Single Sensor Hot-Wire Anemometer Based on Thermal Time Constant Estimation

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***Abstract*—This paper presents a new type of hot-wire anemometer based on its sensor's thermal time constant estimation. In the known types of thermal anemometers, the source of measurement errors is an ambient temperature, which influences the heat exchange between the hot sensor and its surrounding fluid. Therefore, a second sensor is used to compensate the impact of ambient temperature. The solution contemplated here uses the relationship between the sensor's thermal time constant and fluid flow rate. This constant has no relationship to ambient temperature, so there is no need to use a second sensor. The paper presents algorithms for estimating the sensor's time constant, mathematical relationships, the measurement system and sample measurement results obtained.**

Keywords—flow measurement, Hot-wire anemometer, thermal flow sensor, thermal time constant, cyclic heating-cooling mode, least square method

I. Introduction

The basics of measurement of fluid flow rate using a hot-wire anemometer are presented in King's paper [1] describing the process of heat exchange between a hot wire and fluid flowing around this wire. King presents relevant analytical relationships and demonstrates that the amount of heat dissipated by convection from a hot wire to its surrounding fluid depends on the fluid's flow rate and on the difference in temperature of the wire and fluid. Modern hot-wire anemometers typically operate in one of two modes: CCA (constant current anemometer) and CTA (constant temperature anemometer) [2]. Temperature of an anemometer's wire is higher than its surrounding fluid thanks to heating power as a result of electric current flowing through the wire. In a CCA current is constant, which is why the wire's temperature decreases as the surrounding fluid's flow rate increases. In a CTA an electronic circuit with feedback maintains the wire's constant temperature in relation to ambient temperature by appropriately adjusting the wire-heating current. Current adjustment can be implemented using an analogue system with operational amplifiers [3] or a discrete system with pulse-width modulation (PWM) [4], sigma-delta modulation (Σ-Δ) [5] or pulse frequency modulation [6]. The feedback system can cover just the electronic part [4], but there are also thermal-feedback-based solutions that use integrating properties of the thermal part of an anemometer's sensor. This method is used to implement Σ-Δ thermal flow converters [5] and thermal flow-to-frequency heat-balance converters [7]. In feedback systems, microprocessors [8] are also used to adjust the sensor's temperature using the PWM method. There are also thermal anemometers that use various semiconductor components instead of a metal-wire sensor, including thermistors [4, 7], diodes or transistors [9] and integrated circuits [5]. The principle of operation of these systems remains the same, changing only numeric values of factors in King's formulas [1], which have to be determined each time experimentally by calibration. The common property of the solutions mentioned here is the impact of ambient temperature on the measurement result. If the sensor's temperature is appropriately high, changes in fluid temperature can be disregarded; in such case, however, we also need the heat balance to take into account the phenomenon of heat radiation. If the sensor is operating at lower temperatures, then we need to compensate changes in ambient temperature by maintaining a constant difference in temperature of the sensor and fluid. This requires using a second sensor to measure ambient temperature. This paper presents another solution based on an estimation of sensor's thermal time constant, which depends on the fluid flow rate and the sensor's type; it does not, however, depend on the fluid's temperature. This is why there is no need to use a second sensor to compensate the impact of ambient temperature.

II. Heat Exchange Between a Sensor and Fluid in Motion

A. Heat exchange with steady-state thermal balance

A thermal anemometer's sensor operates at the overlap of two areas: of electrical phenomena and of thermal phenomena (Fig. 1). Current i_{sens} flowing through the sensor's resistance R_{sens} generates electric power $P_{el}=i^2_{sens}R_{sens}$ (Fig. 1a). Power P_{el} generated in the sensor increases the sensor's temperature T_{sens} and is transferred by convection to its surrounding fluid flowing at rate ϑ, with ambient temperature T_{amb}.

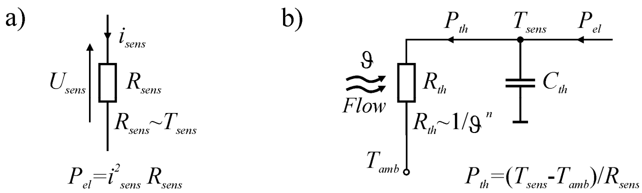

Fig. 1. The principle of operation of a thermal anemometer's sensor: a) electrical phenomena domain, b) thermal phenomena domain.

Thermal power P_{th} transferred from the sensor to its surroundings is equal to:

$$P_{th} = hS_{sens}\left(T_{sens} - T_{amb}\right), \tag{1}$$

where h is the heat transfer coefficient, S_{sens} is the sensor's surface. According to King's Law [1], heat transfer coefficient h is the fluid flow rate function ϑ :

$$h = A + B\vartheta^{n}, \tag{2}$$

where A, B, n are experimentally determined constants. Constant A describes convection in motionless fluid, constant B specifies convection in flowing fluid. King demonstrated that n=0.5 for a sensor in the form of a long thin wire with laminar flow. For sensors of different types, n takes lower values between 0.3 .. 0.45 [3, 4]. In a steady state, when the sensor's temperature T_{sens} and fluid flow rate ϑ are constant over time, electric power P_{el} supplied to the sensor is equal to thermal power P_{th} dissipated to the surrounding area (Fig. 1b):

$$P_{el} = P_{th} = S_{sens}\left(A + B\vartheta^{n}\right)\left(T_{sens} - T_{amb}\right). \tag{3}$$

Ratio of the temperature difference T_{sens}–T_{amb} to power dissipated by the sensor to the surrounding area P_{th} is defined as the sensor's thermal resistance R_{th}:

$$R_{th} = \frac{T_{sens} - T_{amb}}{P_{th}} = \frac{1}{S_{sens}\left(A + B\vartheta^{n}\right)}. \tag{4}$$

Since S_{sens}, A, B, n are constant, if we know ambient temperature T_{amb} and electric power P_{el} necessary to maintain the sensor's temperature T_{sens}, we can use relationship (3) to determine fluid flow rate ϑ. However, this requires using a second sensor to measure ambient temperature T_{amb}. Another solution is to take measurements of the sensor in a dynamic condition.

B. *Idea of the method proposed herein*

The proposed measurement method for a sensor in a dynamic condition is presented on Fig. 2a. The system operates periodically. First, switch SW applies heating current i_{heat}, to the sensor over time t_{heat}, generating energy in the sensor:

$$Q_{heat} = P_{el}t_{heat} = i_{heat}^{2}R_{sens}t_{heat}, \tag{5}$$

as a result of which, the sensor's temperature increases to T_{hot}:

$$T_{hot} = T_{amb} + \frac{Q_{heat}}{C_{th}} \tag{6}$$

where C_{th} is the sensor's heat capacity, C_{th}=$m_{sens}c_{th}$, m_{sens} is the

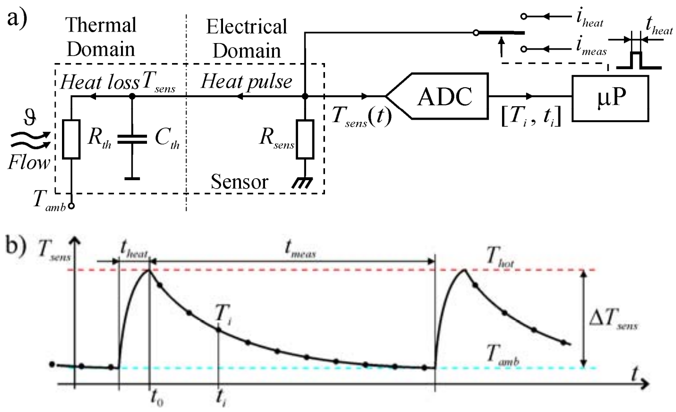

Fig. 2. The measurement principle of a thermal anemometer's sensor under dynamic conditions: a) the measurement system's diagram, b) changes in the sensor's temperature over time.

sensor's mass, c_{th} is specific heat capacity of the sensor's material. Then, at t_0=0, switch SW applies very small measuring current i_{meas} to the sensor, where the sensor's self-heating effect is negligible. The sensor cools down and its temperature $T_{sens}(t)$ decreases from temperature T_{hot} to T_{amb} with exponential relationship (Fig. 2b):

$$T_{sens}(t) = (T_{hot} - T_{amb})\exp(-\frac{t - t_0}{\tau_{th}}) + T_{amb}, \tag{7}$$

where τ_{th} is thermal time constant of the sensor defined as a product of the sensor's thermal resistance R_{th} (4) and its heat capacity C_{th}:

$$\tau_{th} = R_{th}C_{th} = \frac{m_{sens}c_{th}}{S_{sens}\left(A + B\vartheta^{n}\right)}. \tag{8}$$

Equation (8) states that the sensor's thermal time constant τ_{th} depends solely on fluid flow rate ϑ, with remaining factors in equation (8) for a given sensor being constant. The sensor's thermal time constant τ_{th} has no relationship to ambient temperature T_{amb}, which allows us to measure fluid flow rate ϑ using only one sensor. To this end, a microprocessor with ADC (Fig. 2a) measures voltage drop on the sensor's resistance V_{sens}=$R_{sens}i_{meas}$ and calculates and records changes in the cooling sensor's temperature (7) as a sequence of values of temperatures T_i at times t_i. This is the basis for estimating time constant τ_{th} and calculating flow rate ϑ.

III. Estimation of a Sensor's Thermal Time Constant

A. *Simple two point method*

Relationship (7) describing the sensor's temperature T_{sens} as a function of time t has two unknown parameters, i.e. the sensor's thermal time constant τ_{th} and ambient temperature T_{amb}, which can be determined by solving the system of two equations with two variables. The procedure is shown on Fig. 3. A tangent line plotted through point T_1 of time equation (7) intersects the level of final temperature T_{amb} after time equal to

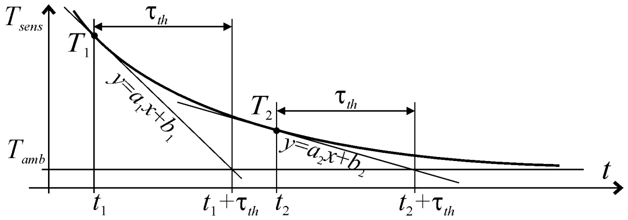

Fig. 3. Determining thermal time constant using two points.

thermal time constant τ_{th} from time t_1, and correspondingly for point T_2:

$$\begin{cases} T_{amb} = T_1 + a_1\tau_{th} \\ T_{amb} = T_2 + a_2\tau_{th} \end{cases}, \tag{9}$$

where a_1, a_2 are slopes of lines tangent at points T_1, T_2. By solving system (9), we arrive at time constant τ_{th}:

$$\tau_{th} = -\frac{T_1 - T_2}{a_1 - a_2}. \tag{10}$$

By substituting (10) into (9), we can also determine ambient temperature T_{amb}. Slopes a_1, a_2 can be determined by numerically differentiating [10] measurement data t_i, T_i collected by the microprocessor (Fig. 2a).

B. Using the least squares method

The method using only two points of function (7) is very sensitive to noise and measurement errors. Better results can be obtained by estimating the time constant using all points of the function. The procedure is shown on Fig. 4. By differentiating (7) and assuming $t_0=t_i$, $T_{hot}=T_i$ we calculate slope a_i of tangent $y=a_ix+b_i$ (Fig. 4a):

$$a_i = \frac{d}{dt}T_{sens}(t)\bigg|_{t=t_i} = -\frac{1}{\tau_{th}}(T_i - T_{amb}). \tag{11}$$

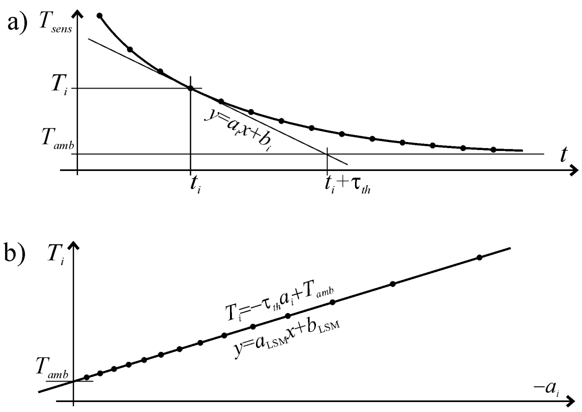

Fig. 4. Determining time constant using the least squares method: a) differentiating of the cooling characteristics (7), b) graph of equation (12).

Relationship (11) can be transformed into a linear function, with a_i, T_i as variables and τ_{th}, T_{amb} as constant factors:

$$T_i = -\tau_{th}a_i + T_{amb}, \tag{12}$$

and plotted on a graph (Fig. 4b). Derivative (11) is numerically calculated using measurement data T_i, t_i [11]. Points a_i, T_i obtained on a graph are approximated by straight line $y=a_{LSM}x+b_{LSM}$ using the least squares method LSM. The estimated value of time constant τ_{th} will be equal to:

$$\tau_{th} = -a_{LSM}. \tag{13}$$

IV. Simulation Studies of the Method Proposed Herein

The correctness of the presented method of time constant estimation using the method of least squares was verified by simulation. Function (7) was used to generate a sequence of values of the sensor's temperature T_i at times t_i evenly spaced in time with sampling period $\Delta t_{sampl}=t_i-t_{i-1}$. Assumed simulation parameters: τ_{th}=3s, T_{hot}=40°C, T_{amb}=25°C, Δt_{sampl}=0.1s. Furthermore, values of temperature T_i were corrupted with Gaussian noise of root mean square ΔT_{RMS}=0.02°C simulating random measurement errors. The simulated changes in temperature are shown on Fig. 5a. Derivative (11) was calculated numerically using a three-point formula for the middle point [11]:

$$a_i = \frac{d}{dt}T_{sens}(t_i) = \frac{1}{2\Delta t_{sampl}}(T_{i+1} - T_{i-1}). \tag{14}$$

Fig. 5b shows the obtained linear relationship (12) approximated using the LSM. Estimated thermal time constant $\tau_{th}=-a_{LSM}$=2.95s, ambient temperature $T_{amb}=b_{LSM}$=25.07°C. The results confirm the correctness of the method presented herein.

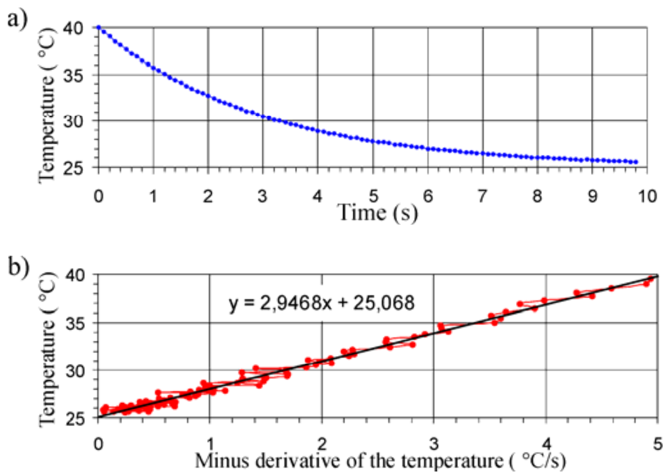

Fig. 5. Simulation studies results: a) changes in sensor's temperature over time, b) linear approximation of relationship (12) using the least squares method.

V. Real-World System Measurements

A measurement system was implemented according to the block diagram shown on Fig. 2a, using a NI USB 6009 measurement card with a 14-bit ADC, connected to a PC via USB. A PTC KTY83/110 thermistor [7] was used as the temperature sensor, placed in an aerodynamic tunnel whose structure is described in the author's previous papers [6]. The algorithm controlling periodic heating of the sensor, temperature measurement, and the data processing algorithm, were implemented in the LabVIEW environment installed on the PC. During a heating cycle, a sensor with resistance $R_{sens}\approx 1\text{k}\Omega$ is supplied with voltage V_{heat}=24V over time t_{heat}=1s, which generates energy $Q_{heat}\approx$0.58J and increases the sensor's temperature by $\Delta T_{\text{sens}}\approx$15K. Measurements were taken in air with flow rate in range of ϑ=0..6m/s. Fig. 6a shows cooling characteristics for various air flow rates, while Fig. 6b contains estimated values of thermal time constant. To determine factors of King's equation (2), relationship (8) was transformed to:

$$\frac{1}{\tau_{th}} = A' + B'\vartheta^{n}\,, \tag{15}$$

where factors A', B' account for values of constant parameters S_{sens}, m_{sens}, c_{th} in equation (8). Then $A'=1/\tau_{th(\vartheta=0)}$ was shifted to the left side, and both sides were expressed as logarithms:

$$\ln\left(\frac{1}{\tau_{th}} - \frac{1}{\tau_{th0}}'\right) = \ln B'\vartheta^{n} = \ln B' + n\ln\vartheta\,. \tag{16}$$

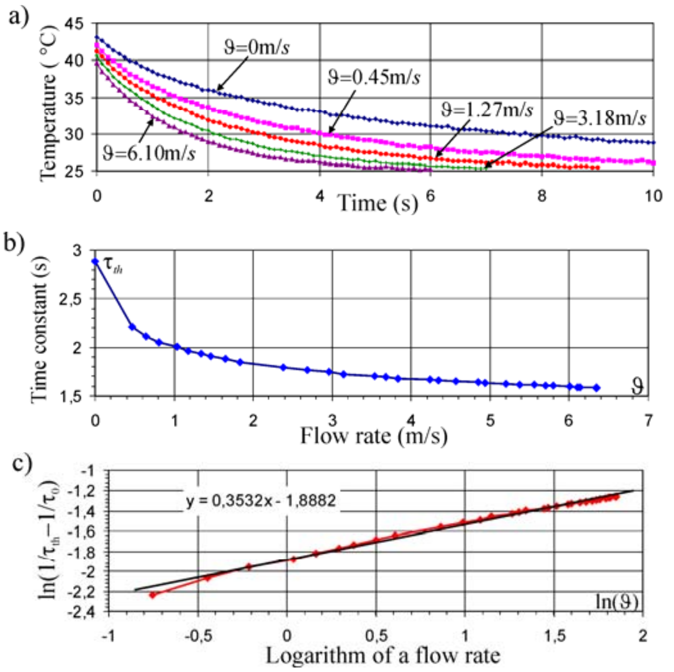

Fig. 6. Real-world experiment results: a) cooling characteristics for various air flow rates, b) estimated values of thermal time constant as a function of air flow velocity, c) linear approximation of the relationship (16).

Measurement data, according to relationship (16), are presented in the form of a graph on Fig. 6c, which was approximated with straight line $y=a_{\text{LSM}}x+b_{\text{LSM}}$ using the LSM. Assuming $b_{\text{LSM}}=\ln B'$, $a_{\text{LSM}}=n$ gives parameters A'=0.3467, B'=0.1513, n=0.3532. Finally, we obtain equation:

$$\frac{1}{\tau_{th}} = 0.3467 + 0.1513\,\vartheta^{0.3532}\,. \tag{17}$$

It should be noted that the value of n=0.5 was given by King for a sensor in the form of a long thin wire. For differently shaped sensors authors give $n\approx$0.31..0.44 [3, 4]; therefore, the resulting value of n=0.3532 can be deemed appropriate.

VI. Summary

The new type of thermal anemometer presented here uses the relationship between a sensor's thermal time constant and flow rate of the surrounding fluid. Compared to previous solutions, the advantage of this solution is the lack of necessity of using a second sensor to measure ambient temperature. The sensor operates periodically and is heated from time to time to a higher temperature, after which it cools down to ambient temperature. Thanks to this, the system conserves energy. Relationships describing converter operation were derived assuming constant fluid flow rate and temperature during measurement, which should equal several time constants of the sensor. The system was examined in air with flow rates up to 6 m/s. Determined factors of King's equation are consistent with literature.

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