistor R?

D2.9 The full-wave rectifier in Figure 2.6(a) is to deliver 0.1 A and 15 V (average) to a load. The ripple voltage is to be no larger than 0.4 V peak-to-peak. The input signal is 120 V (rms) at 60 Hz. Assume diode cut-in voltages of 0.7 V. Determine the required turns ratio, the filter capacitance value, and the diode PIV rating. Verify the design with a computer simulation analysis.

***2.10** Sketch v_o versus time for the circuit in Figure P2.10 with the input shown. Assume $V_\gamma = 0$.

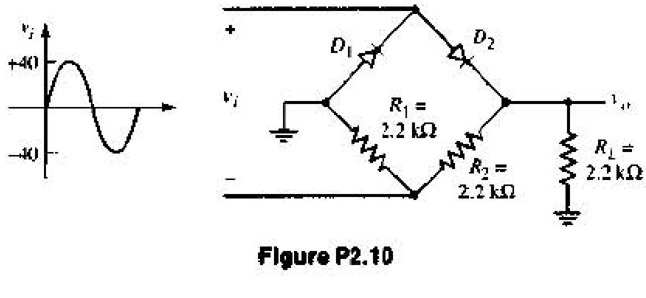


Figure P2.10

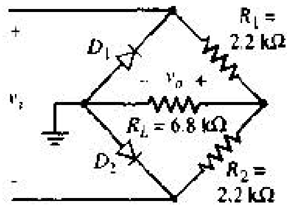
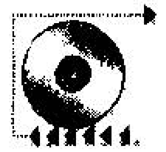


Figure P2.11



*2.11 (a) Sketch v_o versus time for the circuit in Figure P2.11. The input is a sine wave given by $v_i = 10 \sin \omega t$ V. Assume $V_\gamma = 0$. (b) Determine the rms value of the output voltage.

Section 2.2 Zener Diode Circuits

2.12 In the voltage regulator circuit in Figure P2.12, let $V_I = 6.3$ V, $R_i = 12\ \Omega$, and $V_z = 4.8$ V. The Zener diode current is to be limited to the range $5 \le I_Z \le 100$ mA. (a) Determine the range of possible load currents and load resistances. (b) Determine the power rating required for the Zener diode and the load resistor.

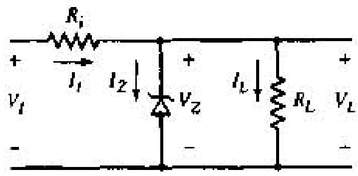


Figure P2.12

*2.13 In the voltage regulator circuit in Figure P2.12, $V_I = 20$ V, $V_Z = 10$ V, $R_i = 222\ \Omega$, and $P_Z(\text{max}) = 400$ mW. (a) Determine I_L, I_Z, and I_I, if $R_L = 380\ \Omega$. (b) Determine the value of R_L that will establish $P_Z(\text{max})$ in the diode. (c) Repeat part (b) if $R_i = 175\ \Omega$.

D2.14 A Zener diode is connected in a voltage regulator circuit as shown in Figure P2.12. The Zener voltage is $V_Z = 10$ V and the Zener resistance is assumed to be $r_z = 0$. (a) Determine the value of R_i such that the Zener diode remains in breakdown if the load current varies from $I_L = 50$ to 500 mA and if the input voltage varies from $V_I = 15$ to 20 V. Assume $I_Z(\text{min}) = 0.1 I_Z(\text{max})$. (b) Determine the power rating required for the Zener diode and the load resistor.

2.15 Reconsider Problem D2.14. (a) Determine the maximum variation in the output voltage if the Zener resistance is $r_z = 2\ \Omega$. (b) Calculate the percent regulation.

2.16 The percent regulation of the Zener diode regulator shown in Figure 2.17 is 5 percent. The Zener voltage is $V_{ZO} = 6$ V and the Zener resistance is $r_z = 3\ \Omega$. Also, the load resistance varies between 500 and 1000 Ω, the input resistance is $R_i = 280\ \Omega$, and the minimum power supply voltage is $V_{PS}(\text{min}) = 15$ V. Determine the maximum power supply voltage allowed. (Ans. $V_{PS}(\text{max}) = 41.3$ V)

*D2.17 A voltage regulator is to have a nominal output voltage of 10 V. The specified Zener diode has a rating of 1 W, has a 10 V drop at $I_Z = 25$ mA, and has a Zener resistance of $r_z = 5\ \Omega$. The input power supply has a nominal value of $V_{PS} = 20$ V and can vary by ±25 percent. The output load current is to vary between $I_L = 0$ and 20 mA. (a) If the minimum Zener current is to be $I_Z = 5$ mA, determine the required R_i. (b) Determine the maximum variation in output voltage. (c) Determine the percent regulation.

*D2.18 Consider the circuit in Figure P2.18. Let $V_\gamma = 0$. The secondary voltage is given by $v_s = V_s \sin \omega t$, where $V_S = 24$ V. The Zener diode has parameters $V_Z = 16$ V at $I_Z = 40$ mA and $r_z = 2\ \Omega$. Determine R_i such that the load current can vary over the range $40 \le I_L \le 400$ mA with $I_Z(\text{min}) = 40$ mA and find C such that the ripple voltage is no larger than 1 V.

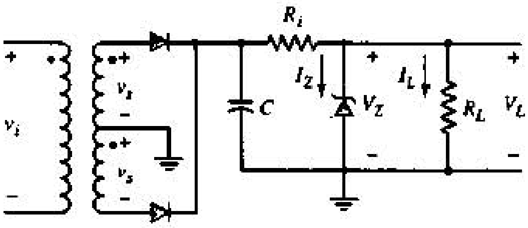


Figure P2.18

*2.19 The secondary voltage in the circuit in Figure P2.18 is $v_s = 12 \sin \omega t$ V. The Zener diode has parameters $V_Z = 8$ V at $I_Z = 100$ mA and $r_z = 0.5\,\Omega$. Let $V_\gamma = 0$ and $R_i = 3\,\Omega$. Determine the percent regulation for load currents between $I_L = 0.2$ and 1 A. Find C such that the ripple voltage is no larger than 0.8 V.

Section 2.3 Clipper and Clamper Circuits

2.20 Consider the circuit in Figure P2.20. Let $V_\gamma = 0$. (a) Plot v_O versus v_I over the range $-10 \le v_I \le +10$ V. (b) Plot i_1 over the same input voltage range as part (a).

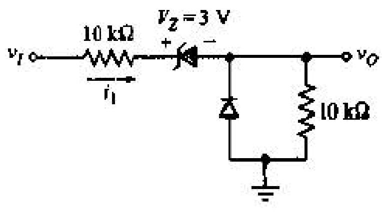


Figure P2.20

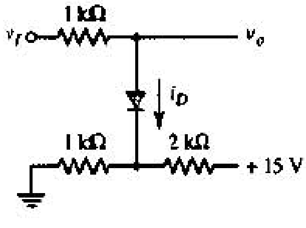


Figure P2.21

2.21 For the circuit in Figure P2.21, (a) plot v_O versus v_I for $0 \le v_I \le 15$ V. Assume $V_\gamma = 0.7$ V. Indicate all breakpoints. (b) Plot i_D over the same range of input voltage. (c) Compare the results of parts (a) and (b) with a computer simulation.

2.22 For the circuit in Figure P2.22, let $V_\gamma = 0.7$ V and assume the input voltage varies over the range $-10 \le v_I \le +10$ V. Plot (a) v_O versus v_I and (b) i_D versus v_I over the input voltage range indicated.

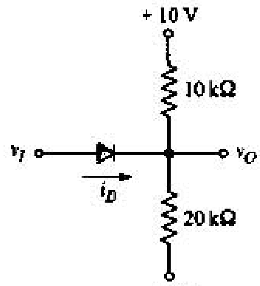
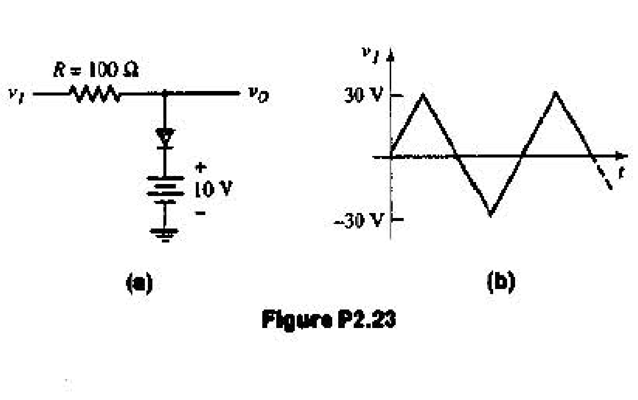


Figure P2.22

*2.23 The diode in the circuit of Figure P2.23(a) has piecewise linear parameters $V_\gamma = 0.7$ V and $r_f = 10\,\Omega$. (a) Plot v_O versus v_I for $-30 \le v_I \le 30$ V. (b) If the triangular wave, shown in Figure P2.23(b), is applied, plot the output versus time.



(a) (b)

Figure P2.23

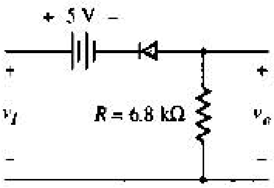
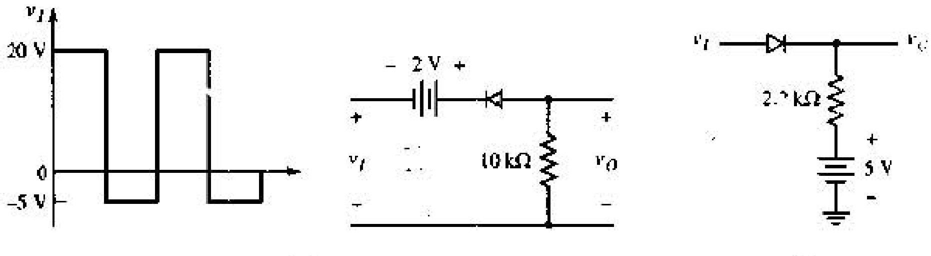


Figure P2.24

2.24 Consider the circuit in Figure P2.24. Sketch v_O versus time if $v_I = 15 \sin \omega t$ V. Assume $V_\gamma = 0.6$ V.

2.25 Plot v_O for each circuit in Figure P2.25 for the input shown. Assume (a) $V_\gamma = 0$ and (b) $V_\gamma = 0.6$ V.

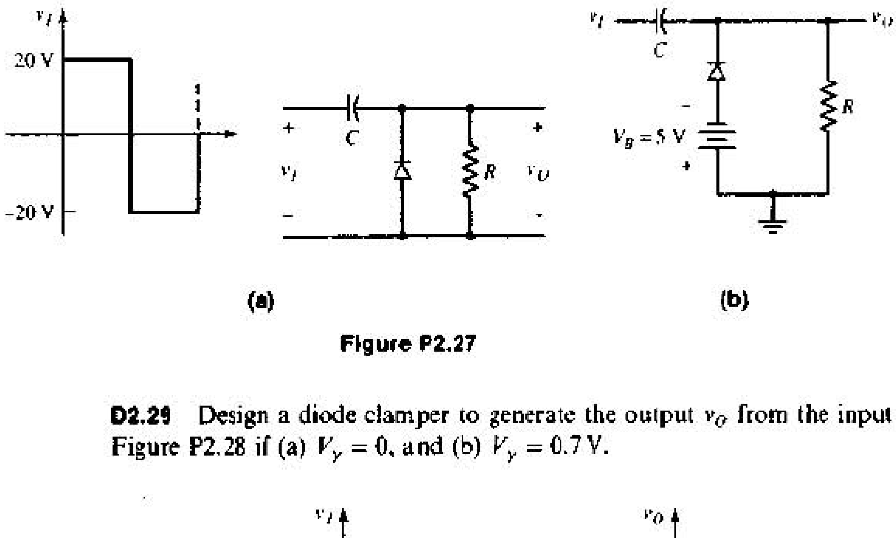


(a) (b)

Figure P2.25

***D2.26** A car's radio may be subjected to voltage spikes induced by coupling from the ignition system. Pulses on the order of ± 250 V and lasting for 120 μs may exist. Design a clipper circuit using resistors, diodes, and Zener diodes to limit the input voltage between +14 V and −0.7 V. Specify power ratings of the components.

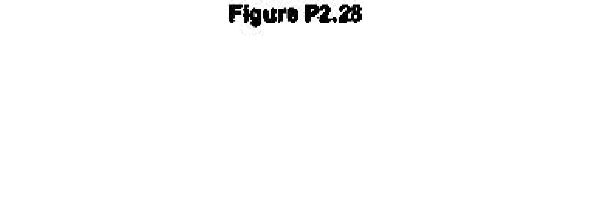
2.27 Sketch v_O versus time for each circuit with the input shown in Figure P2.27. Assume $V_\gamma = 0$ and assume the RC time constant is large.



(a) (b)

Figure P2.27

D2.28 Design a diode clamper to generate the output v_O from the input v_I shown in Figure P2.28 if (a) $V_\gamma = 0$, and (b) $V_\gamma = 0.7$ V.



(a) (b)

Figure P2.28

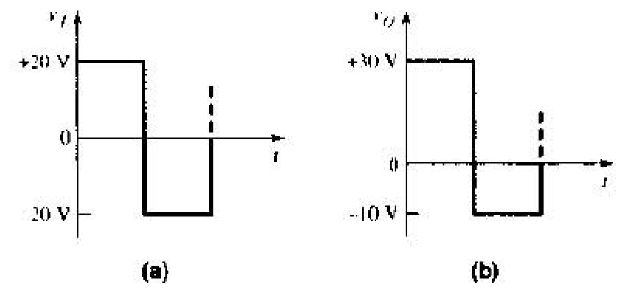


Figure P2.29

D2.29 Design a diode clamper to generate the output v_O from the input v_I in Figure P2.29 if $V_\gamma = 0$.

2.30 For the circuit in Figure 2.27(a), let $V_\gamma = 0$ and $v_I = 10 \sin \omega t$. Plot v_O versus time over 3 cycles of input voltage for (a) $V_B = +3$ V and (b) $V_B = -3$ V.

2.31 Repeat Problem 2.30 for the case when the direction of the diode in the circuit shown in Figure 2.27(a) is reversed.

Section 2.4 Multiple Diode Circuits

2.32 The diodes in the circuit in Figure P2.32 have piecewise linear parameters of $V_\gamma = 0.6$ V and $r_f = 0$. Determine the output voltage V_O and the diode currents I_{D1} and I_{D2} for the following input conditions: (a) $V_1 = 10$ V, $V_2 = 0$; (b) $V_1 = 5$ V, $V_2 = 0$; (c) $V_1 = 10$ V, $V_2 = 5$ V; and (d) $V_1 = V_2 = 10$ V. (e) Compare the results of parts (a) through (d) with a computer simulation analysis.

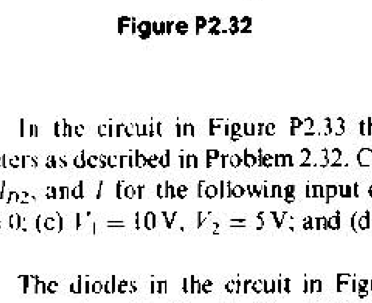


Figure P2.32

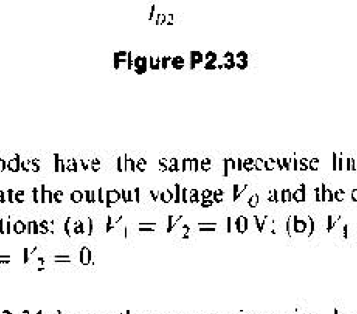


Figure P2.33

2.33 In the circuit in Figure P2.33 the diodes have the same piecewise linear parameters as described in Problem 2.32. Calculate the output voltage V_O and the currents I_{D1}, I_{D2}, and I for the following input conditions: (a) $V_1 = V_2 = 10$ V; (b) $V_1 = 10$ V, $V_2 = 0$; (c) $V_1 = 10$ V, $V_2 = 5$ V; and (d) $V_1 = V_2 = 0$.

2.34 The diodes in the circuit in Figure P2.34 have the same piecewise linear parameters as described in Problem 2.32. Determine the output voltage V_O and the currents I_{D1}, I_{D2}, I_{D3}, and I for the following input conditions: (a) $V_1 = V_2 = 0$; (b) $V_1 = V_2 = 5$ V; (c) $V_1 = 5$ V, $V_2 = 0$; and (d) $V_1 = 5$ V, $V_2 = 2$ V.

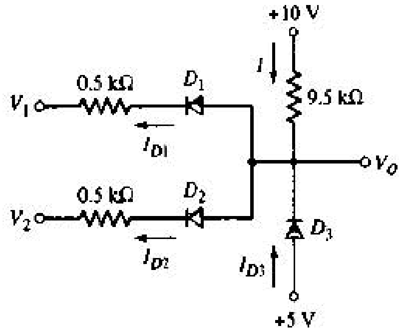


Figure P2.34

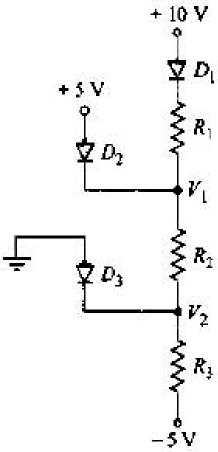


Figure P2.35

2.35 The cut-in voltage for each diode in Figure P2.35 is $V_\gamma = 0.6\,\text{V}$. (a) Find V_1, V_2, and each diode current for $R_1 = 2\,\text{k}\Omega$, $R_2 = 6\,\text{k}\Omega$, and $R_3 = 2\,\text{k}\Omega$. (b) Repeat part (a) for $R_1 = 6\,\text{k}\Omega$, $R_2 = R_3 = 5\,\text{k}\Omega$. (c) Determine R_1, R_2, and R_3 such that each diode current is 0.5 mA.

2.36 (a) For the circuit in Figure P2.36, each diode has $V_\gamma = 0.6\,\text{V}$. Plot v_O versus v_I over the range $0 \le v_I \le 10\,\text{V}$. (b) Compare the results of part (a) with a computer simulation analysis.

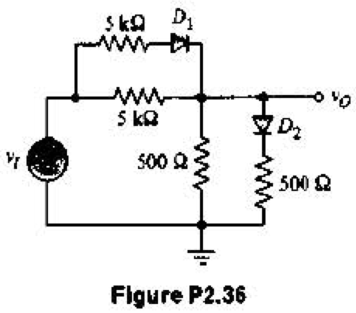


Figure P2.36

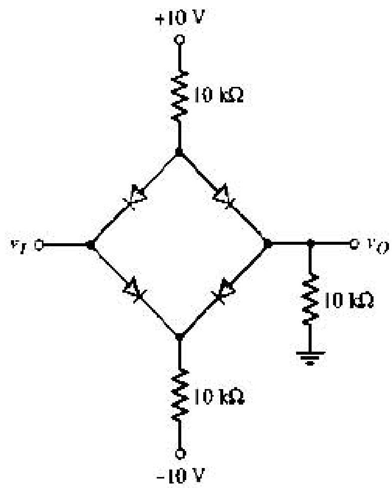


Figure P2.37

***2.37** Assume $V_\gamma = 0.7\,\text{V}$ for each diode in the circuit in Figure P2.37. Plot v_O versus v_I for $-10 \le v_I + 10\,\text{V}$.

2.38 Let $V_\gamma = 0.7\,\text{V}$ for each diode in the circuit in Figure P2.38. (a) Find I_{D1} and V_O for $R_1 = 5\,\text{k}\Omega$ and $R_2 = 10\,\text{k}\Omega$. (b) Repeat part (a) for $R_1 = 10\,\text{k}\Omega$ and $R_2 = 5\,\text{k}\Omega$.

2.39 If $V_\gamma = 0.7\,\text{V}$ for the diode in the circuit in Figure P2.39, determine I_D and V_O.

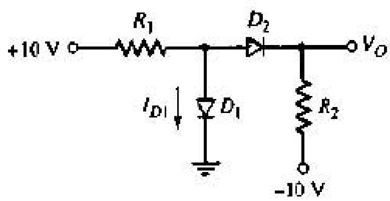


Figure P2.38

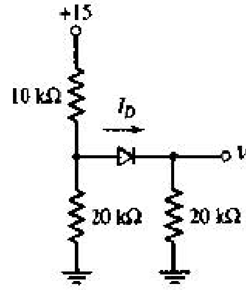


Figure P2.39

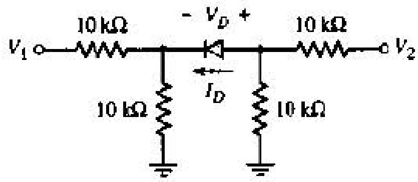


Figure P2.40

2.40 Let $V_\gamma = 0.6$ V for the diode in the circuit in Figure P2.40. Determine I_D and V_D if: (a) $V_1 = 15$ V, $V_2 = 10$ V; and (b) $V_1 = 10$ V, $V_2 = 15$ V.

2.41 (a) Each diode in the circuit in Figure P2.41 has piecewise linear parameters of $V_\gamma = 0$ and $r_f = 0$. Plot v_O versus v_I for $0 \le v_I \le 30$ V. Indicate the breakpoints and give the state of each diode in the various regions of the plot. (b) Compare the results of part (a) with a computer simulation analysis.

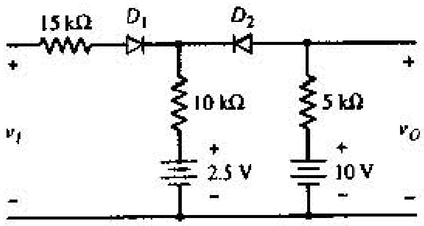


Figure P2.41

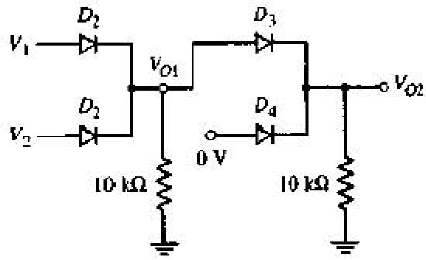


Figure P2.42

2.42 Consider the circuit in Figure P2.42. The output of a diode OR logic gate is connected to the input of a second diode OR logic rate. Assume $V_\gamma = 0.6$ V for each diode. Determine the outputs V_{O1} and V_{O2} for: (a) $V_1 = V_2 = 0$; (b) $V_1 = 5$ V, $V_2 = 0$; and (c) $V_1 = V_2 = 5$ V. What can be said about the relative values of V_{O1} and V_{O2} in their "high" state?

2.43 Consider the circuit in Figure P2.43. The output of a diode AND logic gate is connected to the input of a second diode AND logic gate. Assume $V_\gamma = 0.6$V for each

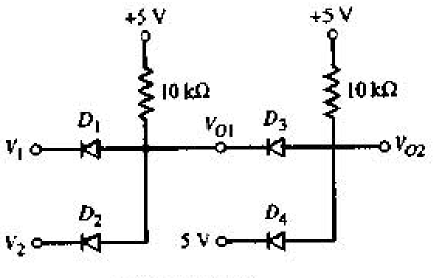


Figure P2.43

diode. Determine the outputs V_{O1} and V_{O2} for: (a) $V_1 = V_2 = 5$ V; (b) $V_1 = 0$, $V_2 = 5$ V; and (c) $V_1 = V_2 = 0$. What can be said about the relative values of V_{O1} and V_{O2} in their "low" state?

2.44 Determine the Boolean expression for V_O in terms of the four input voltages for the circuit in Figure P2.44.

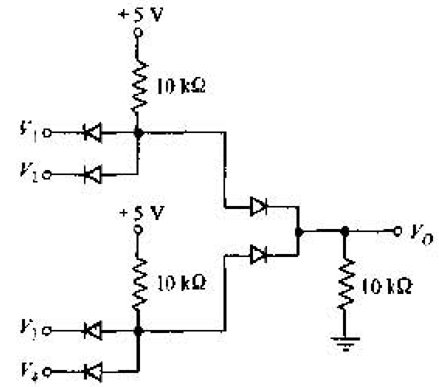


Figure P2.44

Section 2.5 LED and Photodiode Circuits

2.45 Consider the circuit shown in Figure P2.45. The forward-bias cut-in voltage of the diode is 1.5 V and the forward-bias resistance is $r_f = 10\,\Omega$. Determine the value of R required to limit the current to $I = 12$ mA when $V_I = 0.2$ V.

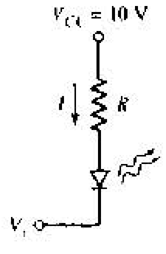


Figure P2.45

2.46 The light-emitting diode in the circuit shown in Figure P2.45 has parameters $V_\gamma = 1.7$ V and $r_f = 0$. Light will first be detected when the current is $I = 8$ mA. If $R = 750\,\Omega$, determine the value of V_I at which light will first be detected.

2.47 If the resistor in Example 2.10 is $R = 2\,k\Omega$ and the diode is to be reverse biased by at least 1 V, determine the minimum power supply voltage required.

2.48 Consider the photodiode circuit shown in Figure 2.39. Assume the quantum efficiency is 1. A photocurrent of 0.6 mA is required for an incident photon flux of $\Phi = 10^{17}$ cm^{-2}-s^{-1}. Determine the required cross-sectional area of the diode.

COMPUTER SIMULATION PROBLEMS

2.49 Correlate the results of Example 2.1 with a computer simulation.

2.50 Consider the voltage doubler circuit shown in Figure 2.13. Assume a 60 Hz sinusoidal input signal and a 1 : 1 transformer turns ratio. Let $R = 5\,k\Omega$ and $C_1 = C_2 = 100\,\mu F$. Plot the steady-state output voltage over two cycles of input voltage.

2.51 Correlate the design results of Example 2.4 with a computer simulation.

2.52 Perform a computer simulation analysis of Exercise 2.13. How much does the output voltage change during each half-cycle?

DESIGN PROBLEMS

[Note: Each design should be correlated with a computer simulation.]

***D2.53** Design a full-wave rectifier to produce a peak output voltage of 9 V, deliver 150 mA to the load, and produce an output with a ripple of not more than 5 percent. A line voltage of 120 V (rms), 60 Hz is available. The only transformers available have turns ratios of $N_1/N_2 = 10$, 15, and 20.

***D2.54** Design a full-wave rectifier to provide a dc output of 28 V at a current of 4 A, with a ripple of not more than 3 percent. A line voltage of 120 V (rms), 60 Hz is available.

***D2.55** Design a full-wave regulated power supply using a 5 : 1 center-tapped transformer and a 7.5 V, 1 W Zener diode. The power supply must provide a constant 7.5 V to a load varying from 120 to 450 Ω. The input voltage is 120 V (rms), 60 Hz.

CHAPTER

3

The Bipolar Junction Transistor

3.0 PREVIEW

In the last chapter, we saw that the rectifying current–voltage characteristics of the diode are useful in electronic switching and waveshaping circuits. However, diodes are not capable of amplifying currents or voltages. The electronic device that is capable of current and voltage amplification, or gain, in conjunction with other circuit elements, is the transistor, which is a three-terminal device. The development of the silicon transistor by Bardeen, Brattain, and Schockley at Bell Telephone Laboratories in the late 1940s started the first electronics revolution of the 1950s and 1960s. This invention led to the development of the first integrated circuit in 1958 and to the operational transistor amplifier (op-amp), which is one of the most widely used electronic circuits.

The bipolar transistor, which is introduced in this chapter, is the first of two major types of transistors. The second type of transistor, the field-effect transistor (FET), is introduced in Chapter 5 and has led to the second electronics revolution in the 1970s and 1980s. These two device types are the basis of modern day microelectronics. Each device type is equally important and each has particular advantages for specific applications.

We begin this chapter with a look at the physical structure and operation of the bipolar transistor. The chapter deals mainly with the transistor characteristics and with the dc analysis and design of bipolar circuits. We continue to use the piecewise linear approximation techniques, developed for the diode, in the bipolar transistor calculations. We discuss how the transistor can be used in switch, digital, and linear amplifier applications.

Much of the material in this chapter may appear to be skewed toward discrete transistor biasing. However, the principal goal of the chapter is to ensure that readers become familiar with transistor characteristics and are able to quickly analyze and design the dc response of bipolar transistor circuits. Integrated circuit biasing is discussed toward the end of the chapter and is emphasized to a greater extent in the later chapters.

3.1 BASIC BIPOLAR JUNCTION TRANSISTOR

The **bipolar junction transistor (BJT)** has three separately doped regions and contains two pn junctions. A single pn junction has two modes of operation—

forward bias and reverse bias. The bipolar transistor, with two pn junctions, therefore has four possible modes of operation, depending on the bias condition of each pn junction, which is one reason for the versatility of the device. With three separately doped regions, the bipolar transistor is a three-terminal device. The basic transistor principle is that *the voltage between two terminals controls the current through the third terminal.*

Our discussion of the bipolar transistor starts with a description of the basic transistor structure and a qualitative description of its operation. To describe its operation, we use the pn junction concepts presented in Chapter 1. However, the two pn junctions are sufficiently close together to be called interacting pn junctions. The operation of the transistor is therefore totally different from that of two back-to-back diodes.

Current in the transistor is due to the flow of both electrons and holes, hence the name **bipolar**. Our discussion covers the relationship between the three terminal currents. In addition, we present the circuit symbols and conventions used in bipolar circuits, the bipolar transistor current-voltage characteristics, and finally, some nonideal current–voltage characteristics.

3.1.1 Transistor Structures

Figure 3.1 shows simplified block diagrams of the basic structure of the two types of bipolar transistor: npn and pnp. The **npn bipolar transistor** contains a thin p-region between two n-regions. In contrast, the **pnp bipolar transistor** contains a thin n-region sandwiched between two p-regions. The three regions and their terminal connections are called the **emitter**, **base**, and **collector**. The operation of the device depends on the two pn junctions being in close proximity, so the width of the base must be very narrow, normally in the range of tenths of a micrometer (10^{-6} m).

The actual structure of the bipolar transistor is considerably more complicated than the block diagrams of Figure 3.1. For example, Figure 3.2 is the cross section of a classic npn bipolar transistor fabricated in an integrated circuit. One important point is that the device is not symmetrical electrically. This asymmetry occurs because the geometries of the emitter and collector regions are not the same, and the impurity doping concentrations in the

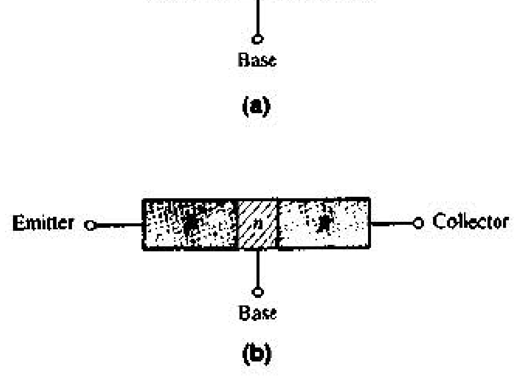


Figure 3.1 Simple geometry of bipolar transistors: (a) npn and (b) pnp

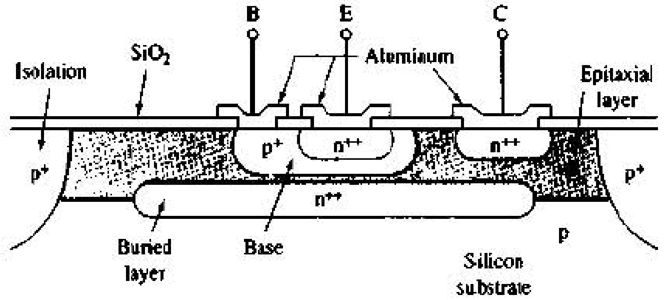


Figure 3.2 Cross section of a conventional integrated circuit npn bipolar transistor

three regions are substantially different. For example, the impurity doping concentrations in the emitter, base, and collector may be on the order of 10^{19}, 10^{17}, and $10^{15}\,cm^{-3}$, respectively. Therefore, even though both ends are either p-type or n-type on a given transistor, switching the two ends makes the device act in drastically different ways.

Although the block diagrams in Figure 3.1 are highly simplified, they are still useful for presenting the basic transistor characteristics.

3.1.2 npn Transistor: Forward-Active Mode Operation

Since the transistor has two pn junctions, four possible bias combinations may be applied to the device, depending on whether a forward or reverse bias is applied to each junction. For example, if the transistor is used as an amplifying device, the **base–emitter (B–E) junction** is forward biased and the **base–collector (B–C) junction** is reverse biased, in a configuration called the **forward-active operating mode**, or simply the **active region**. The reason for this bias combination will be illustrated as we look at the operation of such transistors and the characteristics of circuits that use them.

Transistor Currents

Figure 3.3 shows an idealized npn bipolar transistor biased in the forward-active mode. Since the B–E junction is forward biased, electrons from the emitter are injected across the B–E junction into the base, creating an excess minority carrier concentration in the base. Since the B–C junction is reverse

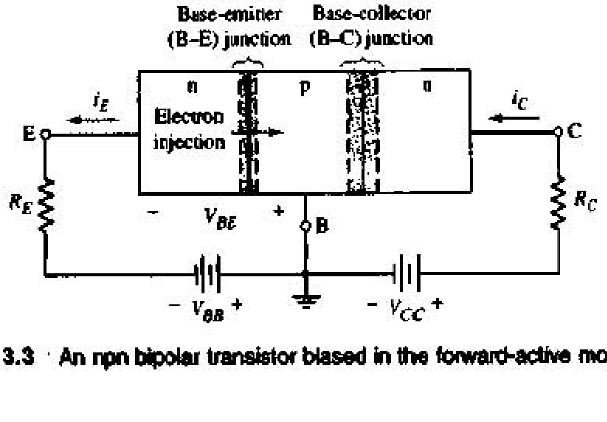


Figure 3.3 An npn bipolar transistor biased in the forward-active mode

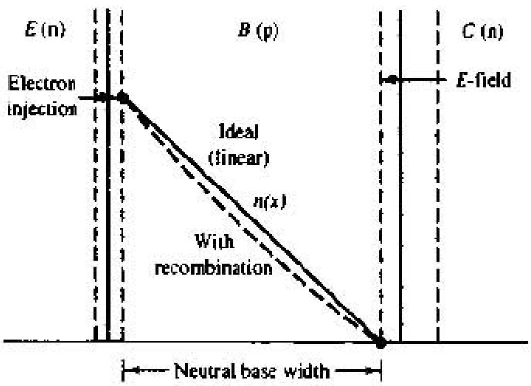


Figure 3.4 Minority carrier electron concentration across the base region of an npn bipolar transistor biased in the forward-active mode

biased, the electron concentration at the edge of that junction is approximately zero.

The electron concentration in the base region is shown in Figure 3.4. Because of the large gradient in this concentration, electrons injected from the emitter diffuse across the base into the B–C space-charge region, where the electric field sweeps them into the collector region. Ideally, as many electrons as possible will reach the collector without recombining with majority carrier holes in the base. Figure 3.4 shows the ideal case in which no recombination occurs, so that the electron concentration is a linear function of distance across the base. However, if some carrier recombination does occur in the base, the electron concentration will deviate from the ideal linear curve, as shown in the figure. To minimize recombination effects, the width of the neutral base region must be small compared to the minority carrier diffusion length.

Emitter Current Since the B–E junction is forward biased, we expect the current through this junction to be an exponential function of B–E voltage, just as we saw that the current through a pn junction diode was an exponential function of the forward-biased diode voltage. We can then write the current at the emitter terminal as

$$i_E = I_S\left[e^{v_{BE}/V_T} - 1\right] \cong I_S e^{v_{BE}/V_T} \tag{3.1}$$

where the approximation of neglecting the (-1) term is usually valid since $v_{BE} \gg V_T$ in most cases. The parameter V_T is the usual thermal voltage. The emission coefficient n that multiplies V_T is assumed to be 1, as we discussed in Chapter 1 in considering the ideal diode equation. The flow of the negatively charged electrons is through the emitter into the base and is opposite to the conventional current direction. The conventional emitter current is therefore out of the emitter terminal.

The multiplying constant, I_S, contains electrical parameters of the junction, but in addition is directly proportional to the active B–E cross-sectional area. Therefore, if two transistors are identical except that one has twice the area of the other, then the emitter currents will differ by a factor of two for the same applied B–E voltage. Typical values of I_S are in the range of 10^{-12} to 10^{-15} A, but may, for special transistors, vary outside of this range.