Диан Минчев Иинев
Гр. 218 ф. № 101311038
ТЕМА № 2 ММЦО

(1) а) $f(x) = |x| \to$ четна $\Rightarrow b_n = 0$

Нека $F(x) = f(x)$ в $[-1;1]$

$F(x+2) = F(x)$

$$a_0 = 2\int_0^1 x\,dx = 2\frac{x^2}{2}\Big|_0^1 = 1$$

$$a_n = 2\int_0^1 x\cos n\pi x\,dx = \frac{2}{n\pi}\int_0^1 x\,d\sin \pi n x =$$

$$= \frac{2}{n\pi}\left(x\sin n\pi x\Big|_0^1 = \frac{2}{n^2\pi^2}(\cos n\pi - \cos 0) =\right.$$

$$= \frac{2}{n^2\pi^2}((-1)^n - 1) = \begin{cases} 0, & n = 2k \\ -\dfrac{4}{(2k-1)^2\pi^2}, & n = 2k-1 \end{cases}$$

$$S = \sum_{n=1}^{\infty}\frac{1}{(2n-1)^2} \qquad |x| = 0$$

1 б) $f(x) = \cos\left(\frac{x}{2}\right) \to$ четна $\cdot\ b_n = 0$

нека $F(x) = f(x)$

$F(x) = f(x + 2\pi)$

$$a_0 = \frac{2}{\pi}\int_0^{\pi}\cos\left(\frac{x}{2}\right)dx = \frac{4}{\pi}\int_0^{\pi}\cos\left(\frac{x}{2}\right)d\frac{x}{2} = \frac{4}{\pi}\sin\frac{x}{2}\Big|_0^{\pi} =$$

$$= \frac{4}{\pi}\left(\sin\frac{\pi}{2} - \sin\pi\right) = \frac{4}{\pi}$$

$$a_n = \frac{2}{\pi}\int_0^{\pi}\cos\frac{x}{2}\cos nx\, dx =$$

$$= \frac{2}{\pi}\int_0^{\pi}\frac{1}{2}\left[\cos\left(\frac{x}{2} - nx\right) + \cos\left(\frac{x}{2} + nx\right)\right]dx =$$

$$= \frac{1}{\pi}\left[\frac{2}{1-2n}\int_0^{\pi}\cos\left(\frac{x}{2} - nx\right)d\left(\frac{1}{2} - n\right)x + \frac{2}{1+2n}\int_0^{\pi}\cos\left(\frac{x}{2} + nx\right)d\left(\frac{1}{2} + nx\right)\right] =$$

$$= \frac{1}{\pi}\left[\frac{2}{1-2n}\sin\left(\frac{1}{2} - n\right)\pi + \frac{2}{1+2n}\sin\left(\frac{1}{2} + n\right)\pi\right] =$$

$$= \frac{1}{\pi}\left[\frac{2(1+2n)\sin\left(\frac{\pi}{2} + n\pi\right) - 2(1-2n)\sin\left(\frac{\pi}{2} + 2\pi\right)}{(1+2n)(1-2n)}\right] =$$

$$= \frac{2}{\pi}\left[\frac{(1+2n)\sin\frac{\pi}{2}\cos n\pi - \cos\frac{\pi}{2}\overset{0}{\sin n\pi}) + (1-2n)\left(\sin\frac{\pi}{2}\cos n\pi + \cos\frac{\pi}{2}\overset{0}{\sin n\pi}\right)}{1-4n^2}\right]$$

$$= \frac{2}{\pi}\left[\frac{(1+2n)\cos n\pi + (1-2n)\cos n\pi}{1-4n^2}\right] =$$

$$= \frac{2}{\pi}\left[\frac{(1+2n+1-2n)\cos n\pi}{1-4n^2}\right] = \frac{4}{\pi}\,\frac{\cos\pi}{1-4n^2} = \frac{4}{\pi}\,\frac{(-1)^n}{1-4n^2} =$$

$$= \frac{4}{\pi}\,\frac{(-1)^{n+1}}{4n^2-1}$$

$$\frac{2}{\pi}\int_0^{\pi} \cos^2\left(\frac{x}{2}\right) dx = \frac{1}{2}\cdot\frac{16}{\pi^2} + \sum_{n=1}^{\infty}\left(\frac{4}{\pi}\cdot\frac{1}{1-4n^2}\right)^2$$

$$\frac{2}{\pi}\int_0^{\pi} \frac{1+\cos x}{2}\, dx = \frac{8}{\pi^2} + \frac{16}{\pi^2}\cdot\sum_{n=1}^{\infty}\frac{1}{(1-4n^2)^2}$$

$$\frac{1}{\pi}\left(\int_0^{\pi} dx + \int_0^{\pi}\cos x\, dx\right) = \frac{8}{\pi^2} + \frac{16}{\pi^2}\cdot\sum_{n=1}^{\infty}\frac{1}{(1-4n^2)^2} \to S$$

$$\frac{1}{\pi}\left(x\Big|_0^{\pi} + \sin x\Big|_0^{\pi}\right) = \frac{8}{\pi^2} + \frac{16}{\pi^2}\cdot S$$

$$1 = \frac{8}{\pi^2} + \frac{16}{\pi^2}\cdot S$$

$$S = \frac{8}{\pi^2}\cdot\frac{\pi^2}{16} = \frac{1}{2}$$

(2) $f(x) = e^{-2|x|}$

$$f_c(x) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} \hat{f}_c(\lambda) \cos \lambda x \, \partial \lambda$$

$$\hat{f}_c(\lambda) = \sqrt{\frac{2}{\pi}} \int_0^{+\infty} f(x) \cos \lambda x \, \partial x =$$

$$= \sqrt{\frac{2}{\pi}} \cdot \frac{1}{\lambda} \int_0^{\infty} e^{-2x} \cos \lambda x \, \partial \lambda x =$$

$$= \sqrt{\frac{2}{\pi}} \cdot \frac{1}{\lambda} \int_0^{\infty} e^{-2x} \partial \sin \lambda x = \sqrt{\frac{2}{\pi}} \frac{1}{\lambda} \Big(\underbrace{e^{-2x} \sin \lambda x \Big|_0^{\infty}}_{0} - \int_0^{\infty} \sin \lambda x \, \partial e^{-2x} \Big) =$$

$$= \frac{1}{\lambda} \sqrt{\frac{2}{\pi}} \int_0^{\infty} \sin \lambda x \, e^{-2x} \partial x = \sqrt{\frac{2}{\pi}} \cdot \frac{(-2)}{\lambda} \int_0^{\infty} e^{-2x} \partial \cos \lambda x =$$

$$= \frac{-2}{\lambda^2} \sqrt{\frac{2}{\pi}} \Big(e^{-2x} \cos \lambda x \Big|_0^{\infty} - \int_0^{\infty} \cos \lambda x \, \partial e^{-2x} \Big) =$$

$$= \frac{-2}{\lambda^2} \sqrt{\frac{2}{\pi}} \Big(-1 + 2 \int_0^{\infty} e^{-2x} \cos \lambda x \, \partial x = \frac{2}{\lambda^2} \sqrt{\frac{2}{\pi}} \; \frac{-4}{\lambda^2} \; \hat{f}_c \, x$$

$$\hat{f}_c(\lambda) = \frac{2}{\lambda^2} \sqrt{\frac{2}{\pi}} - \frac{4}{\lambda^2} \hat{f}_c(\lambda)$$

$$\frac{\lambda^2 + 4}{\lambda^2} \hat{f}_c(\lambda) = \frac{2}{\lambda^2} \sqrt{\frac{2}{\pi}}$$

$$f(x) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} \sqrt{\frac{2}{\pi}} \cdot \frac{2}{\lambda^2 + 4} \cos \lambda x \, \partial x =$$

$$= \frac{4}{\pi} \int_0^{\infty} \frac{\cos \lambda x}{\lambda^2 + 4} \partial x$$

(3) $x(n) = \left(\frac{2}{3}\right)^n u(n)$

$$x_e(n) = \frac{1}{2}\left[\left(\frac{2}{3}\right)^n u(n) + \left(\frac{2}{3}\right)^{-n} u(-n)\right] = \begin{cases} \frac{1}{2}\left(\frac{2}{3}\right)^n & \text{за } n>0 \\ 1 & \text{за } n=0 \\ \frac{1}{2}\left(\frac{2}{3}\right)^{-n} & \text{за } n<0 \end{cases}$$

$$x_e = \frac{\left(\frac{2}{3}\right)^{|n|} + \delta n}{2}$$

$$x_o(n) = \frac{1}{2}\left[\left(\frac{2}{3}\right)^n u(n) - \left(\frac{2}{3}\right)^{-n} u(-n)\right] = \begin{cases} \frac{1}{2}\left(\frac{2}{3}\right)^n & \text{за } n>0 \\ 0 & \text{за } n=0 \\ -\frac{1}{2}\left(\frac{2}{3}\right)^{-n} & \text{за } n<0 \end{cases}$$

$$x_o = \frac{1}{2}\left(\frac{2}{3}\right)^{|n|} \operatorname{sgn}(n)$$

$$S = \sum_{-\infty}^{+\infty} x(n) = \sum_{-\infty}^{+\infty}\left[x_e(n) + x_o(n)\right] =$$

$$= \sum_{-\infty}^{+\infty} x_e(n) + \sum_{-\infty}^{+\infty} x_o(n)$$

$$\sum_{-\infty}^{+\infty} x_e(n) = \frac{1}{2}\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n + 1 + \frac{1}{2}\sum_{n=-\infty}^{-1}\left(\frac{2}{3}\right)^{-n} =$$

за ~~m~~ = −n

$$= \frac{1}{2}\left(\sum_{n=0}^{\infty}\left(\frac{2}{3}\right)^n - 1\right) + 1 + \frac{1}{2}\left(\sum_{m=0}^{\infty}\left(\frac{2}{3}\right)^m - 1\right) =$$

$$= \frac{1}{2}\left(\frac{1}{1-\frac{2}{3}} - 1\right) + 1 + \frac{1}{2}\left(\frac{1}{1-\frac{2}{3}} - 1\right) =$$

$$= \frac{1}{\cancel{2}}\cdot\cancel{2} + 1 + \frac{1}{\cancel{2}}\cdot\cancel{2} = 3$$

$$\sum_{-\infty}^{+\infty} x_o(n) = \frac{1}{2}\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n + 0 - \frac{1}{2}\left(\sum_{n=-\infty}^{-1}\left(\frac{2}{3}\right)^{-n}\right) =$$

$m = -n$

$$= \frac{1}{2}\sum_{n=0}^{\infty}\left(\frac{2}{3}\right)^n - 0 + 0 - \frac{1}{2}\sum_{m=0}^{\infty}\left(\frac{2}{3}\right)^m =$$

$$= \frac{1}{2}\cdot 2 + 0 - \frac{1}{2}\cdot 2 = 0$$

$$S = 3 + 0 = 3$$

б) $P = \sum_{-\infty}^{+\infty} x^2(n) = \sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^{2n}$

$$P = \frac{1}{1-\frac{4}{9}} = \frac{9}{5}$$

в) $P_y = \sum_{n=-\infty}^{\infty} y^2(n) = \sum_{-\infty}^{+\infty} [n\,x(n)]^2 =$

$$= \sum_{n=0}^{\infty} n^2 \left(\frac{4}{9}\right)^n$$

$$\sum_{n=0}^{\infty} n a^n = \frac{a}{(1-a)^2} \qquad |a| < 0$$

$$\sum_{n=0}^{\infty} n^2 a^n = \frac{d}{da} \frac{a}{(1-a)^2} = \frac{1+a}{1-a^3}$$

$$\sum_{n=0}^{\infty} n^2 a^n = \frac{a(1+a)}{(1-a)^3}$$

$$P_y = \frac{\left(\frac{4}{9}\right)\left(\frac{13}{9}\right)}{\left(\frac{5}{9}\right)^3} = \frac{468}{125}$$

4) $\begin{cases} y(n) - y(n-1) + y(n-2) = x(n) - x(n-1) \\ x(n) = \left(\frac{1}{2}\right)^n u(n), \quad y(-1) = 0, \quad y(-2) = 1 \end{cases}$

$y_h(n)$

$z^n - z^{n-1} + z^{n-2} = 0 \;/\; z^{n-2}$

$z^2 - z + 1 = 0$

$D = b^2 - 4ac - 1 - 4.1.1 = -3$

$z_{1/2} = \frac{1 \pm i\sqrt{3}}{2}$

$y_h(n) = c_1 \cdot \left(\frac{1 - i\sqrt{3}}{2}\right)^n + c_2 \left(\frac{1 + i\sqrt{3}}{2}\right)^n$

$y(0) - y(-1) + y(-2) = x(0) - x(-1)$

$y(0) - 0 + 1 = 1$

$y(0) = 0$

$y(1) - y(0) + y(-1) = x(1) - x(0)$

$y(1) = -\frac{1}{2}$

$y_p(n) = c \cdot \left(\frac{1}{2}\right)^n$

$c\left(\frac{1}{2}\right)^n = c\left(\frac{1}{2}\right)^{n-1} + c\left(\frac{1}{2}\right)^{n-2} = \left(\frac{1}{2}\right)^n - \left(\frac{1}{2}\right)^{n-1} \;/ : \left(\frac{1}{2}\right)^{n-2}$

$\frac{c}{4} - \frac{c}{2} + c = \frac{1}{4} - \frac{1}{2}$

$\frac{3}{4}c = -\frac{1}{4} \Rightarrow c = -\frac{1}{3}$

$$y(n) = C_1\left(\frac{1-i\sqrt{3}}{2}\right)^n + C_2\left(\frac{1+i\sqrt{3}}{2}\right)^n - \frac{1}{3}\cdot\left(\frac{1}{2}\right)^n$$

$$y(0) = C_1 + C_2 - \frac{1}{3} = 0 \Rightarrow C_1 + C_2 = \frac{1}{3}$$

$$y(1) = C_1\left(\frac{1-i\sqrt{3}}{2}\right) + C_2\left(\frac{1+i\sqrt{3}}{2}\right) - \frac{1}{6} = -\frac{1}{2}$$

$$C_1\left(\frac{1-i\sqrt{3}}{2}\right) + C_2\left(\frac{1+i\sqrt{3}}{2}\right) = -\frac{4}{6} = -\frac{1}{3}$$

$$\begin{cases} C_1 + C_2 = \frac{1}{3} \\ C_1\left(\frac{1-i\sqrt{3}}{2}\right) + C_2\left(\frac{1+i\sqrt{3}}{2}\right) = -\frac{1}{3} \end{cases}$$

$$C_1 = -C_2 + \frac{1}{3}$$

$$-C_2\left(\frac{1-i\sqrt{3}}{2}\right) + \frac{1}{3}\cdot\frac{1-i\sqrt{3}}{2} + C_2\frac{1+i\sqrt{3}}{2} = -\frac{1}{3}$$

$$C_2\left(\frac{\cancel{-1} + i\sqrt{3} + \cancel{1} + i\sqrt{3}}{\cancel{2}}\right) = -\frac{1}{3} - \frac{1}{3}\cdot\frac{1-i\sqrt{3}}{2}$$

$$C_2\, i\sqrt{3} = -\frac{1}{3} - \frac{1}{3}\cdot\frac{1-i\sqrt{3}}{2} = -\frac{2}{6} - \frac{1-i\sqrt{3}}{6} = \frac{-3+i\sqrt{3}}{6}$$

$$C_2 = \frac{1+i\sqrt{3}}{6}$$

$$C_1 = \frac{1}{3} - \frac{1+i\sqrt{3}}{6} = \frac{1-i\sqrt{3}}{6}$$

$$y(n) = \left(\frac{1-i\sqrt{3}}{6}\right)\cdot\left(\frac{1-i\sqrt{3}}{2}\right)^n + \left(\frac{1+i\sqrt{3}}{6}\right)\left(\frac{1+i\sqrt{3}}{2}\right)^n - \frac{1}{3}\cdot\left(\frac{1}{2}\right)^n$$

(5) $\{x(nT)\}_{n=0}^{3} = \{1, 0, 1, 1\}$

ДПФ

$$X(0) = \sum_{n=0}^{3} x(nT) e^{-i0} = 1 + 0 + 1 + 1 = 3$$

$$X(\Omega) = \sum_{n=0}^{3} x(nT) e^{-in\frac{\pi}{2}} = 1 - 1 + i = i$$

$$X(2\Omega) = \sum_{n=0}^{3} x(nT) e^{-i\pi n} = 1 + 1e^{-i2\pi} + 1e^{-i3\pi} =$$

$$= 1 + 1 - 1 = 1$$

$$X(3\Omega) = \sum_{n=0}^{3} x(nT) e^{-i\frac{3\pi}{2}n} = 1 + 1e^{-i3\pi} + 1e^{-i\frac{3\pi}{2}} =$$

$$= 1 - 1 - i = -i$$

$$\{X(k\Omega)\}_{k=0}^{3} = \{3, i, 1, -i\}$$

ОДПФ

$$x(nT) = \frac{1}{N} \sum_{k=0}^{N-1} X(k\Omega) e^{ik\Omega nT}$$

$$x(0) = \frac{1}{4} \sum_{k=0}^{3} X(k\Omega) e^{0} = \frac{1}{4}(3 + i + 1 - i) = \frac{1}{4} \cdot 4 = 1$$

$$x(T) = \frac{1}{4} \sum_{k=0}^{3} x(k\Omega)\, e^{i\frac{k\pi}{2}} =$$

$$= \frac{1}{4}\left(3 + i e^{\frac{i\pi}{2}} + 1 e^{i\pi} - i e^{i\frac{3\pi}{2}}\right) =$$

$$= \frac{1}{4}(3 - 1 - 1 - 1) = 0$$

$$x(2T) = \frac{1}{4} \sum_{k=0}^{3} x(k\Omega)\, e^{ik\pi} =$$

$$= \frac{1}{4}\left(3 + i e^{i\pi} + e^{i2\pi} - i e^{i3\pi}\right) =$$

$$= \frac{1}{4}(3 - \cancel{i} + 1 + \cancel{i}) = \frac{1}{4} \cdot 4 = 1$$

$$x(3T) = \frac{1}{4} \sum_{k=0}^{3} x(k\Omega)\, e^{i\frac{3k\pi}{2}} =$$

$$= \frac{1}{4}\left(3 + i e^{i\frac{3\pi}{2}} + e^{i 3\pi} - i e^{i\frac{9\pi}{2}}\right) =$$

$$= \frac{1}{4}(3 + 1 - 1 + 1) = \frac{1}{4}\, 4 = 1$$

$$\{x(nT)\}_{n=0}^{3} = \{1, 0, 1, 1\}$$

Персевал

$$\sum_{n=0}^{3} x^2(nT) = \frac{1}{4}\sum_{k=0}^{3} |X(k\Omega)|^2$$

$$\sum_{n=0}^{3} x^2(nT) = 1^2 + 0 + 1^2 + 1^2 = 3$$

$$\sum_{n=0}^{3} |X(k\Omega)|^2 = 3^2 + \left(\sqrt{1^2}\right)^2 + 1^2 + \left(\sqrt{1-4^2}\right)^2 =$$

$$= 9 + 1 + 1 + 1$$

$$3 = \frac{12}{4}$$

$$3 = 3$$

(6) $X_2(k\Omega) = \sum_{n=0}^{N-1} x_2(nT)\, e^{-ik\Omega nT}$

$N = 6$

$\Omega = \frac{2\pi}{NT} = \frac{2\pi}{6T} = \frac{\pi}{3T}$; $e^{-ik\frac{\pi}{3T}\cdot nT} = e^{-i\frac{k\pi n}{3}}$

$X_2(k\Omega) = \sum_{n=0}^{5} x_2(nT)\, e^{-i\frac{kn\pi}{3}}$

$X_2(0) = \sum_{n=0}^{5} x_2(nT)\, e^{-i\frac{kn\pi 0}{3}}$

$X_2(0) = 0_4 - 2 \cdot - 1 + 0 + 1 + 1 = -1$

$X_2(1) = 0 - 2e^{-i\frac{\pi}{3}} - e^{-i\frac{2\pi}{3}} + 0 + 1e^{-i\frac{4\pi}{3}} + e^{-i\frac{5\pi}{3}} =$ 2

$= -2\,\frac{1 - i\sqrt{3}}{2} - \left(\frac{-1 - i\sqrt{3}}{2}\right) + \left(\frac{-1 + i\sqrt{3}}{2}\right) + \left(\frac{1 + i\sqrt{3}}{2}\right)$

$= \frac{-2 + i2\sqrt{3} + 1 + i\sqrt{3} - 1 + \sqrt{3} + 1 + i\sqrt{3}}{2} = \frac{-1 - i5\sqrt{3}}{2}$

$X_2(2) = \sum_{n=0}^{5} x(nT)\, e^{-i\frac{2n\pi}{3}} =$

$= -2e^{-i\frac{2\pi}{3}} - e^{-i\frac{4\pi}{3}} - e^{-i\frac{8\pi}{3}} + e^{-i\frac{10\pi}{3}} =$

$=$

$$= -2\left(\frac{-1-i\sqrt{3}}{2}\right) - \left(\frac{1+i\sqrt{3}}{2}\right) + \left(\frac{1+i\sqrt{3}}{2}\right) + \left(\frac{-1-i\sqrt{3}}{2}\right) =$$

$$= \frac{2+i2\sqrt{3} - 1 - i\sqrt{3} + 1 + i\sqrt{3} - 1 - i\sqrt{3}}{2} = \frac{1+i\sqrt{3}}{2}$$

$$X_2(3) = \sum_{n=0}^{5} x(nT) e^{-in\pi} = -2e^{-\pi} - 1e^{-2\pi} + e^{-5\pi} =$$

$$= (-2)(-1) - 1 + 1 - 1 = 1$$

$$X_2(4) = \sum_{n=0}^{5} x(nT) e^{-i\frac{4n\pi}{3}} =$$

$$= -2e^{-i\frac{4\pi}{3}} - 1e^{-i\frac{8\pi}{3}} + 1e^{-i\frac{16\pi}{3}} + e^{-i\frac{20\pi}{3}} =$$

$$= -2\left(\frac{-1+i\sqrt{3}}{2}\right) - \left(\frac{-1-i\sqrt{3}}{2}\right) + \frac{-1+i\sqrt{3}}{2} + \frac{-1-i\sqrt{3}}{2} =$$

$$= \frac{2-i2\sqrt{3} + 1 + i\sqrt{3} - 1 + i\sqrt{3} - 1 - i\sqrt{3}}{2} = \frac{1-i\sqrt{3}}{2}$$

$$X_2(5) = \sum_{n=0}^{5} x(nT) e^{-\frac{5n\pi}{3}} =$$

$$= -2e^{-i\frac{5\pi}{3}} - e^{-i\frac{10\pi}{3}} + e^{-i\frac{20\pi}{3}} + e^{-i\frac{25\pi}{3}} =$$

$$= \frac{-2 - i2\sqrt{3} + 1 - i\sqrt{3} - 1 - i\sqrt{3} + 1 - i\sqrt{3}}{2} =$$

$$= \frac{-1-i5\sqrt{3}}{2}$$

$$\left\{X_2(k\Omega)\right\}_{k=0}^{5} = \left\{-1, \frac{-1+5i\sqrt{3}}{2}, \frac{1+i\sqrt{3}}{2}, 1, \frac{1-\sqrt{3}}{2}, \frac{-1-5i\sqrt{3}}{2}\right\}$$

$$r_{1/2}(n) = \frac{1}{N} F_D^{-1} \left[\underbrace{X_1(k\Omega) X_2(k\Omega)}_{Y(k\Omega)} \right]$$

$$X(k) = \left\{ 1\cdot(-1), \frac{5+i\sqrt{3}}{2} \cdot \frac{-1+i5\sqrt{3}}{2}, \frac{-1+i\sqrt{3}}{2} \cdot \frac{1-i\sqrt{3}}{2}, 1, \frac{-1-i\sqrt{3}}{2} \cdot \frac{1-i\sqrt{3}}{2}, \right.$$
$$\left. \frac{5-i\sqrt{3}}{2} \cdot \frac{-1-5i\sqrt{3}}{2} \right\}$$

$$X(k) = \{-1, -5, +6i\sqrt{3}, -1, 1, -1, -5-6i\sqrt{3}\}$$

$$F_D^{-1} = \frac{1}{6} \sum_{k=0}^{5} e^{\frac{ikn\pi}{3}}$$

$$y(0) = \frac{1}{6} \sum_{k=0}^{5} Y(k) = \frac{1}{6}(-1-5+i6\sqrt{3}-1+1-1-5-i6\sqrt{3}) = \frac{1}{6}\cdot(-12) = -2$$

$$y(1) = \frac{1}{6} \sum_{k=0}^{5} Y(k)\, e^{\frac{ik\pi}{3}} =$$

$$= \frac{1}{6}\left(-1 + (-5+i6\sqrt{3})e^{\frac{i\pi}{3}} + (-1)e^{\frac{i2\pi}{3}} + e^{i\pi} + (-1)e^{-i\frac{4\pi}{3}} + (-5-6i\sqrt{3})\cdot\right.$$
$$\left.\cdot e^{\frac{i5\pi}{3}}\right) =$$

$$= \frac{1}{6}\left(-1 + \frac{-23+i\sqrt{3}}{2} + \frac{1-i\sqrt{3}}{2} - 1 + \frac{1-i\sqrt{3}}{2} + \frac{-23-i\sqrt{3}}{2}\right) =$$

$$= \frac{1}{6}\left(-2 + \frac{-44}{2}\right) = -\frac{24}{6} = -4$$

$$y(2) = \frac{1}{6}\sum_{k=0}^{5} Y(k)\, e^{i\frac{2k\pi}{3}} =$$

$$= \frac{1}{6}\Big(-1 + (-5 + i\sqrt{3})e^{i\frac{2\pi}{3}} + (-1)e^{i\frac{4\pi}{3}} - e^{i\frac{6\pi}{3}} + (-1)e^{-i\frac{8\pi}{3}} +$$

$$+(-5 - i6\sqrt{3})e^{i\frac{10\pi}{3}}\Big) =$$

$$= \frac{1}{6}\Big(-1 + \frac{-13 - i11\sqrt{3}}{2} + \frac{1 + i\sqrt{3}}{2} + 1 + \frac{1 - i\sqrt{3}}{2} + \frac{-13 + i11\sqrt{3}}{2}\Big) =$$

$$= \frac{1}{6}\Big(-\frac{24}{2}\Big) = -2$$

$$y(3) = \frac{1}{6}\sum_{k=0}^{5} Y(k)\, e^{i\pi} =$$

$$= \frac{1}{6}\Big(-1 + (-5 + i6\sqrt{3})e^{i\pi} - e^{i2\pi} + e^{i3\pi} - e^{i4\pi} + (-5 - i6\sqrt{3})e^{i5\pi}\Big) =$$

$$= \frac{1}{6}\Big(-1 + 5 - i6\sqrt{3} - 1 - 1 - 1 + 5 + i6\sqrt{3}\Big) = \frac{1}{6}\cdot 6 = \underline{1}$$

$$y(4) = \frac{1}{6}\sum_{k=0}^{5} Y_k\, e^{i\frac{4k\pi}{3}} =$$

$$= \frac{1}{6}\Big(-1 + (-5 + i6\sqrt{3})e^{i\frac{4\pi}{3}} - e^{i\frac{8\pi}{3}} + e^{i4\pi} + (-1)e^{i\frac{16\pi}{3}} +$$

$$+(-5 - i6\sqrt{3})e^{i\frac{20\pi}{3}}\Big) =$$

$$= \frac{1}{6}\Big(-1 + \frac{23 - i\sqrt{3}}{2} + \frac{1 - i\sqrt{3}}{2} + 1 + \frac{1 + i\sqrt{3}}{2} + \frac{23 + i\sqrt{3}}{2}\Big) =$$

$$= \frac{1}{6}\cdot\frac{48}{2} = 4$$

$$y(5) = \frac{1}{6}\sum_{k=0}^{5} Y(k)\, e^{\frac{i5k\pi}{3}} =$$

$$= \frac{1}{6}\left(-1 + (5 + i6\sqrt{3})e^{\frac{i5\pi}{3}} + (-1)e^{\frac{i10\pi}{3}} + (-1)e^{\frac{i20\pi}{3}} + (-5 - i6\sqrt{3})e^{\frac{i25\pi}{3}}\right) =$$

$$= \frac{1}{6}\left(-1 + \frac{13 + i11\sqrt{3}}{2} + \frac{1 + i\sqrt{3}}{2} - 1 + \frac{1 - i\sqrt{3}}{2} + \frac{13 - i11\sqrt{3}}{2}\right) =$$

$$= \frac{1}{6}\left(-2 + \frac{28}{2}\right) = \frac{1}{6}\cdot 12 = 2$$

$$r_{12}(n) = \left\{-\frac{2}{6}; -\frac{4}{6}; -\frac{2}{6}; \frac{1}{6}; \frac{4}{6}; \frac{2}{6}\right\} =$$

$$= \left\{-\frac{1}{3}; -\frac{2}{3}; -\frac{1}{3}; \frac{1}{6}; \frac{2}{3}; \frac{1}{3}\right\}$$

$$(x_1 * x_2)n = F_D^{-1}\left[x_1(k)\, x_2(k)\right]$$

$$(x_1 * x_2)n = \{-2; -4; -2; 1; 4; 2\}$$