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ММЦО

1.

а.) $f(x) = x\left(\frac{\pi}{2} - x\right) \quad x \in \left[0; \frac{\pi}{2}\right]$ $\quad F(-x) = F(x)$

$F(x+2l) = F(x)$

$$b_n = \frac{1}{l}\int_{-l}^{l} F(x)\sin\frac{n\pi x}{l}\,dx \qquad b_n = \frac{2}{l}\int_0^{l} f(x)\sin\frac{n\pi x}{l}\cdot dx$$

$$b_n = \frac{2}{\frac{\pi}{2}}\int_0^{\pi/2} x\left(\frac{\pi}{2} - x\right)\sin\frac{n\pi x}{\frac{\pi}{2}}\,dx = \frac{4}{\pi}\int_0^{\pi/2}\left(x\frac{\pi}{2} - x^2\right)\sin 2nx\,dx =$$

$$= \frac{4}{\pi}\Big(\underbrace{\int_0^{\pi/2} x\frac{\pi}{2}\sin 2nx\,dx}_{A} - \underbrace{\int_0^{\pi/2} x^2\sin 2nx\,dx}_{B}\Big)$$

A.1

$$2\int_0^{\pi/2} x\sin nx\,dx = \frac{2}{2n}\left(x(-\cos 2nx)\right)\Big|_0^{\frac{\pi}{2}} - \int_0^{\pi/2}(-\cos 2nx)\,dx) =$$

$$= \frac{1}{n}\left(x(-\cos 2nx)\right)\Big|_0^{\frac{\pi}{2}} + \underbrace{\frac{1}{2n}(\sin n\pi - \sin 0)}_{0}\Big) =$$

$$= -\frac{1}{n}\left(\frac{\pi}{2}(-1)^n\right) = -\frac{\pi}{2n}(-1)^n \Rightarrow 2$$

$$= \frac{\pi}{2n}(-1)^{n+1}$$

B.1

$$= \frac{4}{\pi}\int_0^{\pi/2} x^2\sin 2nx\,dx = -\frac{4}{\pi}\cdot\frac{1}{2n}\int_0^{\pi/2} x^2\,d(-\cos 2nx) =$$

$$= -\frac{2}{n\pi}\Big(x^2(-\cos 2nx)\Big|_0^{\frac{\pi}{2}} + \underbrace{\int_0^{\pi/2}\cos(2nx\cdot 2x\,dx}_{C}\Big) \Rightarrow 1$$

8 c.) $\int_0^{\pi/2} \cos(2nx)\,2x\,dx = 2\int_0^{\pi/2} \cos(2nx)\,x\,dx = \frac{2}{2n}\int_0^{\pi/2} \cos(2nx)\cdot x\,d2nx =$

$= \frac{1}{n}\int_0^{\pi/2} x\,d\sin 2nx = \frac{1}{n}\left(x\sin 2nx\Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sin 2nx\,dx\right) =$

$= \frac{1}{n}\left(x\sin 2nx\Big|_0^{\frac{\pi}{2}} + \frac{1}{2n}(\cos 2nx)\Big|_0^{\frac{\pi}{2}}\right)$

враховуючи це є (1)

$= \frac{2}{n\pi}\left(x^2(-\cos 2nx)\Big|_0^{\frac{\pi}{2}} + \frac{1}{n}\left[x\cdot\sin 2nx\Big|_0^{\frac{\pi}{2}} + \frac{1}{2n}(\cos 2nx)\Big|_0^{\frac{\pi}{2}}\right]\right) =$

$= -\frac{2}{n\pi}\left(\frac{\pi^2}{4}(-\cos n\pi) + \frac{1}{n}\left[\underbrace{\frac{\pi}{2}\sin n\pi}_{0} + \frac{1}{2n}(\cos n\pi - 1)\right]\right) =$

$= -\frac{2}{n\pi}\left(\frac{\pi^2}{4}(-\cos n\pi) + \frac{1}{2n^2}(\cos n\pi - 1)\right) =$

$= -\frac{2}{n\pi}\left[-\frac{\pi^2}{4}(-1)^n + \frac{1}{2n^2}((-1)^n - 1)\right]$ (3)

от (2) и (3)

$\Rightarrow -\frac{2}{n\pi}\left[-\frac{\pi^2}{4}(-1)^n + \frac{1}{2n^2}((-1)^n - 1)\right] + \frac{\pi}{2n}(-1)^{n+1} =$

$= \frac{\cancel{2}^{1}}{n\cancel{\pi}}\cdot\frac{\pi^{\cancel{2}}}{\cancel{4}\,2}(-1)^n - \frac{\cancel{2}}{n\cdot\pi}\cdot\frac{1}{\cancel{2}n^2}((-1)^n - 1) + \frac{\pi}{2n}(-1)^{n+1} =$

$= \frac{\pi}{2}(-1)^n - \frac{1}{n^3\pi}((-1)^n - 1) + \frac{\pi}{2n}(-1)^{n+1}$

8.) $f(x) = \sin\left(\frac{x}{\sqrt{2}}\right)$ $-\pi \le x \le \pi$; $\sum_{n=1}^{\infty} \frac{n^2}{(2n^2-1)^2}$

$a_0 = \frac{2}{\pi}\int_0^{\pi} \sin\left(\frac{x}{\sqrt{2}}\right)dx = \frac{2\sqrt{2}}{\pi}\int_0^{\pi} \sin\left(\frac{x}{\sqrt{2}}\right)\frac{dx}{\sqrt{2}} = \frac{2\sqrt{2}}{\pi}\left(-\cos\left(\frac{x}{\sqrt{2}}\right)\right)\Big|_0^{\pi} =$

$= -\frac{2\sqrt{2}}{\pi}\left(\cos\left(\frac{x}{\sqrt{2}}\right)\right)\Big|_0^{\pi} = -\frac{2\sqrt{2}}{\pi}\left(\cos\frac{\pi}{\sqrt{2}} - \cos 0\right) = -\frac{2\sqrt{2}}{\pi}\left(\cos\frac{\pi}{\sqrt{2}} - 1\right)$

$$a_n = \frac{2}{\pi}\int_0^{\pi} \sin\left(\frac{x}{\sqrt{2}}\right)\cos nx\,dx \Rightarrow$$

$$\Rightarrow \frac{1}{2}\cdot\frac{2}{n}\Bigg[\underbrace{\int_0^{\pi}\sin\left(\frac{x}{\sqrt{2}}+nx\right)dx}_{A} + \underbrace{\int_0^{\pi}\sin\left(\frac{x}{\sqrt{2}}-nx\right)dx}_{B}\Bigg]$$

A.) $$\frac{1}{\pi}\int_0^{\pi}\sin x\left(\frac{1+\sqrt{2}\cdot n}{\sqrt{2}}\right)dx = \frac{\sqrt{2}}{(1+\sqrt{2}\cdot n)\cdot\pi}\int_0^{\pi}\sin x\left(\frac{1+n\sqrt{2}}{\sqrt{2}}\right)dx\cdot\left(\frac{1+\sqrt{2}\cdot n}{\sqrt{2}}\right) =$$

$$= -\frac{\sqrt{2}}{(1+\sqrt{2}\cdot n)\cdot\pi}\left(\cos x\left(\frac{1+\sqrt{2}\cdot n}{\sqrt{2}}\right)\right)\Bigg|_0^{\pi} =$$

$$= \frac{-\sqrt{2}}{(1+\sqrt{2}\cdot n)\pi}\left(\cos\left(\frac{\pi}{\sqrt{2}}+\pi\cdot n\right)-1\right)$$

B.) $$\frac{1}{\pi}\int_0^{\pi}\sin x\left(\frac{1-\sqrt{2}\cdot n}{\sqrt{2}}\right)dx = \frac{-\sqrt{2}}{(1-\sqrt{2}\cdot n)\cdot\pi}\left(\cos x\left(\frac{1-\sqrt{2}\cdot n}{\sqrt{2}}\right)\right)\Bigg|_0^{\pi} =$$

$$= -\frac{\sqrt{2}}{(1-\sqrt{2}\,n)\cdot\pi}\left(\cos\left(\frac{\pi}{\sqrt{2}}-\pi\cdot n\right)-1\right)$$

$$a_n = -\frac{\sqrt{2}}{(1+\sqrt{2}\cdot n)\pi}\left(\cos\frac{\pi}{2}+\cos\pi\cdot n+1\right) - \frac{\sqrt{2}}{(1-\sqrt{2}\cdot n)\cdot\pi}\left(\cos\frac{\pi}{\sqrt{2}}-\cos n\pi+1\right)$$

$$\frac{1}{\pi}\int_{-\pi}^{\pi} f^2(x)\,dx = \frac{2}{\pi}\int_{-\pi}^{\pi}\sin^2\frac{x}{\sqrt{2}}\cdot dx = \frac{2}{\pi}\int_0^{\pi}\frac{1-\cos 2\frac{x}{\sqrt{2}}}{2}\,dx =$$

$$= \frac{1}{\pi}\int_0^{\pi} 1-\cos 2\frac{x}{\sqrt{2}}\,dx = x - \frac{\sqrt{2}}{2\pi}\left(\sin\frac{2x}{\sqrt{2}}\right)\Bigg|_0^{\pi} = 1 - \frac{\sqrt{2}}{2\pi}\cdot\sin\frac{2\pi}{\sqrt{2}}$$

$$\frac{1}{\pi}\int_{-\pi}^{\pi} f^2(x)\,dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty} a_n^2 + b_n^2$$

2. $\underbrace{\int_0^\infty g(z)\sin(zx)\,dz}_{\hat{g}(s)=\frac{\pi}{2}\sin s} = f(x), \quad f(x)=\begin{cases}\frac{\pi}{2}\sin x \Rightarrow x\in[0;\pi]\\ 0\ ,\ x>\pi\end{cases}$

$$g(x)=\int_0^\infty \hat{g}(\lambda)\sin\lambda x\cdot d\lambda=\frac{\pi}{2}\left(\int_0^\pi \cos(\lambda-\lambda x)\,d\lambda-\int_0^\pi \cos(\lambda+\lambda x)\,d\lambda\right)=$$

$$=\frac{\pi}{2}\left(\frac{\sin\lambda(1-x)}{(1-x)}\Big|_0^\pi-\frac{\sin\lambda(1+x)}{(1+x)}\Big|_0^\pi\right)=$$

$$=\frac{\pi}{2}\left(\frac{\sin(\pi-\pi x)}{(1-x)}-\frac{\sin(\pi+\pi x)}{(1+x)}\right)=\frac{\pi}{2}\left(\frac{\sin\pi x}{(1-x)}+\frac{\sin\pi x}{(1+x)}\right)=$$

$$=\frac{\pi}{2}\sin\pi x\left(\frac{1}{1-x}+\frac{1}{1+x}\right)=\frac{\pi\cdot\sin\pi x}{(1-x^2)}$$

3. $x(n)=\left(\frac{4}{3}\right)^n\cdot u(n)$

a.) $S=\sum_{n=0}^{\infty}\left(\frac{4}{3}\right)^{-n}=\sum_{n=0}^{\infty}\left(\frac{3}{4}\right)^n \Rightarrow S=\frac{1}{1-\frac{3}{4}}=4$

b.) $P=\sum_{n=-\infty}^{n=\infty}x^2(n)=\sum_{n=-\infty}^{0}\left(\frac{4}{3}\right)^{2n}$

$$P=\sum_{n=0}^{\infty}\left(\frac{4}{3}\right)^{-2n}=\sum_{n=0}^{\infty}\left(\frac{3}{4}\right)^{2n}=\frac{1}{1-\frac{9}{16}}=\frac{16}{7}$$

b.) $P_y=\sum_{n=-\infty}^{n=\infty}y^2(n) \qquad y(n)=nx(n)$

$P=\sum_{n=-\infty}^{n=\infty}[n\cdot x(n)]^2=\sum_{n=0}^{\infty}n^2\left(\frac{9}{16}\right)^n, \quad \sum_{n=0}^{\infty}n\cdot a^n=\frac{a}{(1-a)^2} \qquad |a|<1$

$$\sum_{n=0}^{\infty}n^2\cdot a^{n-1}=\frac{d}{da}\cdot\frac{a}{(1-a)^2}=\frac{1+a}{(1-a)^3}$$

$$\sum_{n=0}^{\infty}n^2\cdot a^n=\frac{a\cdot(1+a)}{(1-a)^3}$$

$$P=\sum_{n=0}^{\infty}n^2\left(\frac{9}{16}\right)^n=\frac{\left(\frac{9}{16}\right)\cdot\left(1+\frac{9}{16}\right)}{\left(1-\frac{9}{16}\right)^3}=\frac{144}{5}$$

4.

$$\begin{cases} y(n)-\frac{1}{4}y(n-1)-\frac{1}{8}y(n-2)=\underbrace{x(n)-x(n-1)}_{\delta(n)} \\ x(n)=\left(\frac{1}{4}\right)^n\cdot u(n) \quad n\geq 0 \quad u(n)=1 \\ y(-1)=0,\ y(-2)=2 \end{cases}$$

$$y_p(n)=C_1\cdot\left(\frac{1}{4}\right)^n=\frac{3}{2}\cdot\left(\frac{1}{4}\right)^n$$

$$z^2-\frac{1}{4}z-\frac{1}{8}=0$$

$$z_{1,2}=\frac{1}{2},\left(-\frac{1}{4}\right)$$

$$y_h(n)=C_1\cdot\left(\frac{1}{2}\right)^n+C_2\cdot\left(-\frac{1}{4}\right)^n$$

$$C_1\left(\frac{1}{4}\right)^n-\frac{1}{4}C_1\left(\frac{1}{4}\right)^{n-1}-\frac{1}{8}C_1\cdot\left(\frac{1}{4}\right)^{n-2}=\frac{1}{4}^n-\frac{1}{4}^{n-1} \quad \Big/ \frac{1}{4}^n$$

$$C_1-C_1-2C_1=1-4$$

$$-2C_1=-3 \Rightarrow C_1=\frac{3}{2}$$

$y(-1)=0 \quad y(-2)=2$

$$y(n)=C_1\cdot\left(\frac{1}{2}\right)^n+C_2\left(-\frac{1}{4}\right)^n+\frac{3}{2}\cdot\left(\frac{1}{4}\right)^n$$

$$y(-1)=C_1\cdot\left(\frac{1}{2}\right)^{-1}+C_2\left(-\frac{1}{4}\right)^{-1}+\frac{3}{2}\left(\frac{1}{4}\right)^{-1}$$

$$y(-1)=2C_1-4C_2+6$$

$$0=2C_1-4C_2+6$$

$$2C_1-4C_2=-6$$

$$y(-2)=C_1\left(+\frac{1}{2}\right)^{-2}+C_2\cdot\left(-\frac{1}{4}\right)^{-2}+\frac{3}{2}\left(\frac{1}{4}\right)^{-2}$$

$$2=4C_1-16C_2+24$$

$$4C_1-16C_2=-22$$

$$\begin{cases} 2C_1-4C_2=-6/2 \\ 4C_1-16C_2=-22/2 \end{cases}$$

$$\begin{cases} C_1-2C_2=-3 \\ 2C_1-8C_1=-11 \end{cases} \qquad C_1=-3+2C_2$$

$2\cdot(-3+2C_2)-8C_2=-11$

$-6+4C_2-8C_2=-11$

$-4C_2=-5 \quad C_2=\frac{5}{4}$

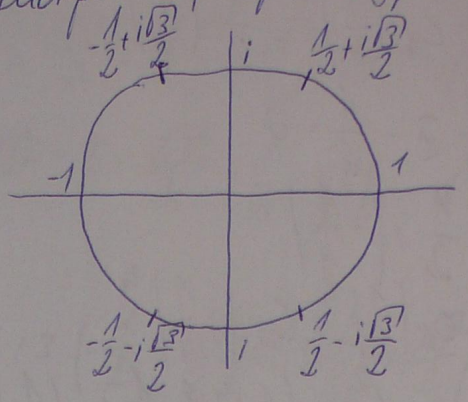
$C_1=-3+2C_2$

$C_1=-3+\frac{\cancel{2}^{1}\cdot 5}{\cancel{4}\,2}$

$C_1=-\frac{1}{2}$

5.

Дискретно преобр. на Фурие



$\{x(nT)\}_{n=0}^{5}=\{1,-1,0,1,0,-1\}$

$x(k\Omega)=\sum_{n=0}^{5}x(nT)\cdot e^{-ik\frac{\pi}{3}\cdot n}$

$k=0$

$x(k\Omega)=\sum_{n=0}^{5}x(nT)\cdot e^{-ik\frac{\pi}{3}\cdot n}=$

$=(1+(-1)+(0)+(1)+(0)+(-1))=0$

$k=1$

$x(\Omega)=\sum_{n=0}^{5}x(nT)\cdot e^{-i\frac{\pi}{3}\cdot n}=1+(-1)\cdot\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)+0+(1)(-1)+0+(-1)\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)=$

$=1-\frac{1}{2}+\cancel{\frac{i\sqrt{3}}{2}}-1-\frac{1}{2}-\cancel{\frac{i\sqrt{3}}{2}}=-1$

$k=2$

$x(2\Omega)=\sum_{n=0}^{5}x(nT)\,e^{-i\frac{2\pi}{3}\cdot n}=\left(1+(-1)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)+0+(1)(1)+0+(-1)\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\right)=$

$=1+\frac{1}{2}+\cancel{\frac{i\sqrt{3}}{2}}+0+1+\frac{1}{2}-\cancel{\frac{i\sqrt{3}}{2}}=3$

$k=3$

$x(3\Omega)=\sum_{n=0}^{5}x(nT)\,e^{-i\frac{3\pi}{3}\cdot n}=(1+(-1)\cdot(-1)+0+(1)(-1)+0+(-1)(-1)=2$

$k=4$

$x(4\Omega)=\sum_{n=0}^{5}x(nT)\cdot e^{-i\frac{4\pi}{3}\cdot n}=1+(-1)\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)+0+(1)\cdot(1)+0+(-1)\cdot\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)=3$

$k=5$

$x(5\Omega)=\sum_{n=0}^{5}x(nT)\,e^{-i\frac{5\pi}{3}\cdot n}=1+(-1)\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)+0+(1)(-1)+0+(-1)\cdot\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)=-1$

$\{x(k\Omega)\}_{k=0}^{5}=\{0,-1,3,2,3,-1\}$

Обратно дискретно преобразуване на Фурие

$n=0$

$$x(nT)=\frac{1}{6}\sum_{k=0}^{5}x(k\Omega)e^{ik\frac{\pi}{3}\cdot n}=\frac{1}{6}(0+(-1)+(3)+2+3+(-1))=1$$

$n=1$

$$x(nT)=\frac{1}{6}\sum_{k=0}^{5}x(k\Omega)\cdot e^{ik\frac{\pi}{3}}=\frac{1}{6}\cdot(0+(-1)\cdot\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)+3\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)+2(-1)+3\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)+$$

$$+(-1)\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)=\frac{1}{6}\cdot\left(0-\frac{1}{2}-\frac{i\sqrt{3}}{2}-\frac{3}{2}+\frac{3\sqrt{3}}{2}i-2-\frac{3\sqrt{3}}{2}i-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)=-1$$

$n=2$

$$x(nT)=\frac{1}{6}\sum_{n=0}^{5}x(k\Omega)e^{ik\frac{2\pi}{3}}=$$

$$=\frac{1}{6}(0+(-1)\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)+3\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)+2\cdot 1+3\cdot\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)+(-1)\cdot\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)=$$

$$=\frac{1}{6}\left(0+\frac{1}{2}-\frac{i\sqrt{3}}{2}-\frac{3}{2}-\frac{3\sqrt{3}}{2}i+2-\frac{3}{2}+\frac{i3\sqrt{3}}{2}+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)=0$$

$n=3$

$$x(nT)=\frac{1}{6}\sum_{k=0}^{5}x(k\Omega)\cdot e^{ik\pi}=\frac{1}{6}(0+(-1)\cdot(-1)+3\cdot 1+2(-1)+3\cdot 1+(-1)\cdot(-1))=1$$

$n=4$

$$x(nT)=\frac{1}{6}\sum_{k=0}^{5}x(k\Omega)e^{ik\frac{4\pi}{3}}=$$

$$=\frac{1}{6}(0+(-1)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)+3\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)+2(1)+3\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)+(-1)\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)=$$

$$=\frac{1}{6}\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}-\frac{3}{2}+\frac{i3\sqrt{3}}{2}+2-\frac{3}{2}-\frac{3\sqrt{3}i}{2}+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)=0$$

$n=5$

$$x(nT)=\frac{1}{6}\sum_{k=0}^{5}x(k\Omega)\cdot e^{ik\frac{5\pi}{3}}=$$

$$=\frac{1}{6}(0+(-1)\cdot\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)+3\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)+2(-1)+3\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)+(-1)\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)=$$

$$=\frac{1}{6}\left(0-\frac{1}{2}+\frac{i\sqrt{3}}{2}-\frac{3}{2}-\frac{i3\sqrt{3}}{2}-2-\frac{3}{2}+\frac{i3\sqrt{3}}{2}-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)=-1$$

$\{x(nT)\}_{n=0}^{5}=\{1,-1,0,1,0,-1\}$ доказва верността на формулата

Дискретно равенство на Парсевал

$$\sum x^2(nT) = \frac{1}{6}\sum |x(k\Omega)|$$

$$(1+(-1)^2+1+(-1)^2) = \frac{1}{6}((-1)^2+(3)^2+(2)^2+(3)^2+(1)^2)$$

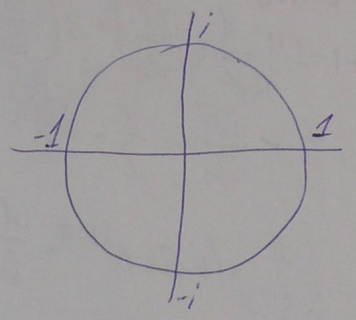
$4 = 4$ доказва вярността на формулата

6.

$$\{x_1(k\Omega)\}_{k=0}^{3} = \left\{3, \frac{1-i}{2}, 0, \frac{-1+i}{2}\right\}$$

$$\{x_2(nT)\}_{n=0}^{3} = \{2, -1, -1, 1\}$$

$$X(k) = \sum_{n=0}^{3} x(n) \cdot e^{-ik\frac{\pi}{2}\cdot n}$$



$k=0$

$$x_2(0) = \sum_{n=0}^{3} x(n) \cdot e^{0} = (2+(-1)+(-1)+1) = 1$$

$k=1$

$$x_2(1) = \sum_{n=0}^{3} x(n) \cdot e^{-i\frac{\pi}{2}\cdot n} =$$

$$= (2+(-1)(-i)+(-1)(-1)+(1)(i)) = 3+2i$$

$k=2$

$$x_2(2) = \sum_{n=0}^{3} x(n) e^{-i\pi \cdot n} = (2+(-1)(-1)+(-1)\cdot 1+1(-1)) = 1$$

$k=3$

$$x_2(3) = \sum_{n=0}^{3} x(n) \cdot e^{-i\frac{3\pi}{2}\cdot n} = (2+(-1)(i)+(-1)(-1)+(1)(-i)) = 3-2i$$

$$X_2: \{1, 3+2i, 1, 3-2i\}$$

$$X_1: \left\{3, \frac{-1-i}{2}, 0, \frac{-1+i}{2}\right\}$$

$$X = X_1 \cdot X_2$$

$$X_1 \left\{3, \frac{-1+i}{2}, 0, \frac{-1-i}{2}\right\}$$

$$X_2 \{1, 3+2i, 1, 3-2i\}$$

$$X = \left\{3, \frac{-5+i}{2}, 0, \frac{-5-i}{2}\right\}$$

$$x(n) = F^{-1}(X) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\cdot e^{ik\frac{2\pi}{N}\cdot n} = \frac{1}{4}\cdot\sum_{n=0}^{3} X(k)\cdot e^{ik\frac{\pi}{2}\cdot n}$$

$n=0$

$$x(n) = \sum_{n=0}^{3} X(k) e^{+ik\frac{\pi}{2}\cdot n}$$

$$x(0) = \left(3 + \left(\frac{-5+i}{2}\right) + \left(\frac{-5-i}{2}\right)\right) = -2$$

$n=1$

$$x(1) = \sum_{n=0}^{3} X(k)\cdot e^{ik\frac{\pi}{2}} = \left(3 + \left(\frac{-5+i}{2}\right)(-i) + \left(\frac{-5-i}{2}\right)(i)\right) = 4$$

$n=2$

$$x(2) = \sum_{k=0}^{3} X(k)\cdot e^{ik\pi} = \left(3 + \left(\frac{-5+i}{2}\right)(-1) + \left(\frac{-5-i}{2}\right)(-1)\right) = 8$$

$n=3$

$$x(3) = \sum_{k=0}^{3} X(k)\cdot e^{ik\frac{3\pi}{2}} = 3 + \left(\frac{-5+i}{2}\right)(-i) + \left(\frac{-5-i}{2}\right)(i) = 4$$

$$\{x(nT)\}_{n=0}^{3} = \{-2, 2, 8, 4\}$$

$$r_{12}(n) = \frac{1}{N} F^{-1}(\overline{X_1}\cdot X_2)$$

$$r_{12}(0) = \frac{1}{4}(-2) = -\frac{1}{2}$$

$$r_{12}(1) = \frac{1}{4}\cdot 2 = \frac{1}{2}$$

$$r_{12}(2) = \frac{1}{4}\cdot 8 = 2$$

$$r_{12}(3) = \frac{1}{4}\cdot 4 = 1$$

$$(x_1 * x_2) = F^{-1}[X_1(k)\cdot X_2(k)]$$

$$X_1(k) = \left\{3, \frac{-1-i}{2}, 0, \frac{-1+i}{2}\right\}$$

$$X_2(k) = \{1, 3+2i, 1, 3-2i\}$$

$$X_1(k)\cdot X_2(k) = \left\{3, \frac{-5i-1}{2}, 0, \frac{5i-1}{2}\right\}$$

$$x(n) = \sum_{k=0}^{3} x(k) \cdot e^{ik\frac{\pi}{2} \cdot n}$$

$n = 0$

$$x(0) = \sum_{n=0}^{3} x(k) \cdot e^{0} = \left(3 - \frac{5i-1}{2} + \frac{5i-1}{2}\right) = 2$$

$n = 1$

$$x(1) = \sum_{n=0}^{3} x(k) \cdot e^{ik\frac{\pi}{2}} = \left(3 + \left(\frac{-5i-1}{2}\right)i + \left(\frac{5i-1}{2}\right)(-i)\right) = 3 + \frac{5-i}{2} + \frac{5+i}{2} = 8$$

$n = 2$

$$x(2) = \sum_{k=0}^{3} x(k) e^{ik\pi} = \left(3 + \left(\frac{-5i-1}{2}\right)(-1) + \left(\frac{5i-1}{2}\right)(-1)\right) = 4$$

$n = 3$

$$x(3) = \sum_{k=0}^{3} x(k) \cdot e^{ik\frac{3\pi}{2}} = \left(3 + \left(\frac{-5i-1}{2}\right)(-i) + \left(\frac{5i-1}{2}\right)(i)\right) = -2$$

$x = \{2, 8, 4, -2\}$

$x^* = \{-2, 4, 8, 2\}$

$r_{12}^* = \frac{1}{4}(-2) = -\frac{1}{2}$

$r_{12}^* = \frac{1}{4} \cdot 4 = 1$

$r_{12}^* = \frac{1}{4} \cdot 8 = 2$

$r_{12}^* = \frac{1}{4} \cdot 2 = \frac{1}{2}$